

The Technology

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Propositional Logic



- Language: propositional variables can be true (1) or false (0)
- Connectives: \Rightarrow implication, \neg negation, \vee disjunction, \wedge conjunction
- Clause: disjunction of variables or their negations (literal)
- Validity: a formula is valid iff it is true for all possible assignments
- Assignment: setting all propositional variables 1 or 0, can also be expressed by showing the true literals
- we write $M \models C$ if the clause C is true by assignment M
- SAT: propositional satisfiability, find an assignment such that for a set of clauses all clauses are valid in the assignment



Unit Propagation



```
 \begin{aligned} & \text{UProp}(N,M) \\ & \text{while (there is a clause } C' \lor L \in N \text{ such that} \\ & M \models \neg C' \text{ and } L \notin M \text{ and } \neg L \notin M) \\ & M := M \cup \{L\}; \\ & \text{return } M; \end{aligned}   \begin{aligned} & \text{UProp}(\{\neg A \lor \neg B \lor E, \quad \neg A \lor B, \quad \neg E, \quad D, \quad A\},\emptyset) \\ & \rightarrow M = \emptyset \\ & \rightarrow M = \{\neg E\} \\ & \rightarrow M = \{\neg E, D\} \\ & \rightarrow M = \{\neg E, D, A\} \\ & \rightarrow M = \{\neg E, D, A, B\} \end{aligned}
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DPLL Procedure



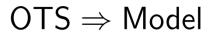
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DPLL(N,M)
if for all C \in N we have M \models C return true;
if there is some C \in N with M \models \neg C return false;
select a variable P occurring in N but not in M;
if (DPLL(N, UProp(N, M \cup \{P\}))) then
  return true;
else
  return DPLL(N, \operatorname{UProp}(N, M \cup \{\neg P\}));
  \neg A \lor \neg B \lor E
                                               \mathrm{DPLL}(N,\emptyset)
  \neg A \lor B
  \neg E
                    \mathrm{DPLL}(N, \mathrm{UProp}(N, \{A\})) \; \mathrm{DPLL}(N, \mathrm{UProp}(N, \{\neg A\}))
  A \vee D
                    \mathrm{DPLL}(N, \{A, B, \neg E\}) \mathrm{DPLL}(N, \{\neg A, D, \neg E\})
```

DPLL is sound and complete and terminating for SAT.

Propositional Logic Formulas



 $XK120 \Rightarrow Model \land Engines$



 $\mathsf{FHC} \Rightarrow \mathsf{Model}$

 $DHC \Rightarrow Model$

 $\mathsf{Model} \Rightarrow \mathsf{OTS} \oplus \mathsf{FHC} \oplus \mathsf{DHC}$

 $3.419CW \Rightarrow Race$

Race \Rightarrow OTS \land Sports

 $3.418C \Rightarrow Engines$

 $3.419C \Rightarrow Engines$

 $3.419CW \Rightarrow Engines$

Reasoning: $XK120 \rightarrow Model$, Engines

FHC $\rightarrow \neg OTS, \neg DHC, \neg 3.419CW$



Challenge: Scalability



- worst case SAT searches 2ⁿ nodes
- before 2009: approx. 1500 nodes
- in 2012: v.control + SPASS-SATT approx. 6000 nodes
- in x years: for a reasonable product approx. 60000 nodes





- Automated Reasoning Lecture:
 http://www.mpi-inf.mpg.de/departments/rg1/teaching/
- contact us on student assistant jobs, bachelor-master-PhD thesis

Thank you for your attention

