Biclique Partition with Application to Display Optimization

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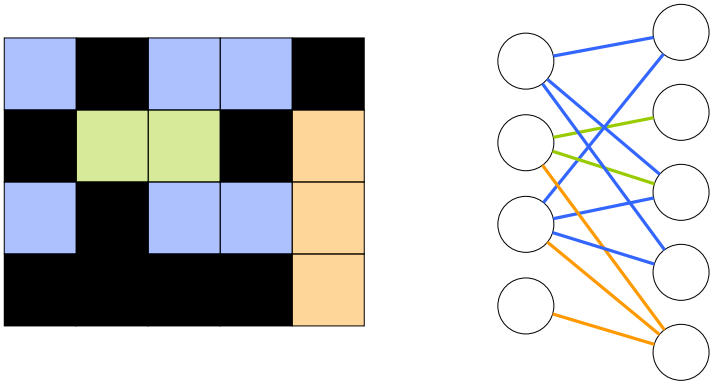

Setting: In the **biclique partition problem**, we are given a bipartite graph as the input and we want to find a partition of the edge set of the graph into a set of bicliques(a.k.a complete bipartite subgraphs). In the optimization version of the problem, we want to find a biclique partition of the smallest possible size, i.e., we want to partition the edges of the graph into minimum number of bicliques. In the parameterized version of the problem, we are given a positive integer k as an additional input and we want to find whether we can partition the edges of the graph into at most k bicliques.

The problem has numerous applications. We will focus on one specific application, namely **display optimization**. In typical display monitors, each pixel of the monitor does not have a dedicated switch. Instead, each

row and each column has a switch. So, if row i and column j is switched on, then pixel (i, j) glows. We can map the display array to a bipartite graph by taking each row as a left vertex and each column as a right vertex. Let us say a black and white image has to be displayed. We will have an edge from vertex i in left side to vertex j in right side if and only if the pixel (i, j) is white in the image. We call this graph, the **image graph**. Because of the row and column addressing of switches, all images cannot be displayed by a single frame. In fact, in one frame, we can only display a part of the picture which corresponds to a biclique of the image graph. Hence, in order to display the picture we need a biclique partition of the image graph. So, minimizing the number of frames required is same as finding a biclique partition of minimum size of the image graph.

Research goals

- In IPEC 2016, we published a paper[3] which gave an algorithm for the parameterized version of biclique partition which runs in time $2^{O(k^2)} \cdot poly(n)$, where n is the input size and k is the parameter as specified above. This drastically improved over the previous best known running time which was $2^{\Omega(2^k)}poly(n)$. The first task of the thesis will be to give an implementation of our algorithm, and evaluate its performance on real images. We may have to combine the algorithm with other heuristics used in display optimization.

- An interesting theoretical open question regarding biclique partition is whether it admits a *polynomial kernel* . If we can find such a kernel, it may also give efficient ways to preprocess an image before running the algorithm on it. So, this will be another goal of the thesis.

Preliminaries The candidate should be familiar with the basic concepts in Algorithms and Computational Complexity and should have basic programming skills (C/C++ preferred).

Our offer We offer a Master's thesis for computer science or math. We will guide you in theoretical and practical questions and offer regular meetings to discuss the state of your thesis.

- Advisor: Davis Issac (Phd student)

- Supervisor: Dr. Andreas Karrenbauer (Senior researcher)

References

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