Improvements to Keyboard Optimization with Integer Programming

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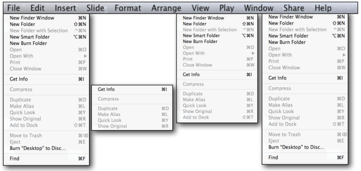
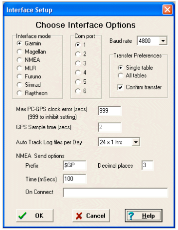
ACM UIST 2014

User Interface Optimization





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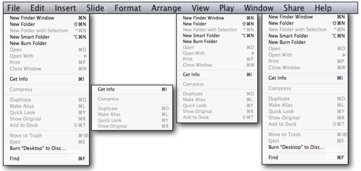
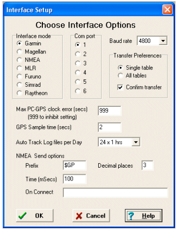
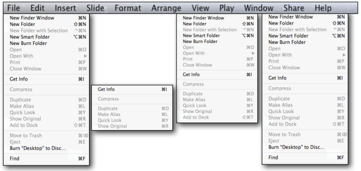


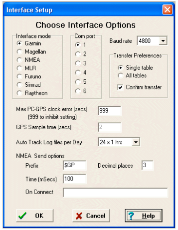


Brute Force

- Explicit enumeration
- Nonpractical for UI
  (immense design spaces $> 10^{26}$)
- No guarantees when stopped prematurely

User Interface Optimization





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Random Search Heuristics

- Random Enumeration
- Practical
- But by chance and no guarantees

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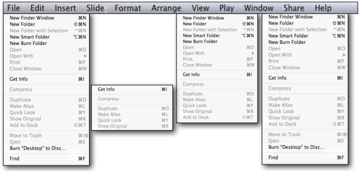

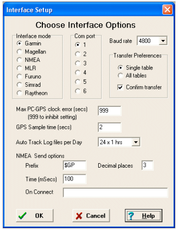

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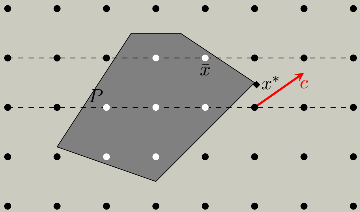
Integer Programming

Given a set of constraints, decide whether an integer point satisfies all of them.

Benefits



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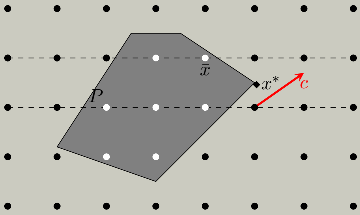
1. Great modeling power
2. Efficient general purpose solvers



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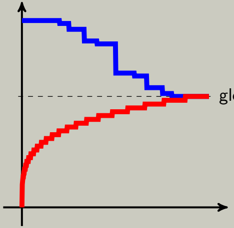

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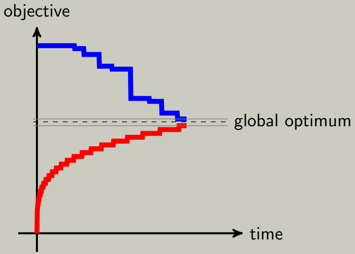
4. Rigorous bounds for the optimum
   even when interrupted prematurely



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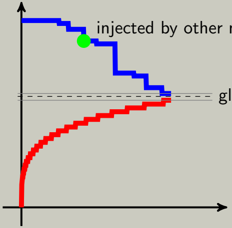

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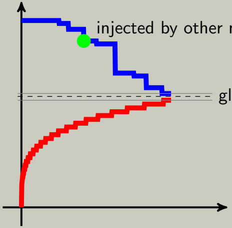
5. Complementary to other methods (e.g., Simulated Annealing)
6. Practical



Overview of the Approach

Steps

1. Definition of constraints and objective
2. Formalization with decision variables
3. Integer Programming Model
4. Refinement by reformulations and linearizations
5. IP-Solver

The Letter Assignment Problem – Constraints and Objective

Constraints: find an assignment (1-to-1 correspondence)

$$\mathcal{L} = \left\{ \begin{matrix} ABCDEFGHI \\ JKLMNOPQR \\ STUVWXYZ \end{matrix} \right\}$$

$$f : \mathcal{L} \rightarrow \mathcal{K}$$

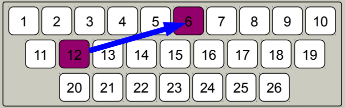
$$= \mathcal{K}$$

The Letter Assignment Problem – Constraints and Objective

Constraints: find an assignment (1-to-1 correspondence)

$$\mathcal{L} = \left\{ \begin{matrix} ABCDEFGHI \\ JKLMNOPQR \\ {\color{red}S}TUVWX{\color{red}Y}Z \end{matrix} \right\} \qquad f : \mathcal{L} \to \mathcal{K}$$

${\color{red}p_{SY}}$



$= \mathcal{K}$

Objective: maximize typing speed, i.e., minimize the expected inter-key intervals

$$\sum_{k,\ell \in \mathcal{L}} {\color{red}p_{k,\ell}} \cdot {\color{blue}t_{f(k),f(\ell)}}$$

bigram probabilities $p_{k,\ell}$

inter-key intervals $t_{i,j}$

The Letter Assignment Problem – Decision Variables

Model assignment by binary decision variables

$$x_{\ell j} = \begin{cases} 1 & \text{if letter } \ell \text{ is assigned to keyslot } j\text{, i.e., } f(\ell) = j \\ 0 & \text{otherwise} \end{cases}$$

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Constraints

- Each letter ℓ is assigned to exactly one keyslot $\rightsquigarrow$ $\forall \ell \in \mathcal{L} : \sum_{j \in \mathcal{K}} x_{\ell,j} = 1$
- Each keyslot j contains exactly one letter $\rightsquigarrow$ $\forall j \in \mathcal{K} : \sum_{\ell \in \mathcal{L}} x_{\ell,j} = 1$

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Objective

expected inter-key interval $\sum_{k,\ell \in \mathcal{L}} p_{k,\ell} \cdot t_{f(k),f(\ell)}$ $\rightsquigarrow$ $\sum_{i,j \in \mathcal{K}} \sum_{k,\ell \in \mathcal{L}} p_{k,\ell} \cdot t_{i,j} \cdot x_{k,i} \cdot x_{\ell,j}$

The Letter Assignment Problem – IP Model

Quadratic Assignment Problem

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n} p_{k\ell} t_{ij} x_{ki} x_{\ell j}$$

$$\text{s.t.} \sum_{\ell=1}^{n} x_{\ell j} = 1 \quad \forall j \in \{1,\dots,n\}$$

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$$x_{\ell j} \in \{0,1\} \quad \forall \ell, j \in \{1,\dots,n\}$$

The Letter Assignment Problem – IP Model $\rightarrow$ IP-Solver

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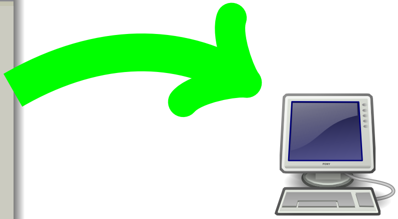

The Letter Assignment Problem – IP Model $\leftrightarrow$ IP-Solver

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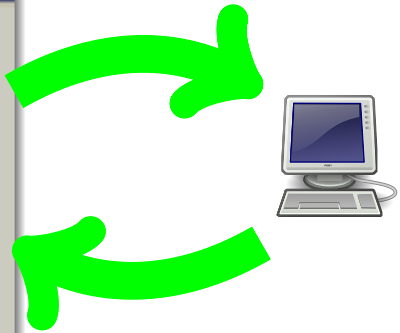

The Letter Assignment Problem – IP Models $\leftrightarrow$ IP-Solver

Reformulations

$$
\begin{aligned}
\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n} & p_{k\ell} t_{ij} x_{ki} x_{\ell j} \\
\text{s.t.} \sum_{\ell=1}^{n} x_{\ell j} &= 1 \quad \forall j \in \{1,\dots,n\} \\
\sum_{j=1}^{n} x_{\ell j} &= 1 \quad \forall \ell \in \{1,\dots,n\} \\
x_{\ell j} &\in \{0,1\} \quad \forall \ell, j \in \{1,\dots,n\}
\end{aligned}
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\min \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{\ell=1}^{n} & p_{k\ell} t_{ij} y_{ijk\ell} \\
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\sum_{k=1}^{n} y_{ijk\ell} &= x_{\ell j} \quad \forall i, j, \ell \in \{1,\dots,n\} \\
\sum_{i=1}^{n} y_{ijk\ell} &= x_{\ell j} \quad \forall j, k, \ell \in \{1,\dots,n\} \\
0 \leq y_{ijk\ell} &= y_{ji\ell k} \quad \forall i, j, k, \ell \in \{1,\dots,n\}
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$$

$$
\begin{aligned}
\min \sum_{\ell=1}^{n}\sum_{j=1}^{n} & w_{ij} \\
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x_{\ell j} &\in \{0,1\} \quad \forall \ell, j \in \{1,\dots,n\} \\
w_{\ell j} &= x_{\ell j} \sum_{k=1}^{n}\sum_{i=1}^{n} p_{k\ell} t_{ij} x_{ki} \quad \forall \ell, j \in \{1,\dots,n\} \\
w_{\ell j} &\geq 0 \quad \forall \ell, j \in \{1,\dots,n\} \\
w_{\ell j} &\geq \sum_{k=1}^{n}\sum_{i=1}^{n} p_{k\ell} t_{ij} x_{ki} - a_{ij} \cdot (1 - x_{\ell j}) \quad \forall \ell, j \in \{1,\dots,n\}
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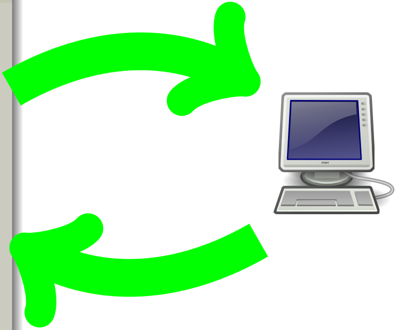
$$



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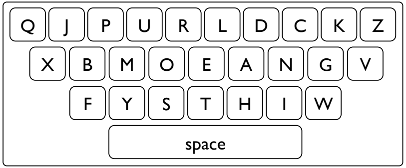
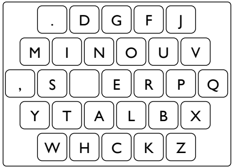
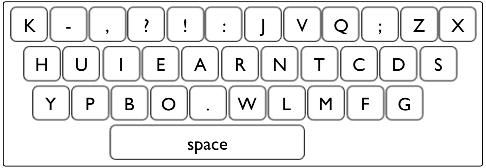
$$

- Models can be solved in parallel
- Complementary to other approaches such as random search heuristics

Results







Virtual 3-row trapezoid	Virtual hexagonal	Physical 3-row standard
26 keys	29 keys	34 keys
Fitts' model	Fitts' model	Hiraga et al. 10-finger model

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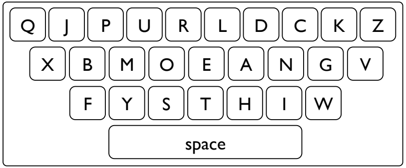

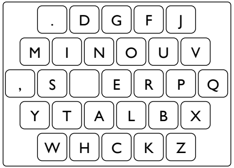

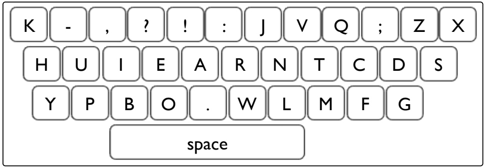

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Keyboards (wpm)

qwerty (33.65)	metropolis (42.04)	qwerty (73.21)
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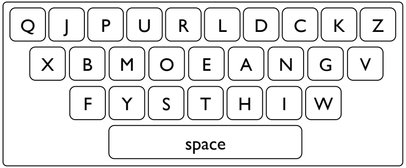

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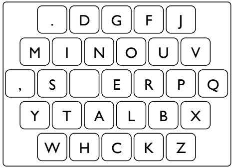

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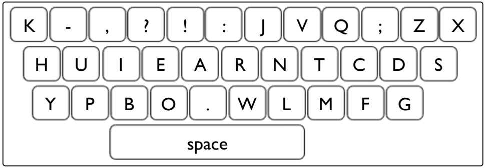

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This (43.10)	This (42.65)	This (78.36)
any (< 44.82)	any (< 45.55)	any (< 79.95)

Summary

Take-home messages

- Integer Programming proved useful for UI Optimization
- IP provides rigorous mathematical guarantees for the optimum
- IP is complementary to other methods (e.g., random search heuristics)

`http://resources.mpi-inf.mpg.de/keyboardoptimization/`

(by Oct 15)

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