# Practical Capture and Reproduction of Phosphorescent Appearance Supplemental Material

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#### 1. Introduction

In this document, we provide further details on our submission "Capture and Reproduction of Phosphorescent Appearance". In short, we provide the spectrum of the light source used in our measurements (Sec. 3), a comparison between Euler integration and a Runge-Kutta solver when used to solve our ODE (Sec. 4), plots demonstrating the decay curves' independence on the previous level of excitation (Sec. 5), a comparison of emission spectra after excitation by different light sources (Sec. 6), additional comparisons between photographed samples and renders (Sec. 7), measured decay curves and corresponding closed-form fits (Sec. 8), measured saturation curves and corresponding ODE simulations with the fitted parameters (Sec. 9, Sec. 10) and typical search ranges for automatically finding the parameters to best fit the user's constraints when manipulating phosphorescence (Sec. 11).

## 2. Verification of the Spectral Acquisition

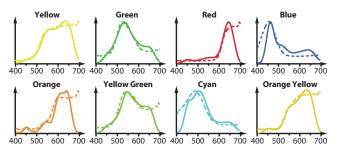
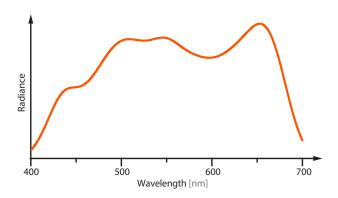


Figure 1: Measured (solid lines) and ground truth (dashed) spectral reflectance for a selection of color checker patches.

To verify that we can measure spectral distributions with our linear variable filter-based setup, we acquired the reflectance of MacBeth Color Checker patches. In Fig. 1, we compare measured spectra to the actual distributions. The spectra are generally reproduced well, problems only arise at the ends of the spectrum were the sensitivity of the used digital camera vanishes. This limitation is shared with other approaches also using consumer cameras.

## 3. Spectrum of the Light Source



**Figure 2:** The spectral intensity distribution of the halogen lamp used by us.

Fig. 2 shows the spectral intensity distribution of the light source which we used (together with the linear variable bandpass filter) to excite our samples in the acquisition of the saturation curves (Sec. 9). Note that, due to the limited spectral sensitivity of the camera which we used (about 400 to 700 nm) the values close to 400 and 700 nm are less reliable.

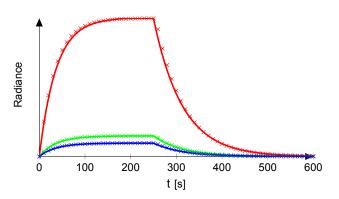
## 4. Comparison of ODE solvers

Fig. 3 shows the change of phosphorescent emission in a typical situation where the light is first switched on (t < 260) and then turned off (t >= 260). As can be seen from the plot, simple Euler integration delivers nearly identical results compared to the more sophisticated Runge-Kutta (2,3) method employed by Matlab's ode23 solver.

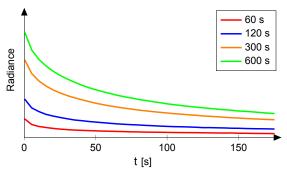
# 5. Saturation-independence of the Decay

In our model, the shape of decay curves is independent of the initial concentration  $r_i(0)$  ("the level of excitation"). To verify the correctness of this property, we measured decay curves of the same material sample after excitation of different duration (Fig. 4). As can be seen

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**Figure 3:** Our ODE for  $n_c = 1$ , solved by both, a Runge-Kutta solver (lines) and the (forward) Euler method (crosses). The different lines correspond to the red, green and blue channel of the final result.



(a) Decay from different excitation levels. For each line, the duration of the preceding excitation period is given.

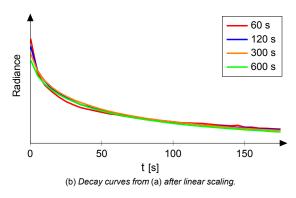
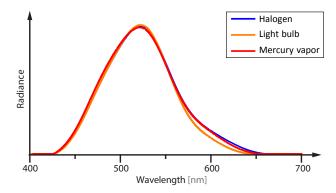


Figure 4: The decay curves from different excitation levels are the same except for linear scaling.

in Fig. 4 b, linear scaling of the individual curves makes them match. Remaining differences can be attributed to slightly different delays between switching the light off and starting the measurement.

#### 6. Stokes Shift

We do not enforce the fulfillment of Stokes shift in our model. One argument for this is its negligible effect in everyday lighting



**Figure 5:** Re-radiation spectrum for phosphorescent stickers after excitation by different types of light sources. The curves have been linearly scaled by their respective average value.

situations: Whether excited by a lamp with smooth spectrum and strongest output in the red range (e.g., Halogen light bulb) or by a gas discharge lamp with very thin blue and green spectral lines (e.g., Mercury vapor), the resulting re-radiation is nearly indistinguishable regarding its spectral distribution as shown in Fig. 5, and even more so when observed by a human with only three types of cone cells.

#### 7. Additional Comparisons to Photographs

Fig. 6 shows how various samples which we used during our measurements decay when the light is turned off (left column) compared to double- (middle column) and single-exponential (right column) simulations using our fitted parameters.

# 8. Fitted Decay Curves

Fig. 7 contains single-exponential and double-exponential fits for the decay behavior we measured (points).

#### 9. Saturation Curves

Fig. 8 contains the data we acquired in our saturation experiments (points) as well as the result of simulating the single- and double-exponential models with the parameters we found. Each line and set of points of one color corresponds to the process of excitation by light of only that wavelength band.

# 10. Parameters for the Double-exponential Fit

The parameters to our double exponential fit are given in Tbl. 1.

## 11. Parameters in the Exhaustive Search

As outlined in the main paper, we compute sample parameters by expressions of the form  $a \cdot 2^b$ , where a is a parameter-dependent scale and the exponent b is sampled equidistantly from a parameter-dependent range  $[b_{\min}, b_{\max}]$ . Tbl. 2 contains the values of a,  $b_{\min}$  and  $b_{\max}$  which we used for our manipulation example.

**Table 1:** Re-radiation and excitation rates in units  $s^{-1}$ . The first column shows the daylight color of the samples, the second the decay gradients (scale of 60 s). The response for some wavelengths was too low to be measured with our setup, thus excitation rates are only given for the first three wavelength bands. For the last three materials, only their decay behavior was measured.

Material k <sub>r,1</sub> k <sub>r,2</sub>	$k_{ m r,1}$	$k_{\rm r,2}$	$\lambda_1$	$k_{ m e,1} \ \lambda_2$	$\lambda_3$	$\lambda_1$	$k_{\mathrm{e},2} \ \lambda_2$	$\lambda_3$	$\lambda_1$	$\lambda_2$	$\lambda_3$	λ4	<b>Λ</b> 1 λ5 ;	٧ رو	λ ,	8 λ <sub>1</sub>	$\lambda_2$	$\lambda_1$ $\lambda_2$ $\lambda_3$ $\lambda_4$ $\lambda_5$ $\lambda_6$ $\lambda_7$ $\lambda_8$ $\lambda_1$ $\lambda_2$ $\lambda_3$ $\lambda_4$ $\lambda_5$ $\lambda_6$ $\lambda_7$	$\lambda_4$	$\mathbf{A}_2$ $\lambda_5$	$\lambda_6$	$\lambda_7$	$\lambda_8$
Blue paint Green paint	0.008	0.116	0.0030	0030 0.0075	0.0008 (0.0007 (0.0007)	0.0420	0.1100	0.0000 (	0.43	5.59 4	4.67 C	0 0.71 C	0.0 0	0.01 0.0	0.0 0.0 0.0 0.0	0.00	9 1.22 1.	2 1.02	0.15	0.0	0.0	0.0	0.0
Orange paint Pink paint		0.091	0.0036	0.0016	0.0000	0.0480	0.0190	0.0000	0.05	2.07 (	0.53	_	0.51 8	8.65 5.4	5.48 2.80		1 0.33	3 0.08	0.0	0.08	1.36	0.86	0.44
Purple paint		0.007	0.0360	0.0440	0.0030		0.0030	0.0003	0.12					0.05 0.0						0.0	0.24		0.0
Red paint		0.093	0.0026	0.0009	0.0000	0.0380	0.0130	0.0000	0.14	3.42 (	0.50	0.0	0.03 6	6.21 7.0	7.03 0.7	0.72 0.02	2 0.55	5 0.08	0.0	0.0	1.00	1.13	0.12
White paint		0.000	0.0390	0.1000	0.0130	0.0025	090000	0.0009	0.08	0.80	0.78	0.19	0.0	0.0	0.0 0.03	3 0.36	6 3.55	5 3.47	0.84	0.0	0.01	0.0	0.14
Yellow paint	0.025	0.414	0.0160	$0.0040 \ 0.0010$	0.0010	0.2500	0.0600	0.0110	0.0	0.0	0.10 1.01		0.18 0	0.15 0.	0.02 0.	0.0 0.0	0.0	0.04	0.44	0.08	0.06	0.01	0.0
Bottle	0.020	0.324							0.0	0.06	0.55 0.79	0 62.	0.34 0	0.08 0.0	0.01 0.	0.0 0.0	0.0	0.02 0.21	0.30	0.13	0.03	0.0	0.0
Gloves	0.178		,	,	,	1	,	,	0.03	0.31 1.87 1.86	1.87	.86	0.71 0	0.24 0.0	0.04 0.0	0 0.14	4 1.30	0 7.78	3 7.76	2.95	0.98	0.18	0.01
Stickers	0.132	0.009	ı	ı		ı			0.0	1.70	11.5	11.5 4.19 1.43	.19	43 0.	0.31 0.0	0.04 0.0	9.7 (	0.0 7.62 51.6 51.5 18.8	51.5	18.8	6.43	1.41 0.18	0.18

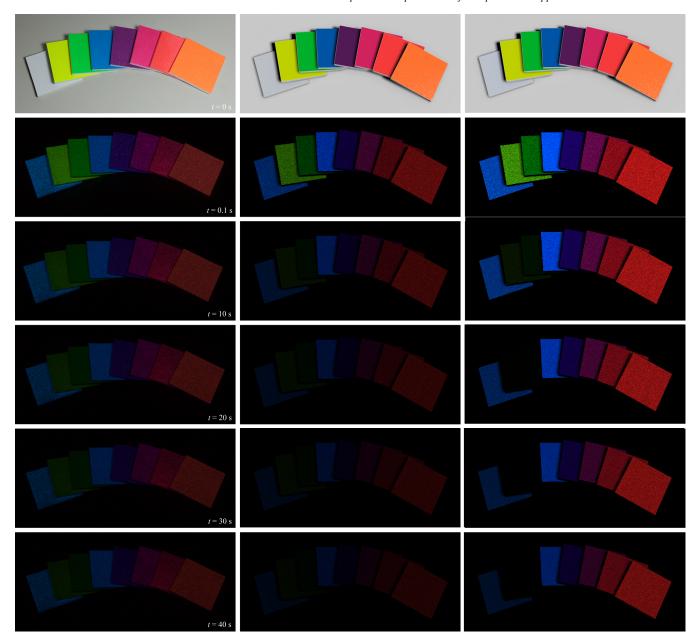


Figure 6: Some of the samples that were used during the data acquisition under the light of the lamp used (first row) and after the light has been turned off (second to sixth row). The left column consists of actual photographs, the second and third column show double- and single-exponential simulations, respectively.

**Table 2:** Parameters used in the computation of sample parameter sets. For  $k_e$  and  $\Lambda$ , which are vectors, the given values are with respect to one component.

Parameter	а	$b_{ m min}$	$b_{\max}$
$k_{\rm r}$	0.001	0	5.0
$k_{\rm e}$	0.001	0	12.0
Λ	0.1	0	9.0

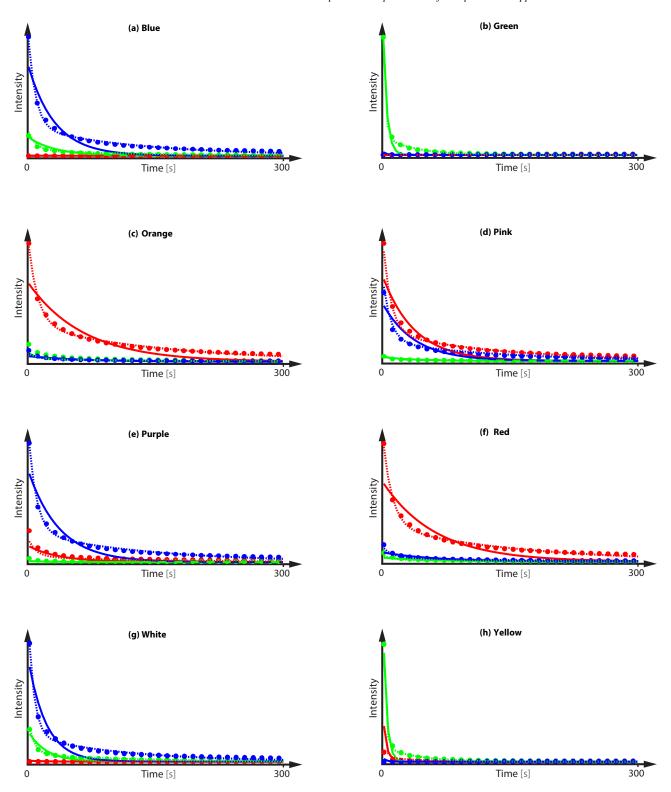


Figure 7: Measured decay of the phosphorescence intensity and fitted single- (solid lines) and double-exponential (dashed) decay functions.

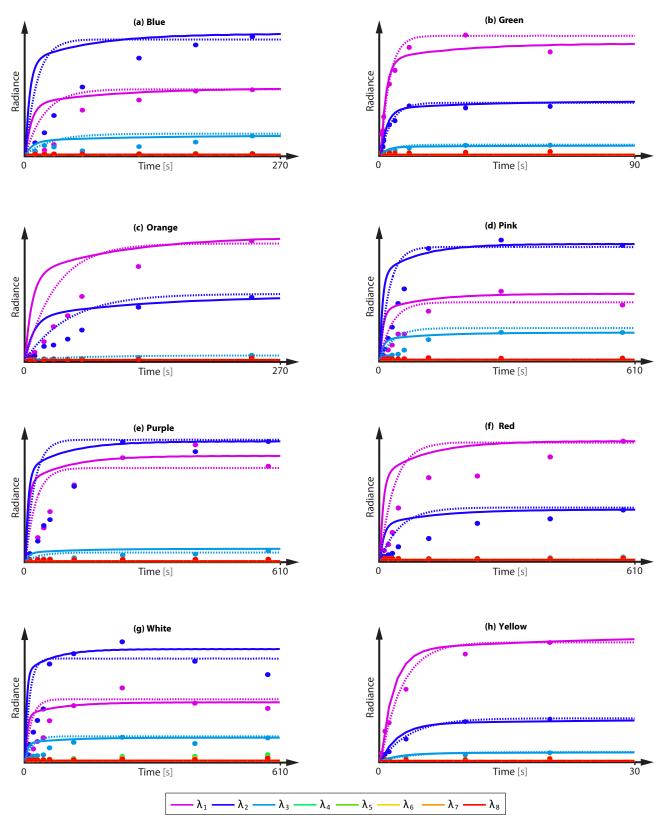


Figure 8: Measured saturation of the phosphorescence intensity and curves simulated using fitted parameters for the single- (solid) and double-exponential (dashed) models.