Satisfiability — Exercises

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Mohan Paturi, UC San Diego

Solve the following problems.

Problem 1: Cnf-Sat

Design an (randomized or deterministic) algorithm for CNF-SAT that runs in time $2^{n(1-\frac{c}{\log(m/n)})}$ for some constant $c > 0$.

Problem 2: Two quantifier blocks

Show that the satisfiability of formulas of the form $\forall\vec{x}\exists\vec{y}\phi(\vec{x},\vec{y})$ where ϕ is a 3-CNF or of the form $\exists\vec{x}\forall\vec{y}\phi(\vec{x},\vec{y})$ where ϕ is a 3-DNF cannot be solved in time $2^{n(1-\varepsilon)}$ for any $\varepsilon > 0$ assuming **SETH**. Here n, the number of variables, is the sum of the number of variables in $\vec{x}$ and $\vec{y}$.

Hint: Reduce k-CNF to formulas of the form $\exists\vec{x}\forall\vec{y}\phi(\vec{x},\vec{y})$ where ϕ is a 3-DNF. Use Sparsification Lemma and the following minimally unsatisfiable formula to reduce k-CNF. The minimally unsatisfiable formula is a conjunction of the following disunctions.

- $\bigvee_{i=1}^{m} p_i$
- $p_i \rightarrow q_j$ for all $1 \leq i, j \leq m$.
- $\bigvee_{i=1}^{m} \bar{q}_i$

Problem 3: k-colorability

Show that the language of graphs that are k-colorable is in the class **SNP**.

Problem 4: Satisfiability of depth-3 circuit

Let C be a depth-3 layered circuit of AND-OR-AND where the bottom-level AND gates have their input from among the literals $x_1, \bar{x}_1, \ldots, x_n, \bar{x}_n$. Assume that C has a total of cn gates. Deisgn an (randomized or deterministic) algorithm for checking the satisfiability of C.