Distributed Computing

Pierre Fraigniaud

INSTITUT
DE RECHERCHE
EN INFORMATIQUE
FONDAMENTALE

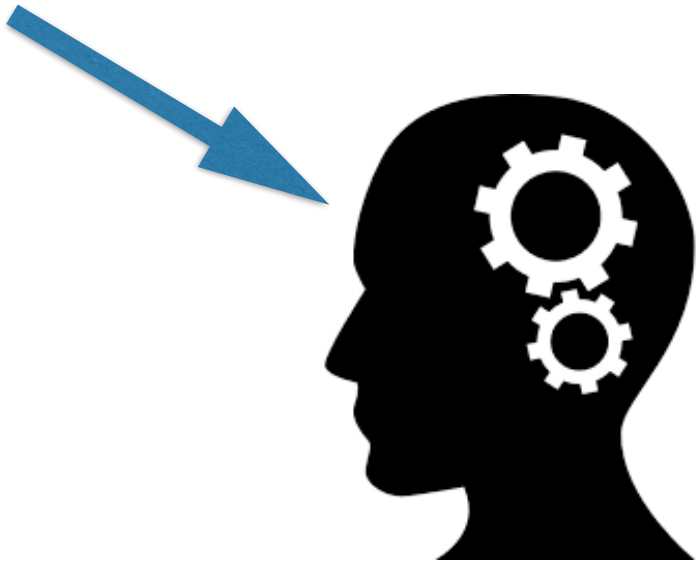
CNRS and Université de Paris

20th Max Planck Advanced Course on the Foundations of Computer Science (ADFOCS)
19 - 23 August 2019, Saarbrücken, Germany

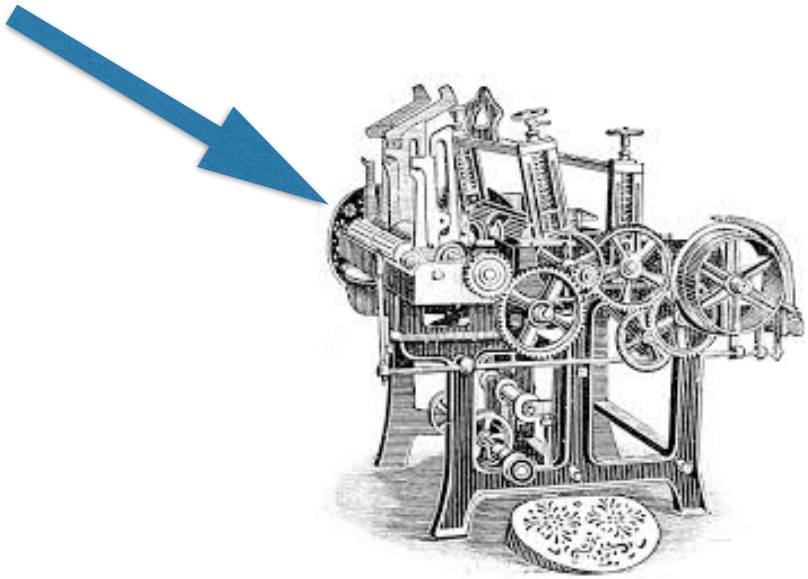
Algorithms vs. programs

Mechanical procedures for solving a given problem

algorithm



program

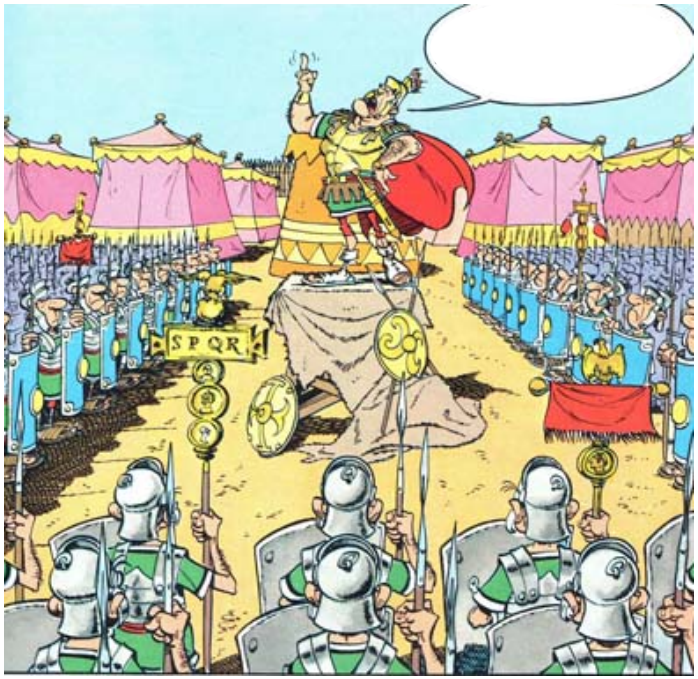


Distributed Algorithm

A collection of autonomous computing entities
collaborating for solving a task
in absence of any coordinator

Parallel vs. Distributed

Parallel computing



Performances

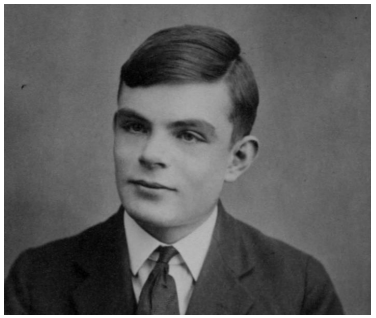
> petaFLOPS (10^{15} op./s)

Distributed computing

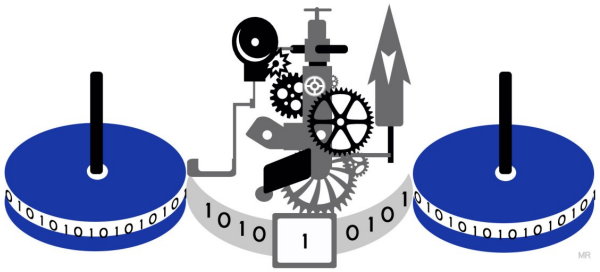
Coping with uncertainty

temporal and spatial

Sequential vs. Distributed

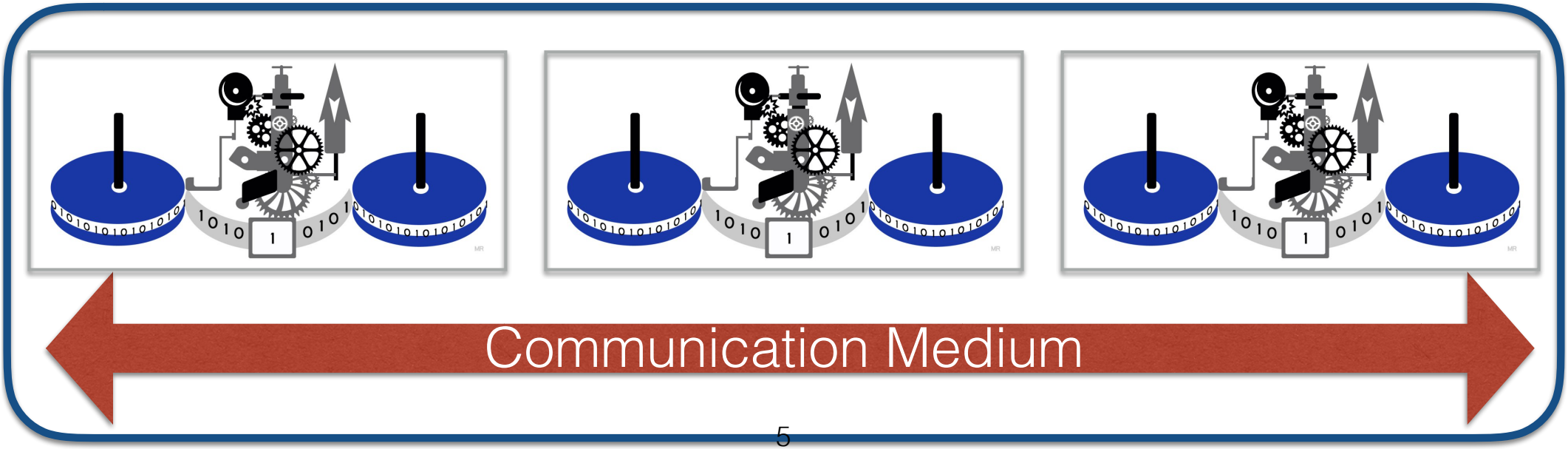


Alan Turing

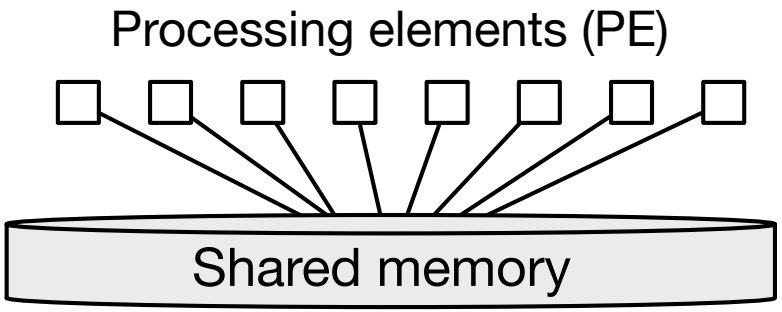


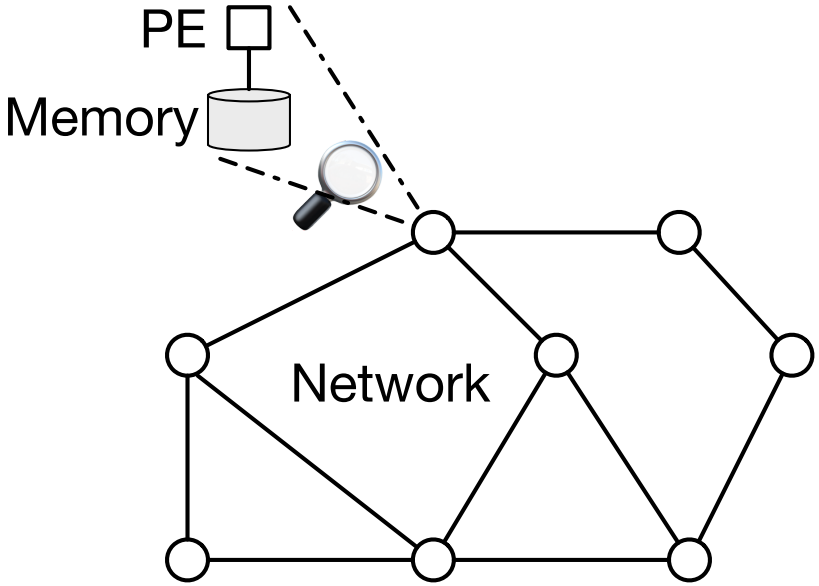
Alonzo Church

Typical model for distributed computing



Communication Medium





Limitations Faced by Distributed Computing: Undecidability  + Uncertainty 

Sources of uncertainties:

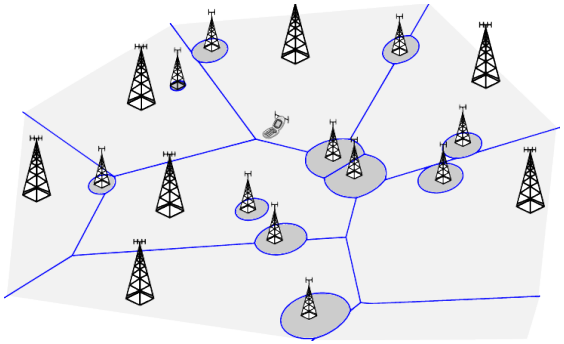
- Spatial: communication network
- Temporal: clock drifts (asynchrony, load, etc.)
- Failures (transient, crash, malicious, etc.)
- Selfish behavior (game theory)
- …

Several Turing machines are weaker than one!

Symmetry Breaking

- Leader election
- Consensus
- Coloring
- Graph problems
- Etc.

Applications :



Frequency assignments

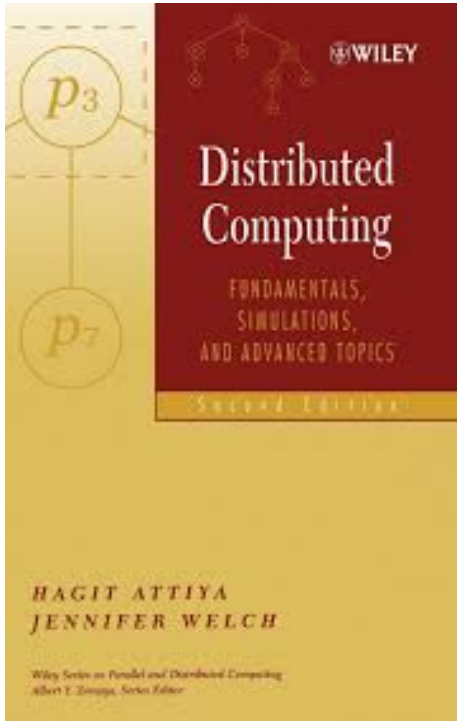
Distributed data-bases consistency

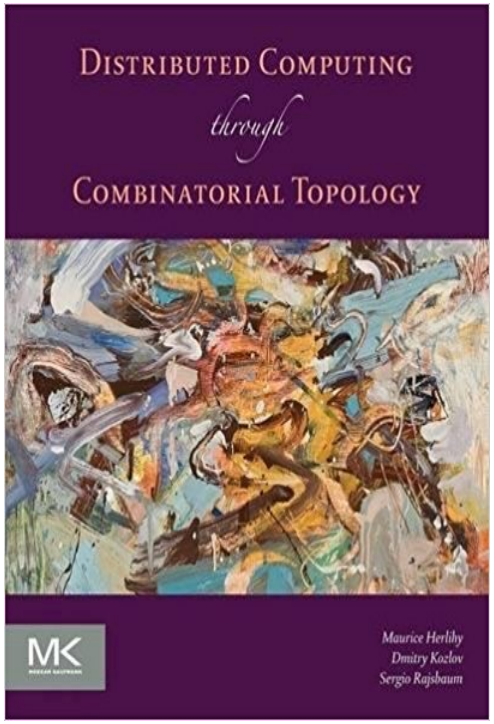
ADFOCS Lectures

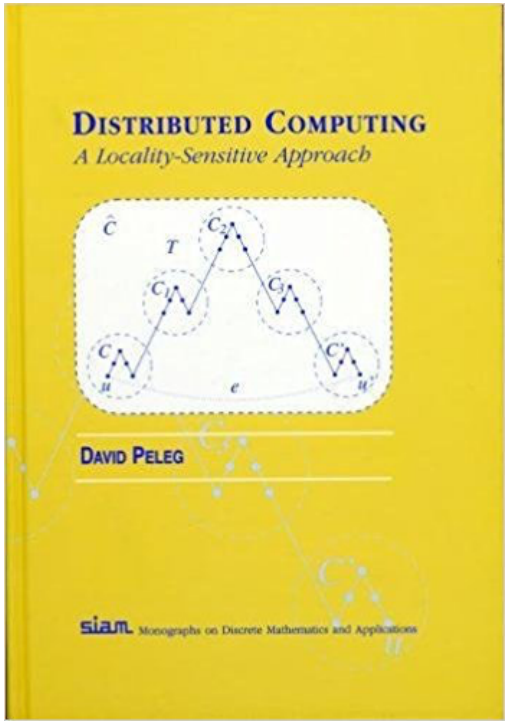
- **Asynchronous Crash-Prone Distributed Computing**
- **Locality in Distributed Network Computing**
- **Congestion-Prone Distributed Network Computing[1]**
- **Other Aspects of Distributed Computing**

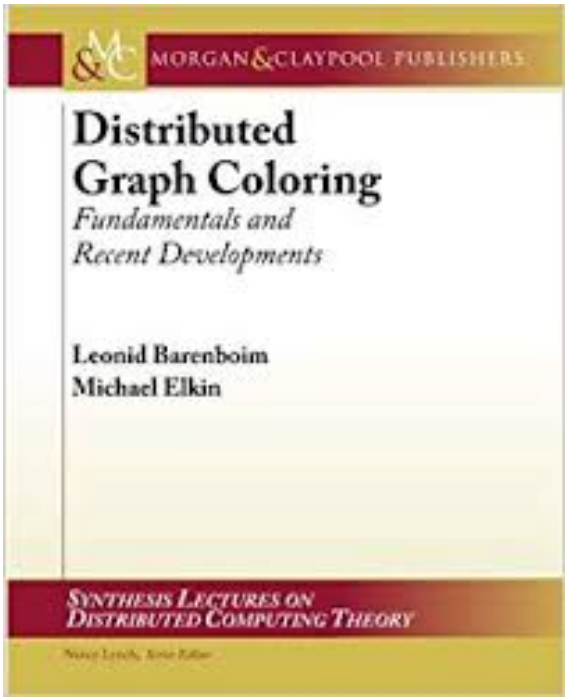
[1] See also lecture by Cristoph Lenzen on Wednesday

Reference books









ADFOCS Lectures

- ☑ **Asynchronous Crash-Prone Distributed Computing**
- ☐ **Locality in Distributed Network Computing**
- ☐ **Congestion-Prone Distributed Network Computing**
- ☐ **Other Aspects of Distributed Computing**

Temporal Uncertainty

Dealing with **asynchronism**:

- clock drifts
- cache misses
- poor load balancing
- etc.

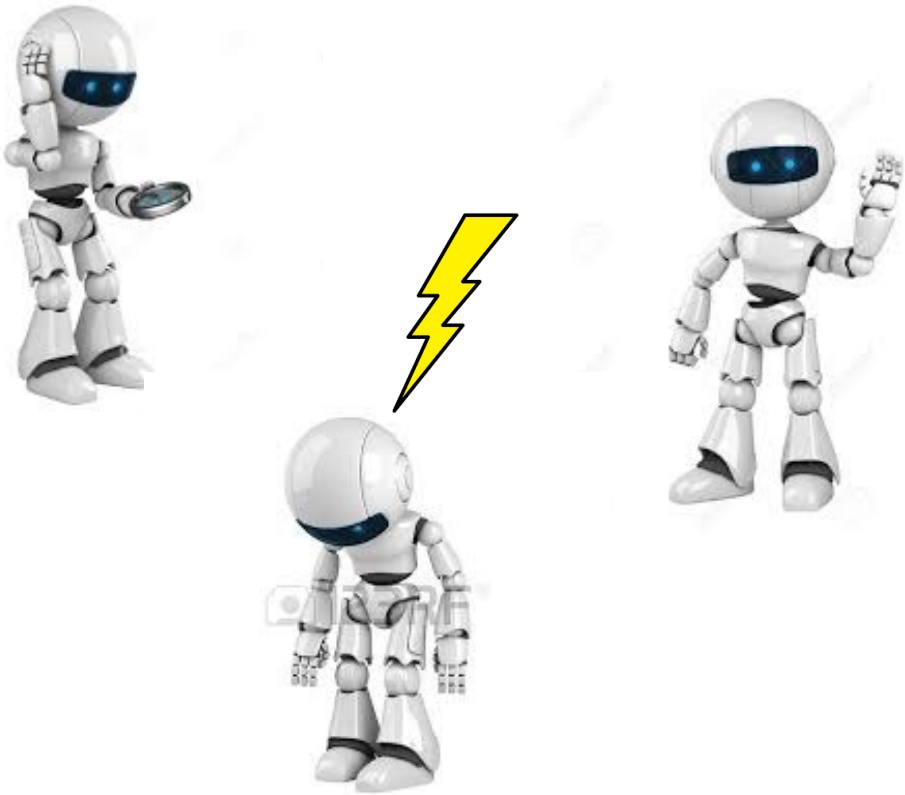
and **failures**:

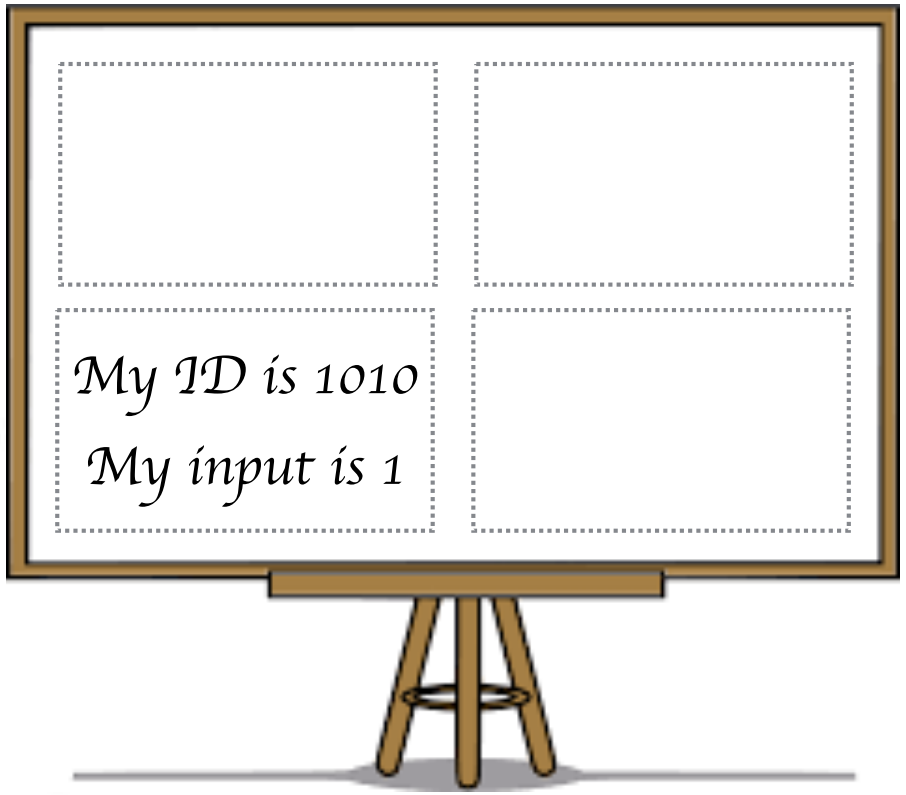
- crash failures
- transient failures
- byzantine (i.e., adversarial) failures
- etc.

Computing Model

Shared memory

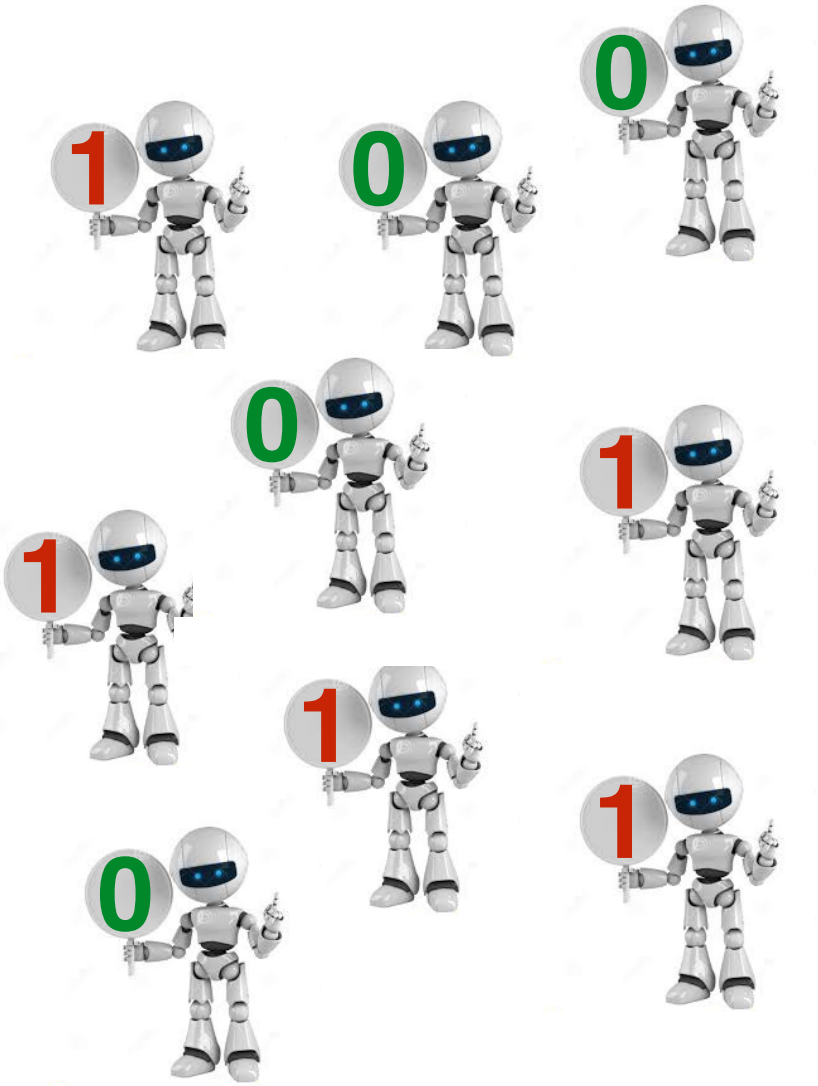
Processing elements, a.k.a. processes





- **write(value)**
- **read(register index)**

Consensus



Distributed replica,
mutual exclusion, etc

- **Termination:** every correct process decides a value 0 or 1.
- **Agreement:** all the decided values are identical.
- **Validity:** every decided value must have been proposed.

Impossibility of Consensus

M. Fischer, N. Lynch, M. Paterson (1985)

Theorem Binary consensus cannot be solved in a shared-memory asynchronous system, even with at most one crash failure.

Dijkstra Prize 2001

Proof
(in the case of any #failures)

- Also known as the **wait-free** model
- **Extension-based proof:** sequence of system configurations for which no processes can decide

$$C^{(0)}, C^{(1)}, C^{(2)}, C^{(3)}, \ldots$$

- Time = **Scheduler**
- **Bivalent** vs. **monovalent** configurations
- Monovalent configuration: **0**-valent or **1**-valent

Claim 1 There exists an initial bivalent configuration

Proof. Assume all init configurations are monovalent.

C_0 = 00…0 is 0-valent

C_n = 11…1 is 1-valent

Let k be smallest index such that C_k is 1-valent

11…100…00
11…110…00

Scheduler crashes process p_k

➡ other processes cannot distinguish C_{k-1} from C_k

Contradiction! ❑

$C \sim_p C'$ if C and C' looks the same from process p

Claim 2 Let C and C' be two monovalent configurations. If $C \sim_p C'$ then C and C' have the same valency.

Proof. The scheduler crashes all processes but p. ❑

A process p is critical for a bivalent configuration C if p taking a step in C results in a monovalent configuration.

Claim 3 For every bivalent configuration C, there exists a process p that is not critical for C.

Proof. Assume every process is critical.

p ➔ 0-valent and q ➔ 1-valent

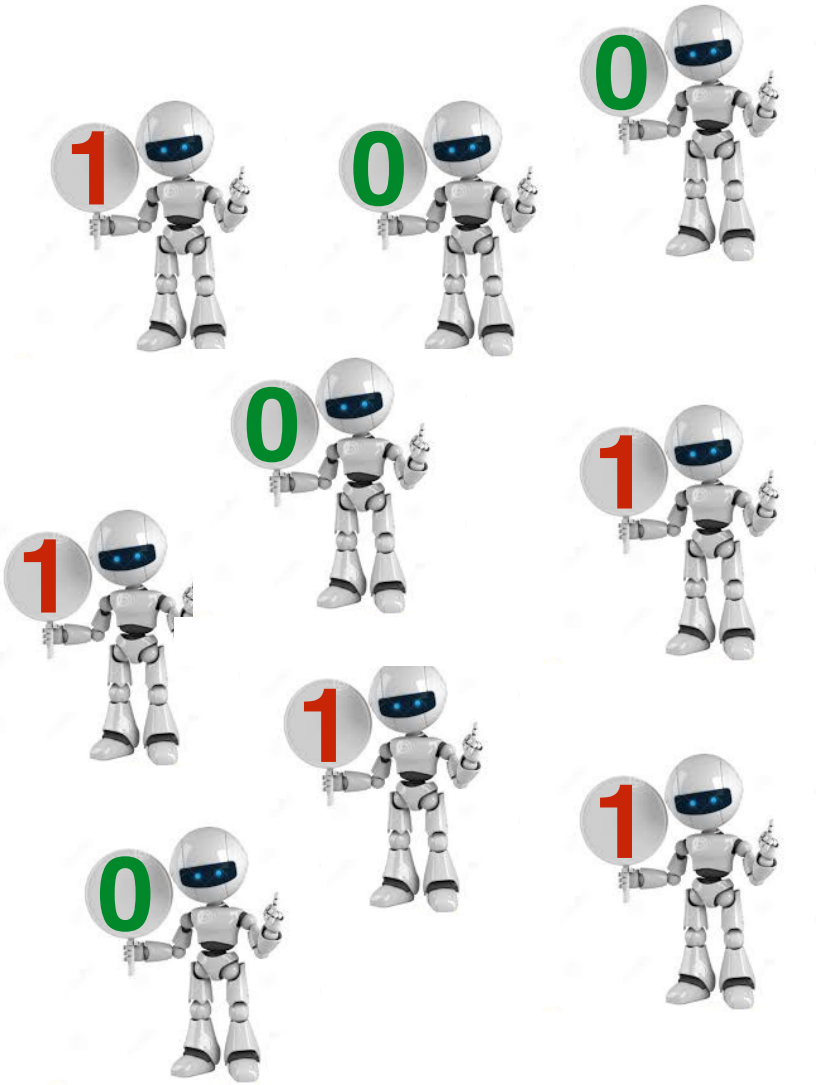
- Case 1: p and q both read, or they read or write in different registers

  ⇒ Cpq = Cqp, contradiction.

- Case 2: p reads or writes in R, and q writes in R

  ⇒ Cq $\sim_q$ Cpq, contradiction with Claim 2. ❑

Weak Consensus



- **Termination:** every correct process decides a value 0 or 1, or ⊥ (i.e., aborts).
- **Agreement:** all the decided values ≠ ⊥ are identical.
- **Validity:** If no processes crash, then at least one process must decide a proposed value.

Property Weak consensus is solvable wait-free in asynchronous shared-memory systems.

The algorithm uses *snapshot* instructions

snapshot = atomic read of the entire memory (i.e., all the registers)

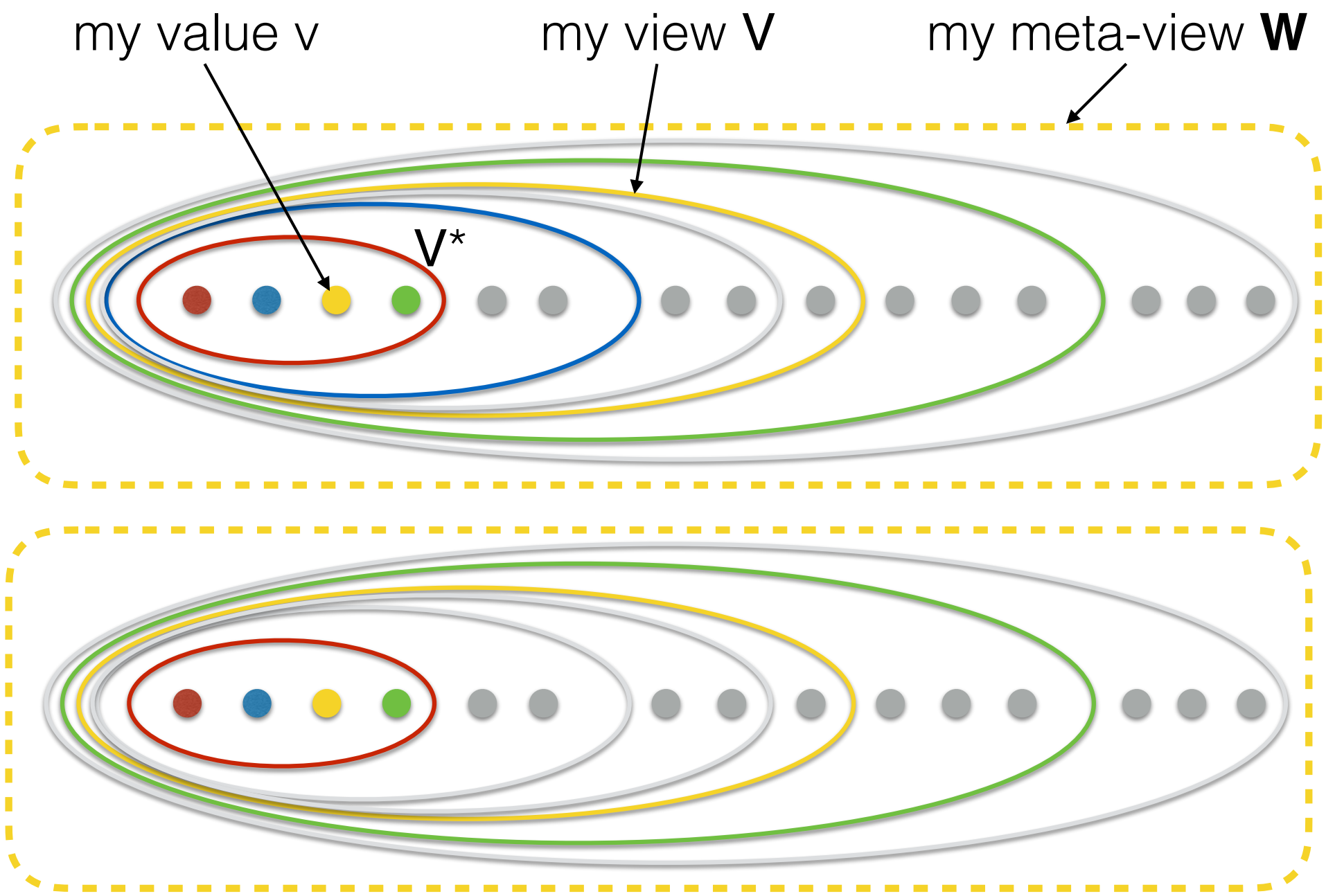
Lemma Atomic snapshot can be implemented wait-free.

Remark *Immediate* snapshot — write-snapshot as a single atomic operation — can also be implemented wait-free.

Algorithm

Algorithm of process p **with input value** v
begin
write (p,v)
snapshot
let $\mathbf{V} = ((p_1,v_{p1}),\ldots,(p_k,v_{pk}))$ */*the view of p*/*
write (p,V)
snapshot
let $\mathbf{W} = ((p_1,\mathbf{V}_{p1}),\ldots,(p_m,\mathbf{V}_{pm}))$ */*the meta-view of p*/*
let $\mathbf{V}^* = \bigcap_{i=1,\ldots,m} \mathbf{V}_{pi}$ */*smallest view in the meta-view of p*/*
if for every $i \in [1,n]$ such that $v_i \in \mathbf{V}^*$, $\mathbf{V}_i \in \mathbf{W}$ holds
then decide smallest value in $\mathbf{V}^*$
else decide $\perp$
end

Intuition



Termination trivially holds

Claim 1 Agreement holds

Proof Assume p decides $v \neq \perp$, and p' decides $v' \neq \perp$ with $v < v'$.

Let $q \neq q'$ such that $V^*_p = V_q$ and $V^*_{p'} = V_{q'}$.

- On the one hand: $v \notin V_{q'}$ since p' decides $v' > v$.

Therefore $V_{q'} \subset V_q$, and thus $v_{q'} \in V_q = V^*_p$

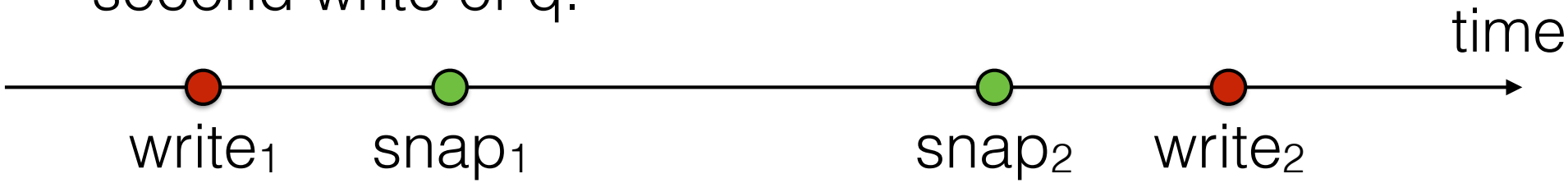
- On the other hand: $V_{q'} \notin \mathbf{W}_p$ as otherwise $V^*_p = V_{q'}$

Contradiction: p does not satisfy the if-condition. ❑

Claim 2 Validity holds

Proof

- If p decides ⊥ then there exists q≠p such that q performed its first write before the first snapshot of p, and p performed its second snapshot before the second write of q.

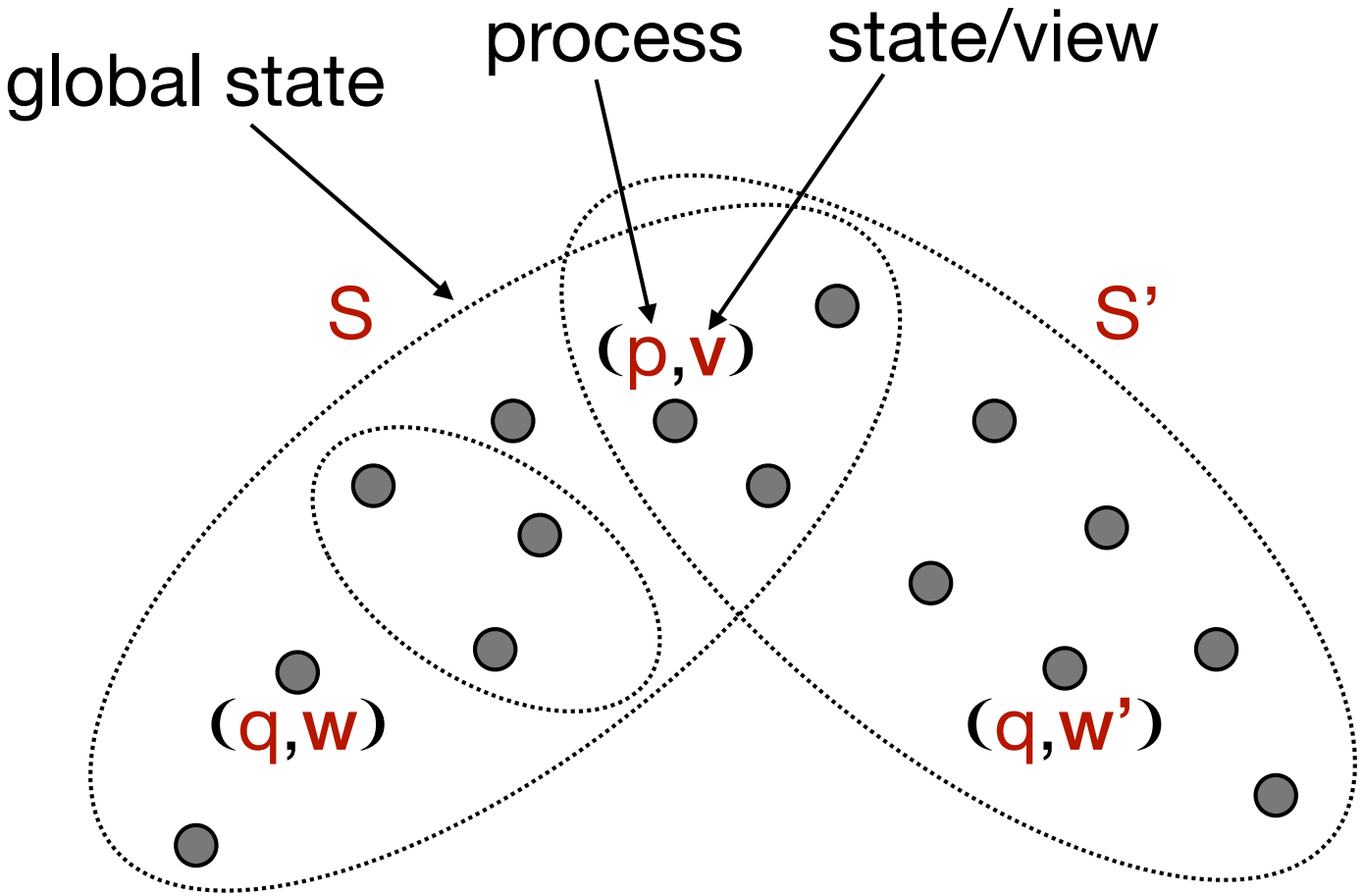


- Assume all proc decide ⊥.

❑

Combinatorial Topology

Configurations



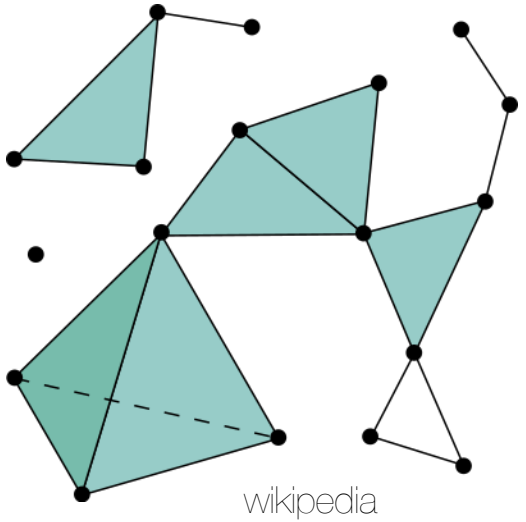
System configurations at time t

Simplexes and Complexes

- A **complex** is defined as a pair $K = (V, \mathcal{S})$ where
  - V is the (finite) set of vertices
  - $\mathcal{S}$ is a collection of non-empty subsets of V, closed under vertex deletion, i.e., $S \in \mathcal{S} \Longrightarrow \forall\, S' \subseteq S,\ S' \in \mathcal{S}$.

    Every $S \in \mathcal{S}$ is a **simplex**.

- **Examples:**
  - $G=(V,E)$ defines the complex $K=(V, E \cup V)$
  - A higher dimensional complex:

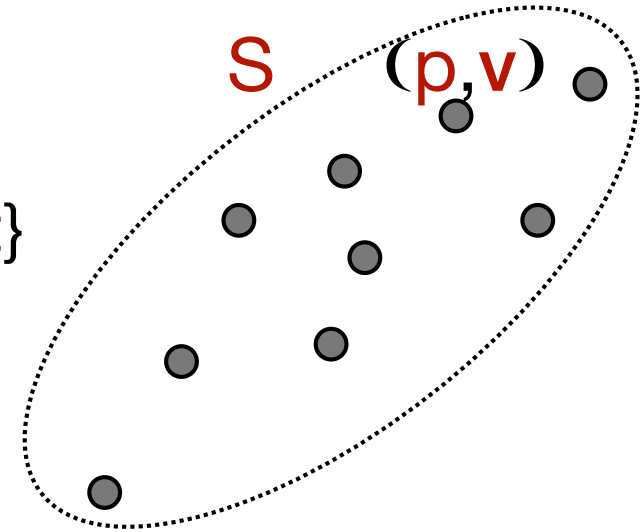


wikipedia

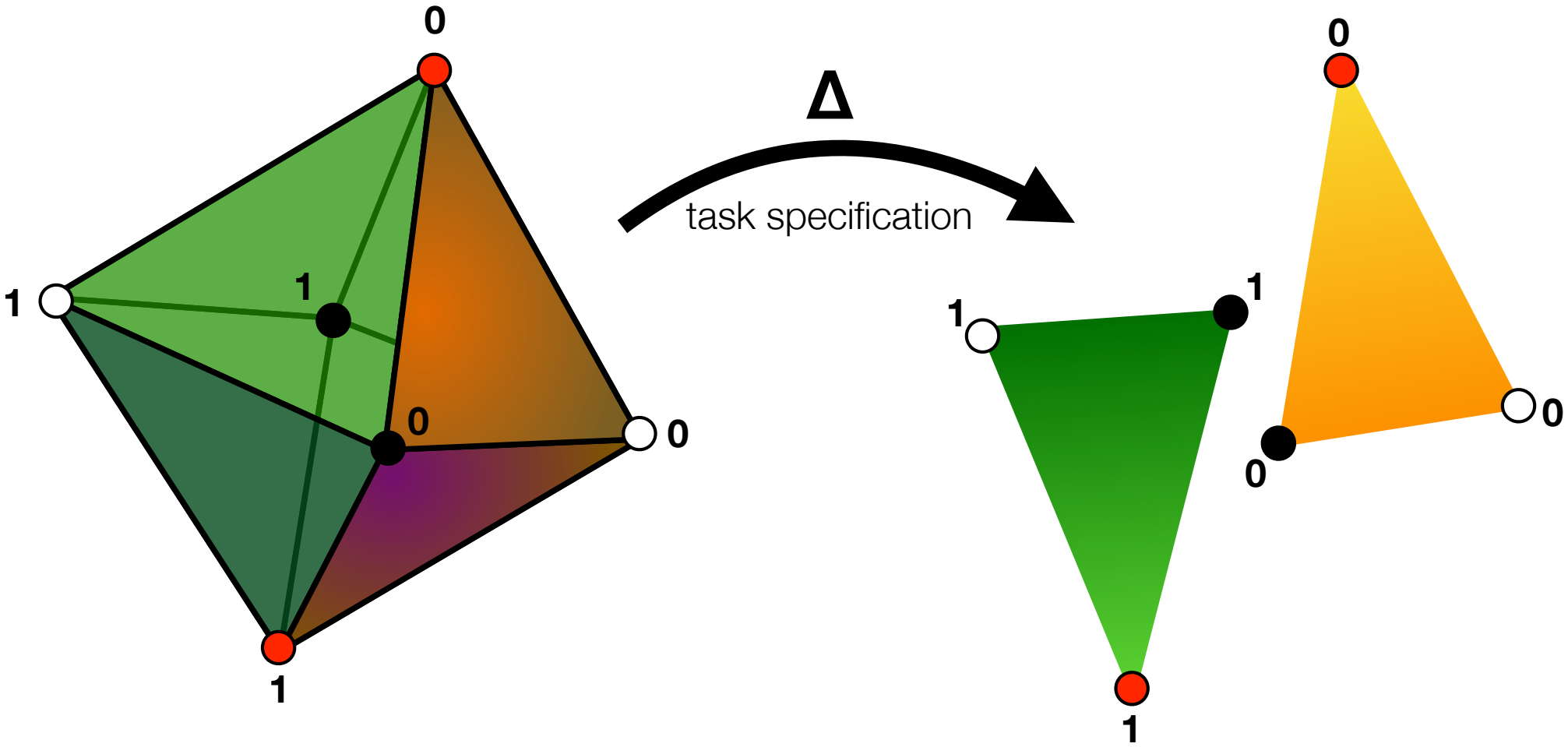
Protocol Complex

The configurations of a distributed system at time t defines the **protocol complex** $\mathbf{P}_t = (V, \mathcal{S})$ with

- $V = \{(p,v)$, p process, v state of p at time t$\}$
- $S \in \mathcal{P}(V)$ belongs to $\mathcal{S}$ if S is a set of views from different processes, corresponding to a same execution $\mathcal{E}$



Input/Output Complexes and Task Specification

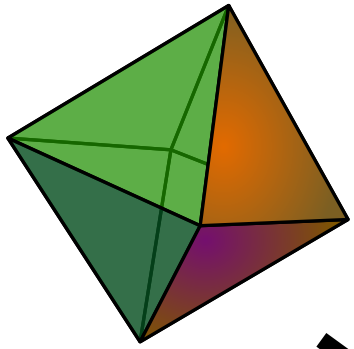


Input binary consensus

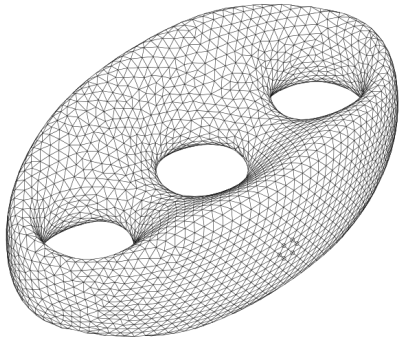
Output binary consensus

Task Solvability

input complex **I**



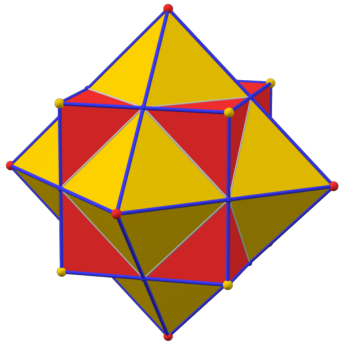
protocol execution ξ



protocol complex **P**

task specification Δ

decision δ



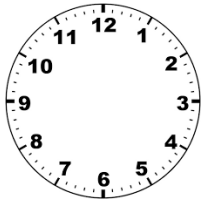
output complex **O**

Theorem Task (**I**,**O**,Δ) is solvable iff there exists δ such that, for every $S \in I$,

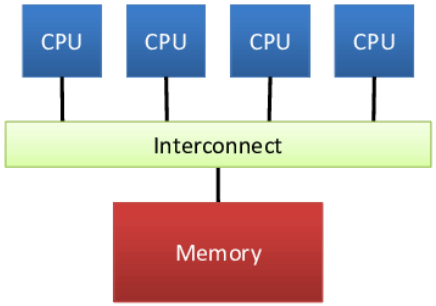
$\delta(\xi(S)) \in \Delta(S)$

Wait-Free Computing

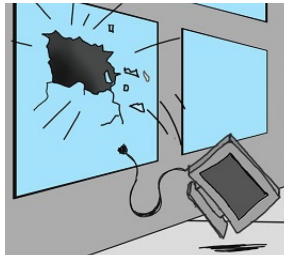
Asynchronous

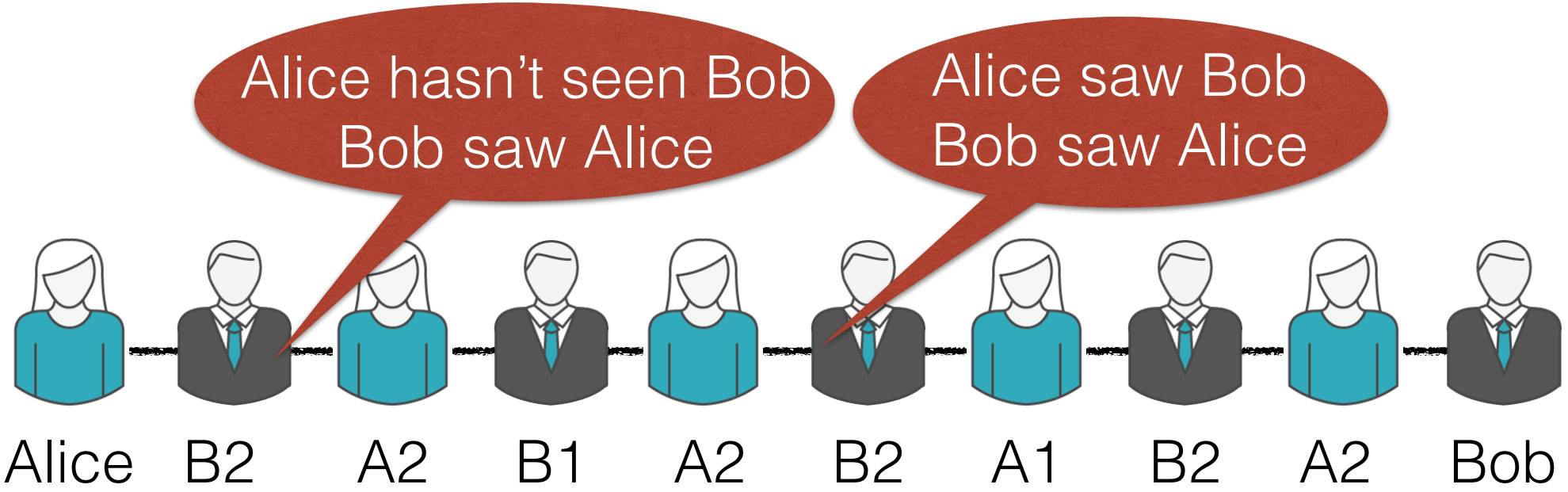


Shared Memory



Crash failures



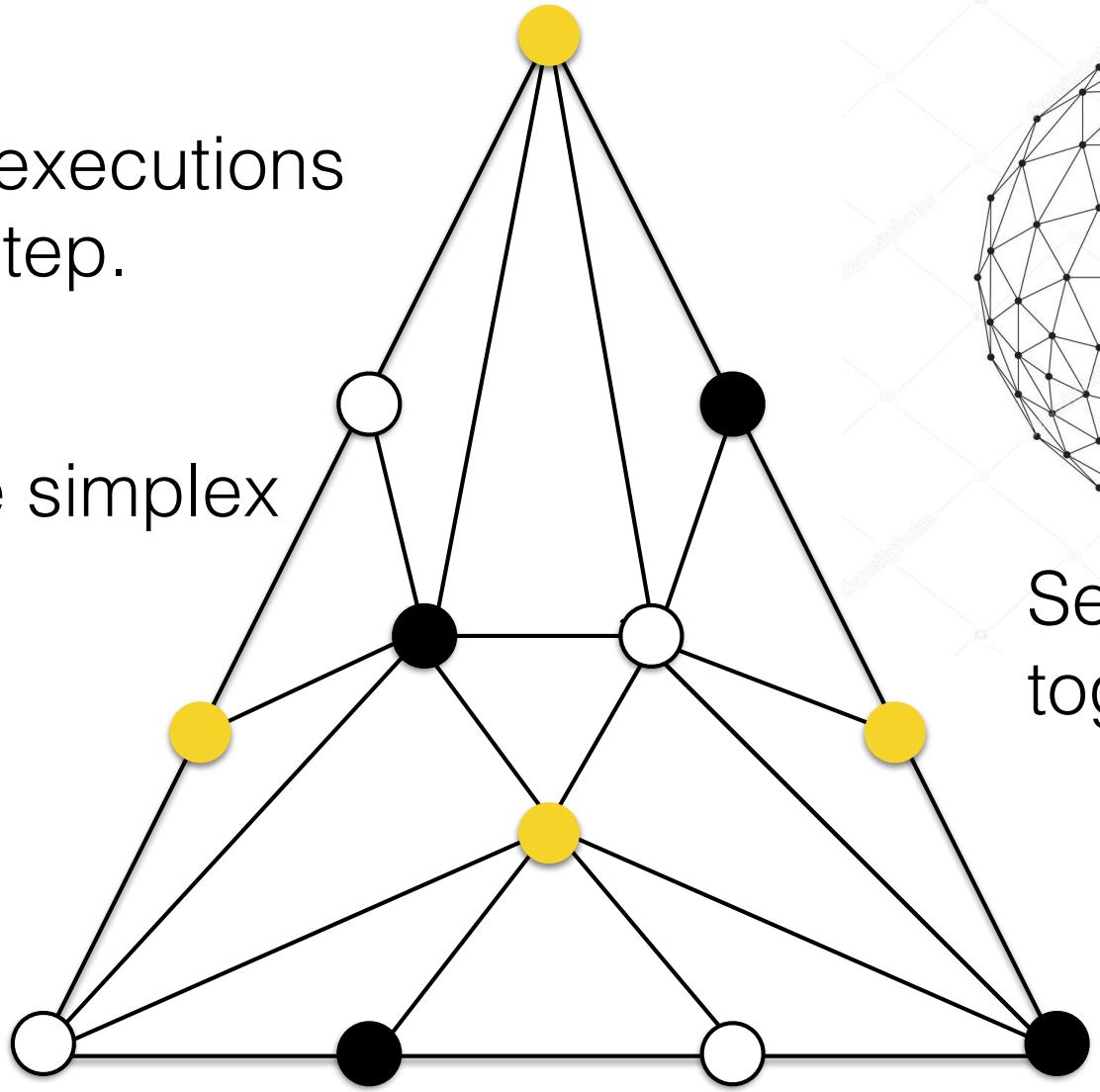


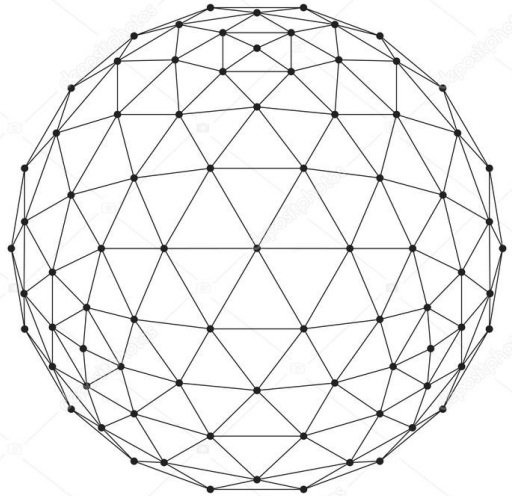
Three Processes

(iterated immediate snapshot)

All possible executions during one step.

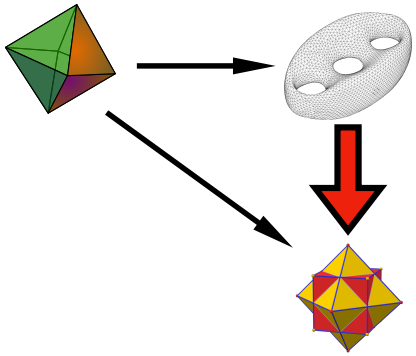
A facet of the simplex





Several facets together

Wait-Free Solvability

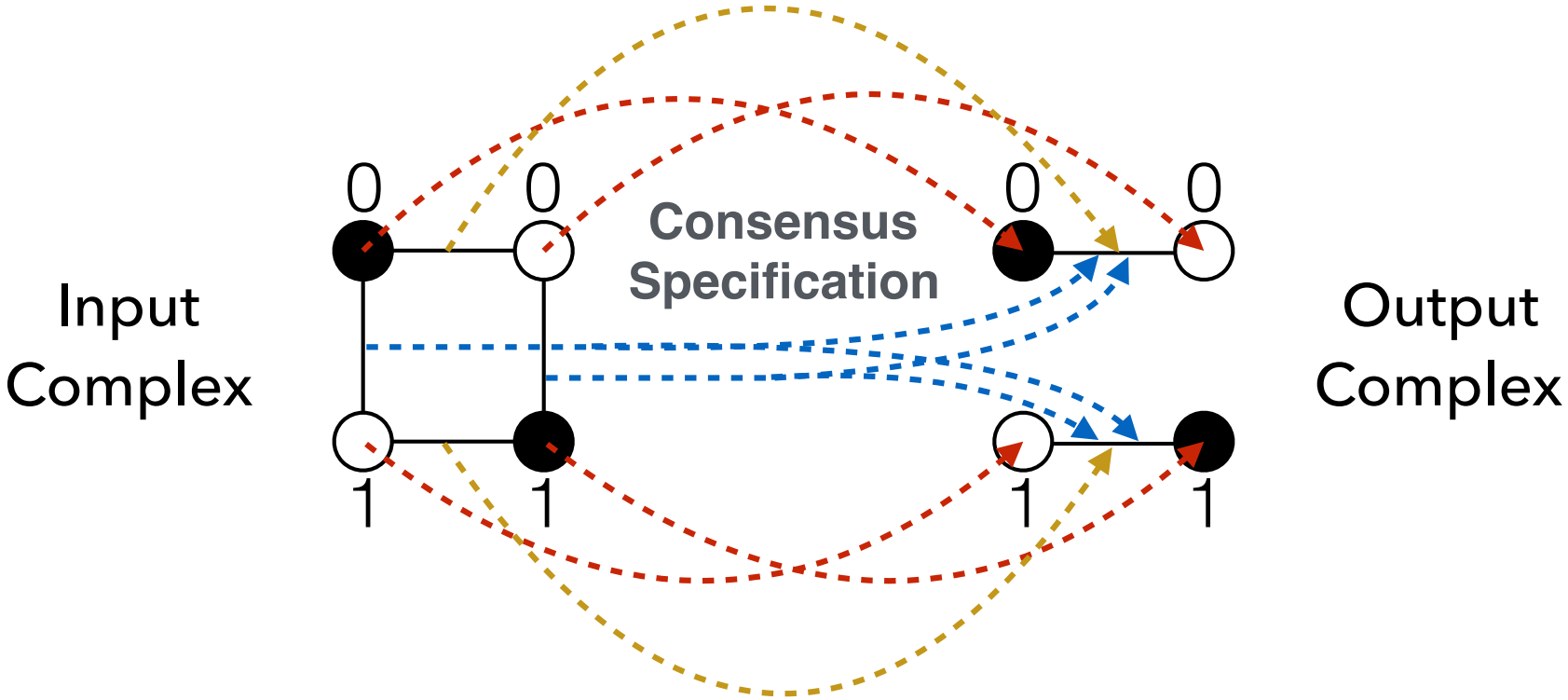


M. Herlihy and N. Shavit (1999)

Theorem A task is solvable in the asynchronous model with crashes if and only if there exists a simplicial map from a *chromatic subdivision* of the input complex to the output complex, respecting the specification of the task.

Gödel Prize 2004

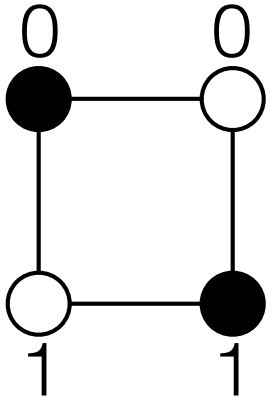
Consensus

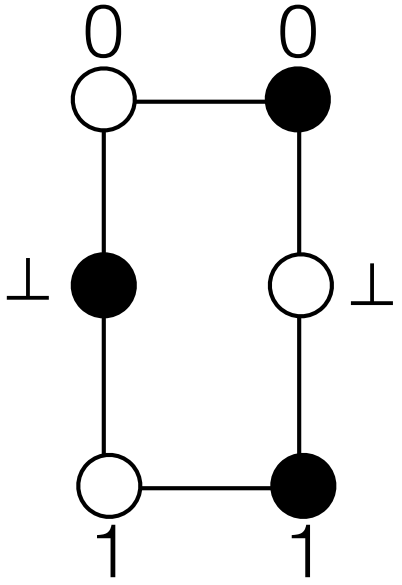


No simplicial map from a subdivision of the input complex to the output complex respecting the specifications of consensus.

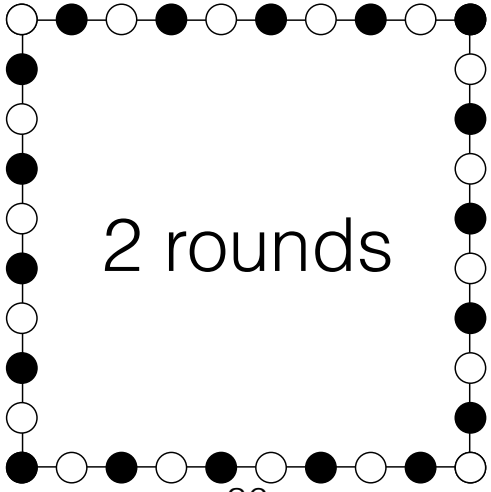
Weak consensus

Input Complex





Output Complex



Variants

k-set agreement

- n processes with input values in {1,…,m}
- objective: agree on at most k proposed values
- remark: (n-1)-set agreement is called set-agreement

t-resilient model

- asynchronous
- t = maximum number of crash failures

Set-agreement solvability

Theorem In the t-resilient model, if $k \geq t+1$, then k-set agreement is solvable.

Proof Algorithm for (t+1)-set agreement in the t-resilient model:

begin

repeat snapshot

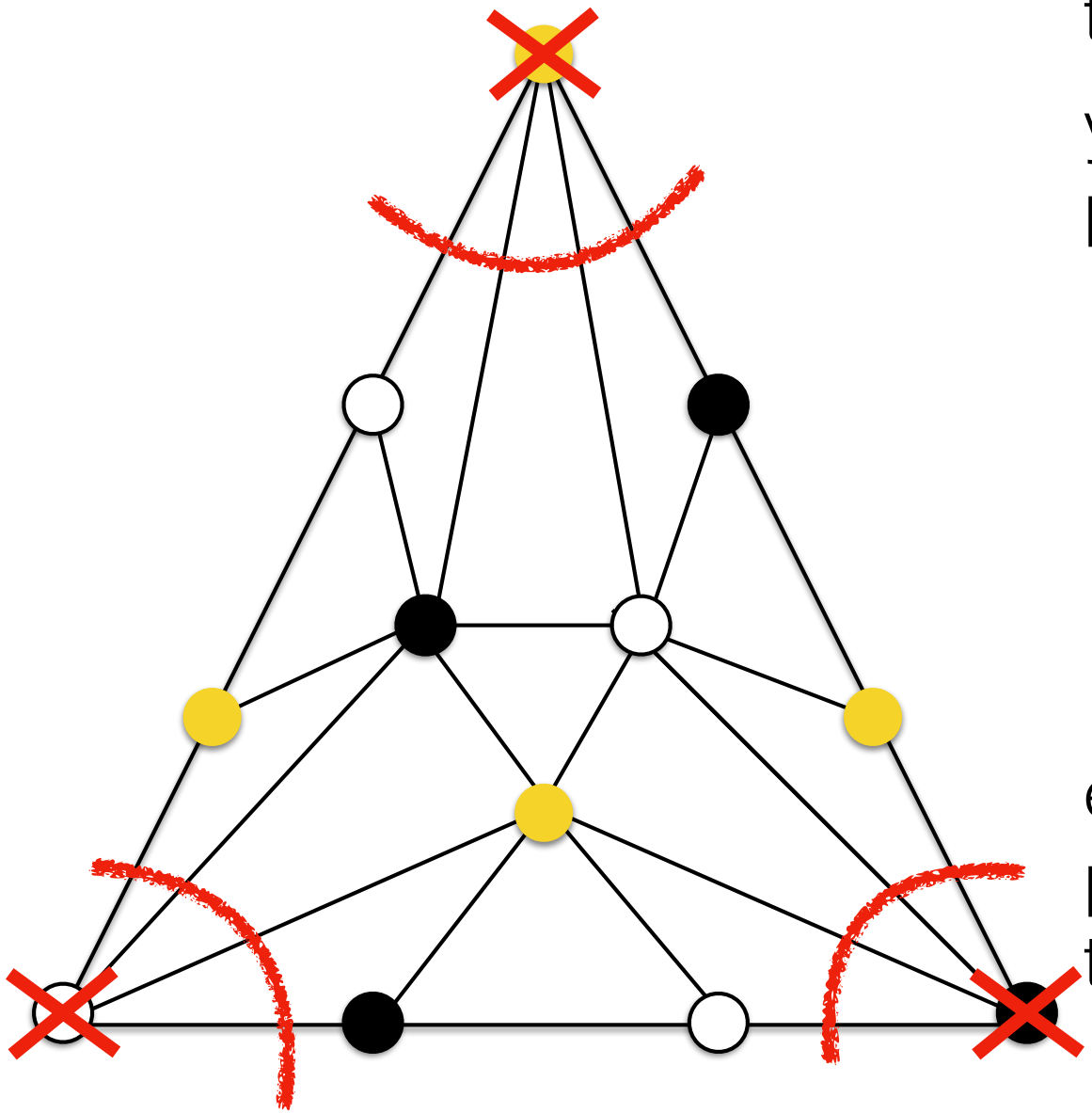
until values from at least n-t processes are seen

decide minimum seen value

end

➥ at most t+1 different views ❑

Topological perspective



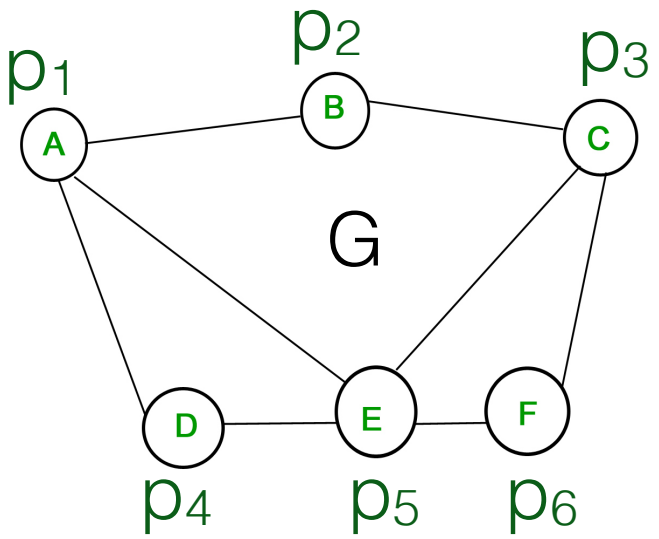
t = 1

yields holes in the protocol complex

enables to map the protocol complex to the output complex

Other applications of topology to distributed computing

- Processes occupy nodes of a graph G
- Synchronous model
- Communication by messages
- No failures



- Graph G is known to every process, including the position of every other process.

Lower bound

A dominating set in G=(V,E) is a set $D \subseteq V$ such that every node not in D has a neighbor in D.

Definition G has dominating number d if the min size of a dominating set in G has cardinality = d.

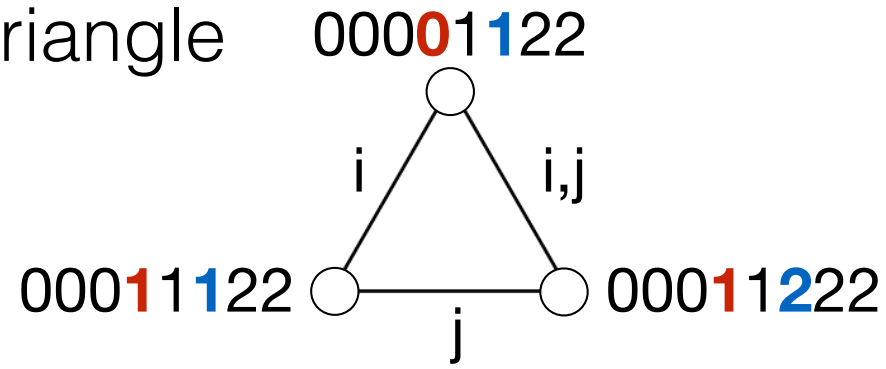
Theorem k-set agreement in G requires at least r rounds where r is the minimum integer such that G^r has dominating number $\leq k$.

Proof for m=3 and k=2

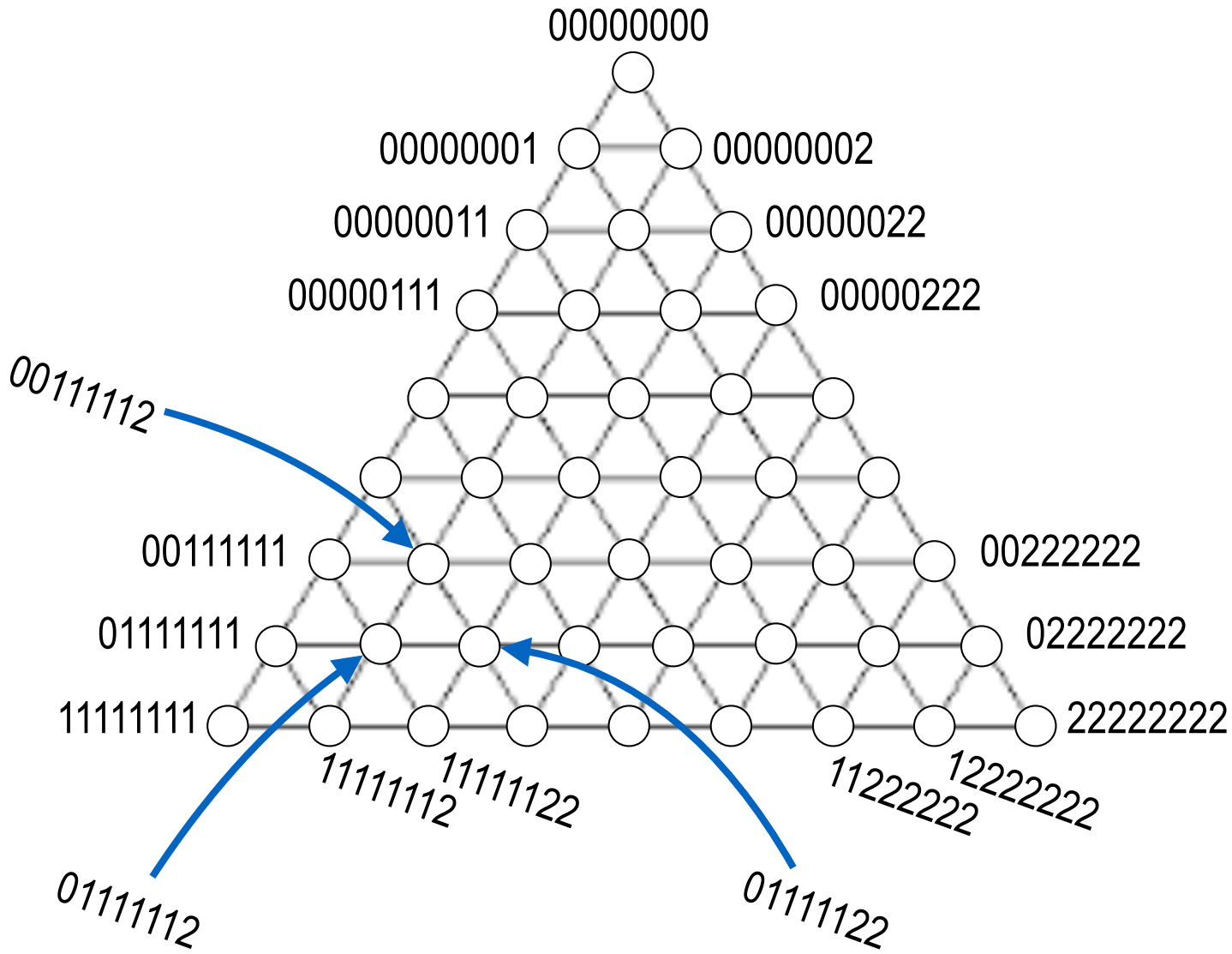
Input configuration: $v_1v_2\ldots v_n$ with $v_i \in \{0,1,2\}$

For every i,j, there exists process q that is not dominated by p_i nor p_j.

➥ triangle

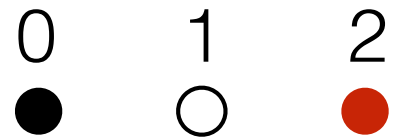


These triangles can be glued together

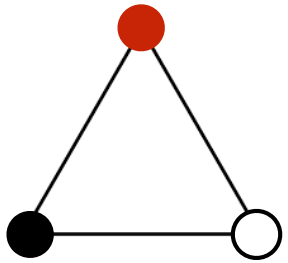

00000000
00000001
00000002
00000011
00000022
00000111
00000222
00111112
00111111
00222222
01111111
02222222
11111111
22222222
11111112
11111122
11222222
12222222
01111112
01111122

Assume existence of an algorithm.

- Colored each node by the discarded color

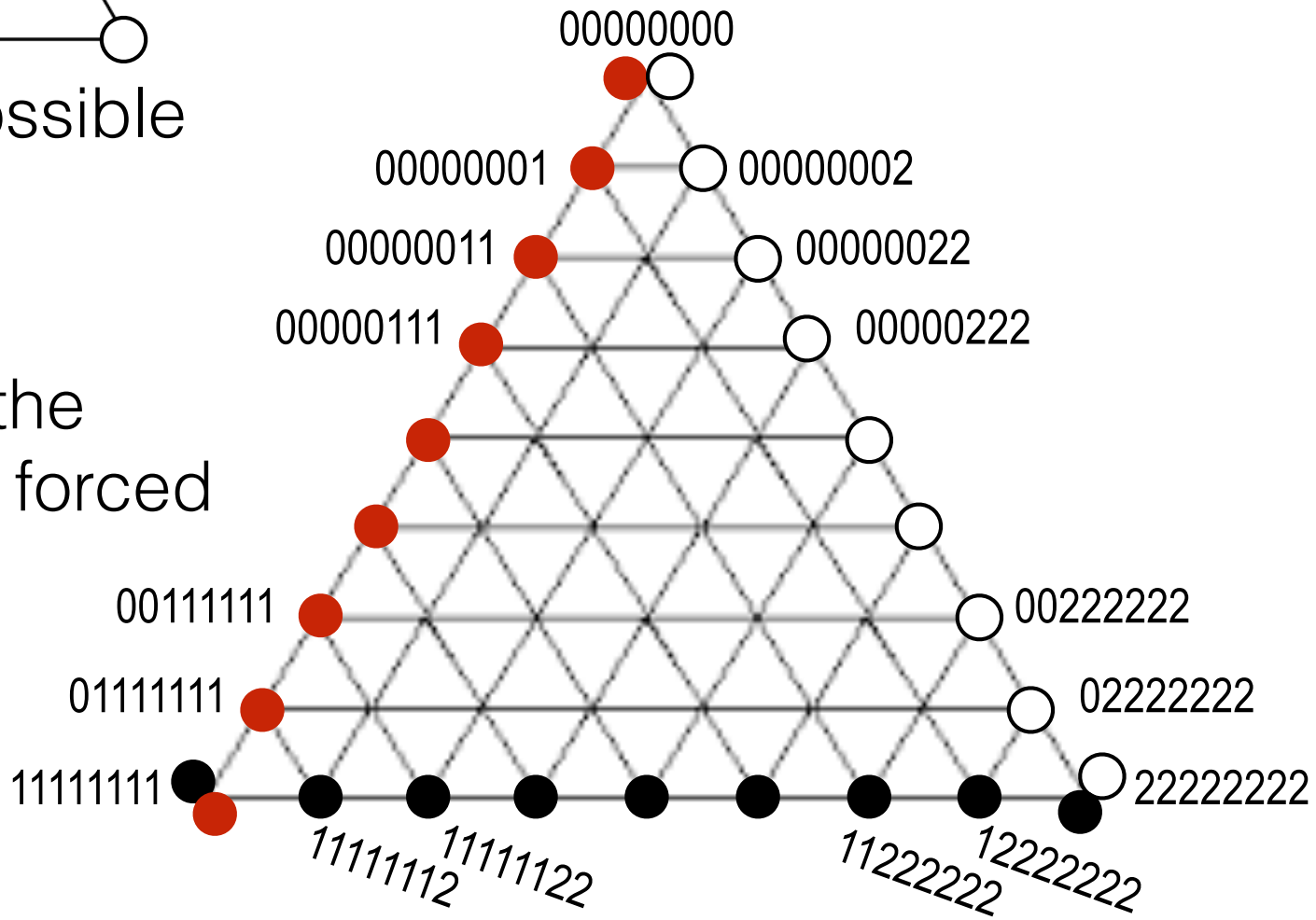


- Remark:



Impossible

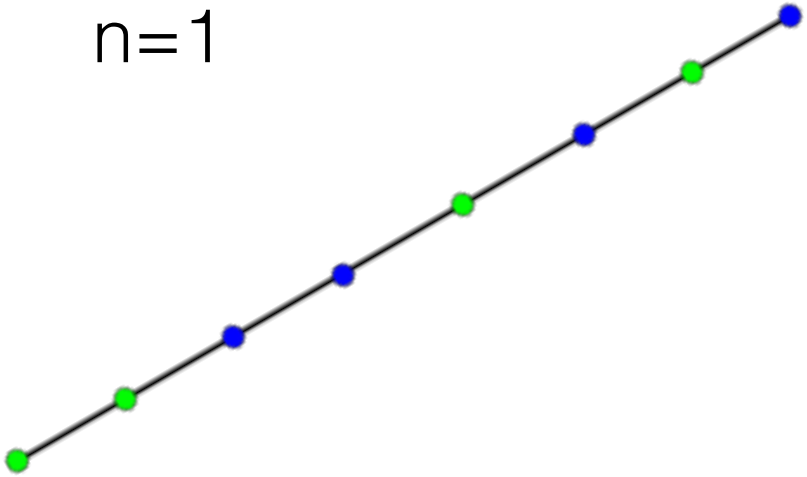
The coloring of the border nodes is forced



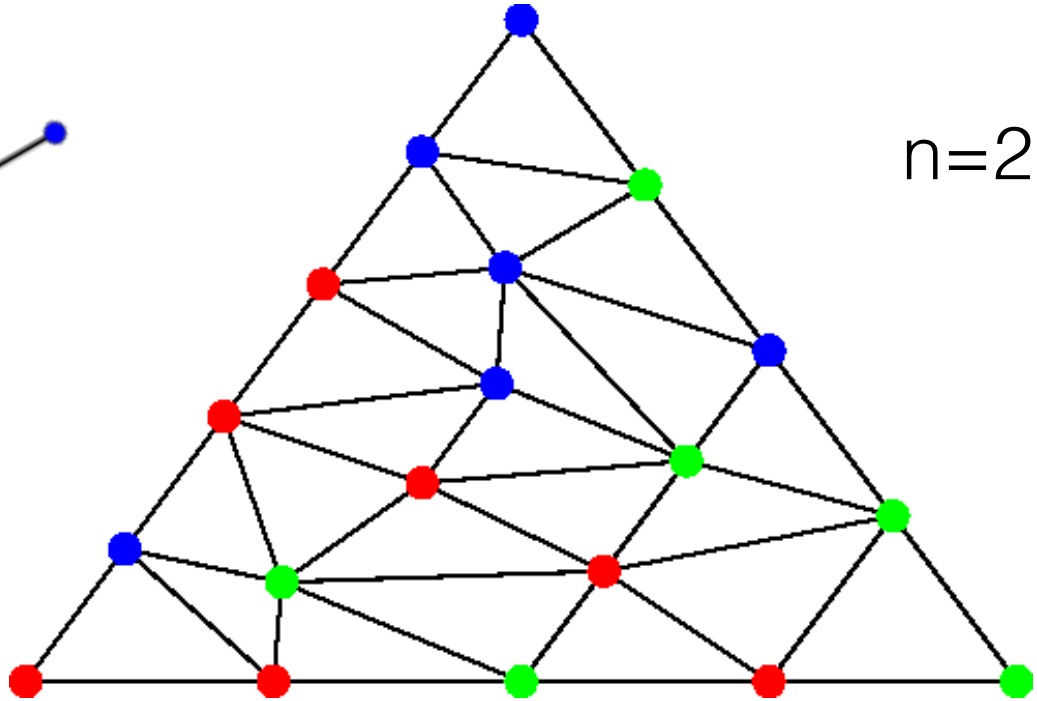
Sperner's Lemma

Lemma Every Sperner coloring of a triangulation of an n-dimensional simplex contains a cell colored with a complete set of colors.

n=1

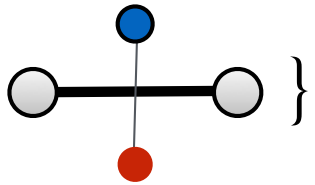


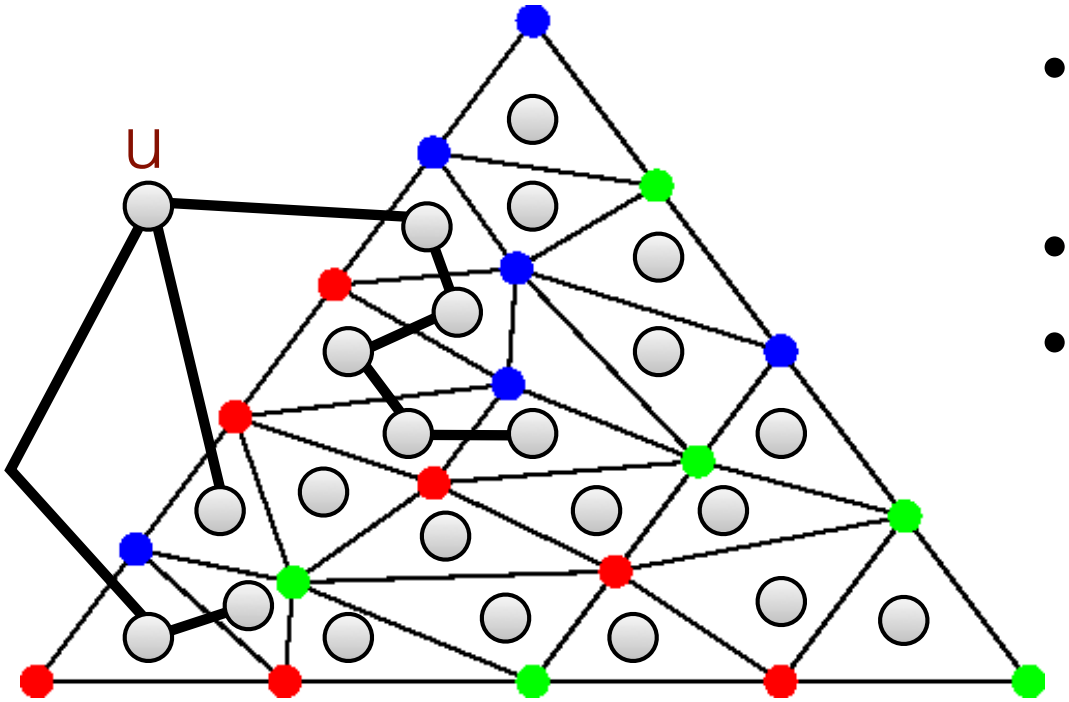
n=2



Proof sketch

V(G) = {○}

E(G) = {  }



- By induction on n: deg(u) is odd
- $\sum_{v \in V(G)} \deg(v) = 2\,|E(G)|$
- triangles with 1 or 2 colors induce nodes with even degrees (0 or 2)

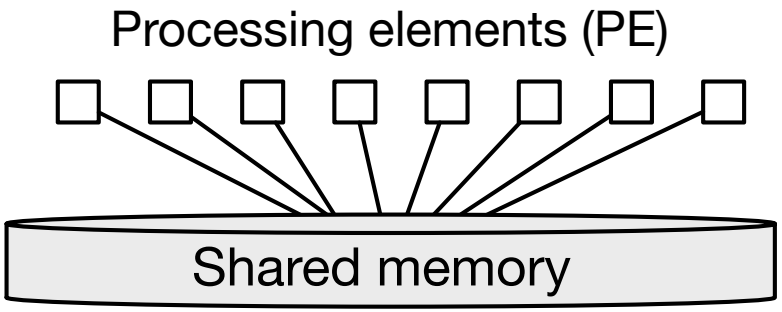
⇩

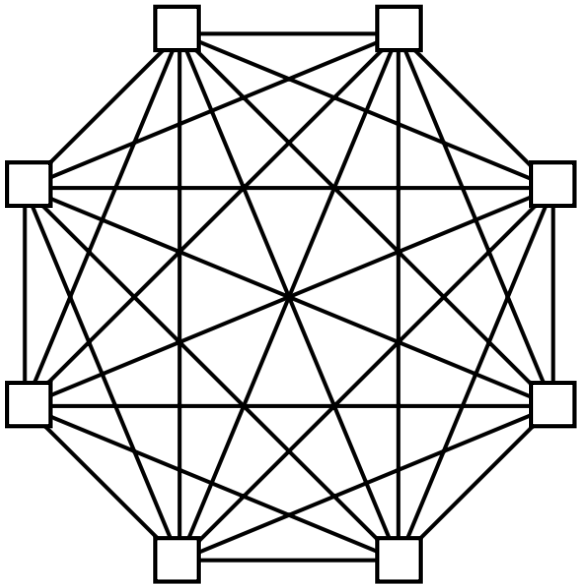
odd number of
3-colored triangles

❏

Concluding remarks

Message Passing vs. Shared Memory





Message passing

Equivalence

H. Attiya, A. Bar-Noy, D. Dolev (1990)

Theorem The message-passing and shared-memory models with crash failures are “essentially” equivalent

Dijkstra Prize 2011

Overcoming impossibility results

- **Failure detectors:** e.g., T. Chandra, V. Hadzilacos, S. Toueg (1995)
- **Randomization:** e.g., Ben-Or Algorithm for consensus (1983)
- **Best-effort algorithms:** e.g., *Paxos* algorithm (1989) by L. Lamport (Turing Award 2014)
- **Build-in atomic objects:** beyond read/write registers, like *test&set*, *compare&swap*, etc.

Open problems

- **Renaming:**
  - n processes start with unique names taken from a large name space [1,N]
  - they must decide new unique names from a name space as small as possible.
  - Result: 2n-1 possible; optimal for infinitely many n, but not for all n.
- **Algebraic topology:**
  - Randomized algorithms
  - Byzantine failures
- **Distributed verification**
  - Proving correctness using formal methods and/or proof assistants
  - Distributed monitoring