$$M = \begin{array}{c} \begin{array}{cc} 1 & \phantom{\underline{m}^T} \end{array} \\ \begin{pmatrix} d & \underline{m}^T \\ \underline{m} & R \end{pmatrix} \end{array}$$

$$M - \frac{1}{d}\begin{pmatrix} d \\ \underline{m} \end{pmatrix}\begin{pmatrix} d & \underline{m}^T \end{pmatrix} = \begin{pmatrix} \underline{0} & \underline{0} \\ \underline{0} & R - \dfrac{\underline{m}\,\underline{m}^T}{d} \end{pmatrix}$$

$$\underline{l}_1 \underline{l}_1^T$$

$\underline{l}_1$ & $\mathcal{L}$

$$M_0 = M$$

$$M_{i+1} = M_i - \underline{l}_i \underline{l}_i^T \qquad \underline{l}_i = \frac{M_i(:,i)}{M(i,i)^{1/2}}$$

$$\cancel{\mathcal{L}\mathcal{L}^T} \qquad M = \sum_i \underline{l}_i \underline{l}_i^T$$

$$= \underbrace{\begin{pmatrix} \ell_1 & \cdots & \ell_n \end{pmatrix}}_{L} \underbrace{\begin{pmatrix} \ell_1^T \\ \vdots \\ \ell_n^T \end{pmatrix}}_{L^T}$$

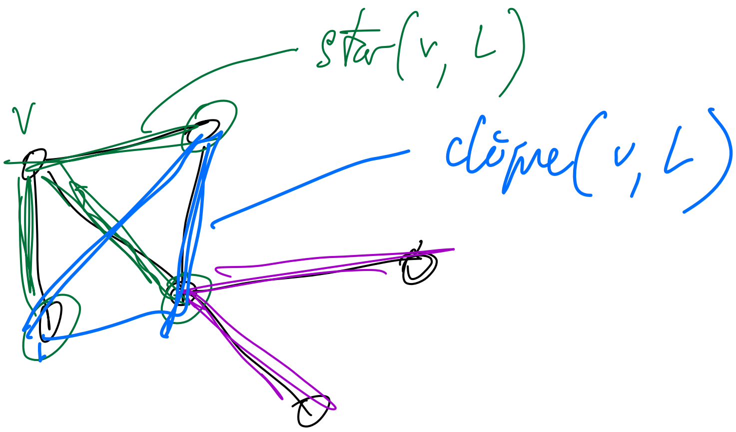
$$M_i(i, i) > 0$$

$$\underbrace{M(1, 1)} \qquad e_1^T M e_1 = 0$$

$$\underbrace{\min_y \begin{pmatrix} y \\ z \end{pmatrix}^T M \begin{pmatrix} y \\ z \end{pmatrix}}_{> 0} = z^T \underbrace{\left(R - \frac{r_2 r_2^T}{d} \right)}_{M_1} z$$

$$\underbrace{\qquad M_1 \qquad}$$

$$L = \begin{pmatrix} d & -\underline{a}^T \\ -\underline{a} & \operatorname{diag}(\underline{a}) + L_{-1} \end{pmatrix}$$



$$\underline{\ell}_1 = \frac{1}{d^{1/2}} \begin{pmatrix} d \\ -\underline{a} \end{pmatrix}$$

$$S_1 = L - \underline{\ell}_1 \underline{\ell}_1^T = \begin{pmatrix} 0 & \underline{0}^T \\ \underline{0} & \operatorname{diag}(\underline{a}) + L_{-1} - \frac{\underline{a}\,\underline{a}^T}{d} \end{pmatrix}$$

$$\operatorname{diag}(\underline{a}) - \frac{\underline{a}\,\underline{a}^T}{d} \Rightarrow \text{Laplacian}$$

$$w_{new}(i,j) = \frac{w(i,v)\, w(j,v)}{d}$$

$$\parallel$$

$$\boxed{\frac{w(i,v)\,w(j,v)}{w(i,v)+w(j,v)} \cdot \left(\frac{w(j,v)}{d} + \frac{w(i,v)}{d} \right)}$$