

$$M = \begin{pmatrix} d & \underline{m}^T \\ \underline{m} & R \end{pmatrix}$$

$$M - \frac{1}{d} \begin{pmatrix} d \\ \underline{m} \end{pmatrix} \begin{pmatrix} d & \underline{m}^T \end{pmatrix} = \begin{pmatrix} \underline{0} & \underline{0} \\ \underline{0} & R - \frac{\underline{m}\underline{m}^T}{d} \end{pmatrix}$$

$\underline{l}, \underline{z} \quad \mathcal{L}$

$$M_0 = M$$

$$M_{i+1} = M_i - \underline{l}_i \underline{l}_i^T \quad \underline{l}_i = \frac{M_i(:, i)}{M_i(i, i)^{1/2}}$$

$$\mathcal{L} \mathcal{L}^T = M = \sum_i \underline{l}_i \underline{l}_i^T$$

$$= \begin{array}{|c|c|c|} \hline l_1 & 0 & 0 \\ \hline \vdots & \ddots & \vdots \\ \hline l_n & 0 & 0 \\ \hline \end{array} \quad \begin{array}{|c|} \hline r_1 \\ \hline \vdots \\ \hline r_n \\ \hline \end{array}$$

$L \qquad R^T$

$$M(\underline{e}, \underline{e}) > 0$$

$$\underline{M}(\underline{1}, \underline{1})$$

$$\underline{e}_1^T M \underline{e}_1 = 0$$

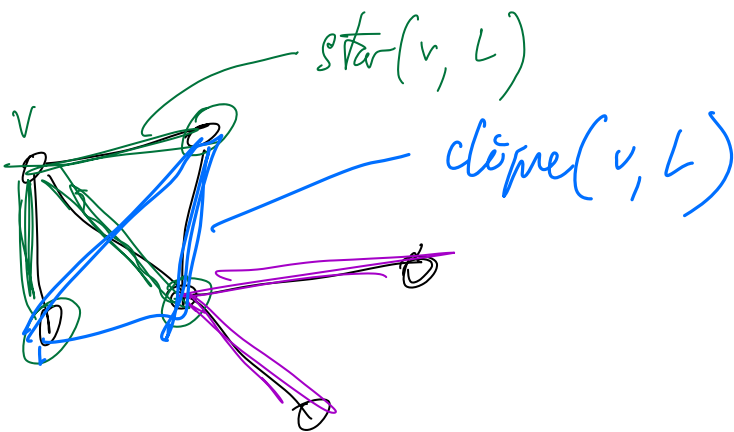
$$\min_y \begin{pmatrix} y \\ \underline{z} \end{pmatrix}^T M \begin{pmatrix} y \\ \underline{z} \end{pmatrix} = \underline{z}^T \underbrace{\left(R - \frac{\underline{u}_1 \underline{u}_1^T}{d} \right)}_{M_1} \underline{z}$$

$$> 0$$

$$M_1$$

$$L = \begin{pmatrix} d & \underline{a}^T \\ \underline{a} & L_{-1} \end{pmatrix}$$

Diagram illustrating the Laplacian matrix L partitioned into blocks. The top-left element is d , the top-right is \underline{a}^T , the bottom-left is \underline{a} , and the bottom-right is L_{-1} . The top row and top-right block are circled in green. The bottom-left block is circled in purple.



$$\underline{e}_{-1} = \frac{1}{d} \begin{pmatrix} d \\ -\underline{a} \end{pmatrix}$$

$$\underline{S}_1 = L - \underline{e}_{-1} \underline{e}_{-1}^T$$

$$= \begin{pmatrix} 0 & \underline{0} \\ \underline{0} & \text{diag}(a) + L_{-1} - \frac{\underline{a} \underline{a}^T}{d} \end{pmatrix}$$

$$\underline{\text{diag}}(a) - \frac{\underline{a} \underline{a}^T}{d} \text{ is Laplacian}$$

$$w_{rev}(i, j) = \frac{w(i, v) w(j, v)}{u \cdot d}$$

$$\frac{w(i, v) w(j, v)}{w(i, v) + w(j, v)} \cdot \left(\frac{w(j, v)}{d} + \frac{w(i, v)}{d} \right)$$