Exercise 1 (Truthfulness for single-minded bidders) Prove that an exact mechanism for combinatorial auctions with single-minded bidders in which losers pay 0 is truthful if it satisfies the following two properties:

- *Monotonicity:* A bidder who wins with bid (S_i^*, v_i^*) keeps winning for any $v_i' > v_i^*$ and for any $S_i' \subset S_i^*$ (for any fixed setting of the other bids).
- *Critical Payment:* A winning bidder pays the minimum value needed for winning: The infimum of all values v_i' such that (S_i^*, v_i') wins.

Exercise 2 (Estimating valuations) Let $\mathcal{M}(L)$ denote an efficiently computable, deterministically truthful mechanism for combinatorial auctions that is parametrized with a positive number L. Suppose $\mathcal{M}(L)$ guarantees an α-approximation of the optimal social welfare, for some $\alpha > 1$, provided that there is at least one bidder whose valuation for the full bundle is at least L (i.e., a bidder i with $v_i(M) \geq L$) and at most one bidder whose valuation for the full bundle exceeds L (i.e, $v_i(M) > L$).

Derive a randomized, efficiently computable, universally truthful mechanism $\mathcal{M}^*$ with unconditioned approximation ratio $O(\alpha)$ (wrt expected social welfare). *Hint:* Use sampling based statistics.

Exercise 3 (Combinatorial auctions with explicit valuations) Consider combinatorial auctions for m items among n bidders where each valuation is represented simply as a vector of $2^m - 1$ numbers (a value for each subset of items). Prove that the optimal allocation can be computed in time that is polynomial in the input length: $n(2^m - 1)$. *Hint:* Use dynamic programming.

(An immediate consequence is that the optimal allocation can be computed in time polynomial in n when $m = \log n$.)