A Game Theoretic Framework for Incentives in P2P Systems

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1 Introduction

Peer-to-Peer(P2P) file-sharing systems organize users into an overlay network to facilitate the exchange of data. However,P2P systems have democratic nature, which means there is no central authority to mandate or coordinate the resource that each peer contributes. Because of voluntary participation, the distributed resources are highly variable and unpredictable. Recently research shows that many users are simply consumers and do not contribute much to the system. Most of users are "free riders" (In Gnutella [1], 25% users share nothing). Users' sessions are relative short, 50% of sessions are shorter than 1 hour. Short session means that a large portion of the data in the system might be unavailable for large period of time. When the growing number of free riders, as the result, the systems lose the sprite of peer to peer and becomes a traditional client server system and various forms of abuse and attack have been observed in practice.

If the peer to peer system is to be a reliable platform for distributed resource sharing, they must provide predictable level of service. So, in order to let peer make contribution as much as possible, there are two economic methods that can be used for incentives:

Monetary payment one pays to consume resources and paid to contribute resource.

Differential service peers that contributes more get better quality of service.

Monetary payment needs a imaginary currency and requires an accounting infrastructure to track various resource transaction and charges for them using micro payments. But as written in [3], it is highly impractical because of network pricing.

So the differential service is choosed to be an incentive model. Actually, some currently P2P implementation have already used differential service model. For example, KaZaA [2] file sharing system uses *participation level*

$$Participation\ level = \frac{upload\ in\ MB}{download\ in\ MB \times 100}$$

to model how active a peer contributes to the system. In general, system could use *reputation index* and reputation reflects the overall contribution to the system.

In general, players are strategy players because users compete for shared but limited resources and at the same time they restrict others download from their server by deny access or not contribute anything. Because all peers are strategic and rational player, it is very intuitive to model the interaction of peers in game theory. It is actually a non-cooperative game among peers: each player wants to maximize his benefit and minimize this expense. In the jargon of Game Theory, each peer wants to maximize his utility value. Section 3 will discuss how to use Game Theory [4] to model P2P system. In section 2, a fraction of Game Theory that is essential for understanding section 3 is introduced(especially: Nash equilibrium and best response function).

		Player 2	
		L	R
Player 1	T	2,2	0,3
	B	3,0	1,1

Figure 1: Two players' game.

2 Bootstrapping Game Theory

2.1 Jargon of strategy game

We use the notion of *strategic game* from Game Theory [4] to model behavior in which peers interact with each other. Precisely, a strategic game consists of

- a set of peers(players);
- for each peer(player), a set of *actions* (sometimes called *strategies*);
- for each peer(player), a *utility function* that gives the peer's utility to each list of the peers' actions.

A list of actions, one for each player in the game, is called an *action profile* (or, sometimes, a *strategy profile*).

For simplicity, we can represent a strategic game with two players in a table, like figure 1. This table represents a strategic game in which player 1's actions are T and B and player 2's actions are L and R. The first number in each box is player 1's utility to the pair of actions that define the box, while the second number in each box is player 2's utility to the pair of actions that define the box. Thus, for example, if player 1 chooses the action B and player 2 chooses the action L then player 1's utility is 3 and player 2's utility is 0.

2.2 Nash equilibrium

A peer(player) that is strategy or rational player if we can model the player with following two assumptions:

- Each player chooses the action that is best for her, given her beliefs about the other players' actions.
- Every player's belief about the other players' actions is correct.

The notion of equilibrium that embodies these two principles is called *Nash equilibrium* (after John Nash, who suggested it in the early 1950s). (The notion is sometimes referred to as a "Cournot-Nash equilibrium".) Precisely:

Definition 1 ***Nash equilibrium***
A Nash equilibrium of a strategic game is an action profile (list of actions, one for each player) with the property that no player can increase her utility by choosing a different action, given the other players' actions.

So now the question is how to finding Nash equilibria. Conside the example showed in figure 1. There are four action profiles ((T,L), (T,R), (B,L), and (B,R)); we can examine each in turn to check whether it is a Nash equilibrium.

(T,L) By choosing B rather than T, player 1 obtains a utility of 3 rather than 2, given player 2's action. Thus (T,L) is not a Nash equilibrium. Player 2 also can increase her utility (from 2 to 3) by choosing R rather than L.

(T,R) By choosing B rather than T, player 1 obtains a utility of 1 rather than 0, given player 2's action. Thus (T,R) is not a Nash equilibrium.

(B,L) By choosing R rather than L, player 2 obtains a utility of 1 rather than 0, given player 1's action. Thus (B,L) is not a Nash equilibrium.

(B,R) Neither player can increase her utility by choosing an action different from her current one. Thus this action profile is a Nash equilibrium.

So this game has a unique Nash equilibrium, (B,R).

2.3 Finding Nash equilibria: best response functions

In a game in which each player has infinitely many possible actions, it is not possible to find a Nash equilibrium by examining all action profiles in turn. To develop an alternative method of finding Nash equilibria, we first reformulate the definition of a Nash equilibrium for a two-player game.

Call the action of player 1 that maximizes her utility, given that player 2's action is a_2, player 1's *best response* to a_2. Similarly, call the action of player 2 that maximizes her utility, given that player 1's action is a_1, player 2's *best response* to a_1.

Given this definition of best responses, a pair (a_1, a_2) of actions is a Nash equilibrium if and only if

- Player 1's action a_1 is a best response to player 2's action a_2;
- and player 2's action a_2 is a best response to player 1's action a_1.

So, in order to find a Nash equilibrium we need to find a pair (a_1, a_2) of actions such that a_1 is a best response to a_2, and vice versa.

If we denote player 1's best response to a_2 by $b_1(a_2)$ and player 2's best response to a_1 by $b_2(a_1)$ then we can write the condition for a Nash equilibrium more compactly: the pair (a_1, a_2) of actions is a Nash equilibrium if and only if $a_1 = b_1(a_2)$ and $a_2 = b_2(a_1)$.

Consider the example in figure 1 using the method of finding the players' best response functions and then solving the two simultaneous equations.

Player 1's best response to L is B, and her best response to R is also B. Similarly, player 2's best response to T is R and her best response to B is R. Thus we have

$$b_1(L) = B$$

$$b_1(R) = B$$

$$b_2(T) = R$$

$$b_2(B) = R$$

We see that the only pair of actions (a_1, a_2) with the property that $a_1 = b_1(a_2)$ and $a_2 = b_2(a_1)$ is (B,R): the Nash equilibrium that we found previously.

As we are armed with the basic principal of Game Theory, we can start to discuss how to use Game Theory to model the interaction among peers.

3 Incentive model

3.1 Basic notion

We use N to denote the number of peers in the system. Using $P_1, P_2, P_3, \cdots, P_N$ to denote peers. Utility function for P_i is U_i. Contribution of P_i is D_i. D_0 is absolute measure of contribution. D_0 could be set different in different implementation. For example, one could set D_0 to 20MB per week. Then it is possible to normalize the contribution and get dimensionless contribution:

$$d_i = \frac{D_i}{D_0} \tag{1}$$

The unit cost is c_i for each unit that peer contributes. So the total cost is:$c_i D_i$.

3.2 Benefit matrix

Each peer's contribution to the system will potentially benefits all other peers, but in different degree. In order to express this different degree, a $N \times N$ benefit matrix B is introduced. In this matrix, each element B_{ij} denotes how much the contribution made by P_j is worth to P_i. Normally, $B_{ij} \geq 0$. If P_i is not interested in P_j's contribution, then B_{ij} would be 0. And $B_{ii} = 0$ for all i, because no peer will interest his own contribution.

After normalization, we get dimensionless equations:

$$b_{ij} = \frac{B_{ij}}{c_i} \tag{2}$$

$$b_i = \sum_j b_{ij} \tag{3}$$

$$b_{av} = \frac{1}{N} \sum_i b_i \tag{4}$$

The b_i is the total benefit that peer P_i can get from the system. b_{av} is the average benefit that all peers can get from the system.

3.3 Model differential service using probability

As said in the beginning, we use differential service model to incentive users' contribution. In differential service model, a peer reward other peers in proportion to their contribution. To implement this idea, we use probability: P_j accepts a request for a file from peer P_i with probability $p(d_i)$ and rejects it with probability $1 - p(d_i)$.

So if P_i's contribution to the system is very high, its request is more likely to be accepted. Because the only property for the probability function $p(d_i)$ is that

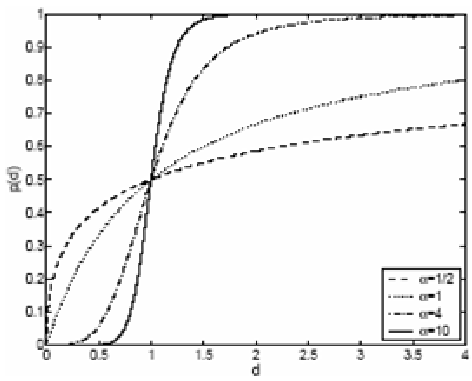

Figure 2: $p(d)$ plotted as a function of d for different value of α.

it is a monotonically increasing function of the contribution that peer made, it is possible to choose any probability function that satisfy this requirement. The function which used by the paper is the following:

$$p(d) = \frac{d^\alpha}{1 + d^\alpha} \tag{5}$$

When a peer made 0 contribution, the probability would be 0. When a peer made infinitive contribution, the probability would be approach to 1. Figure 2 plots the probability function that according to different value of α. To reduce complexity, in the following discussion, α is set to 1. So the probability function is

$$p(d) = \frac{d}{1 + d} \tag{6}$$

3.4 Utility function

The utility is define to contain both of peer's contribution and peer's expense to join the system:

$$U_i = -c_i D_i + p(d_i) \sum_j B_{ij} D_j, B_{ii} = 0 \tag{7}$$

After normalization with dimensionless parameter,

$$u_i = \frac{U_i}{c_i D_0} \tag{8}$$

The utility function is rewritten as

$$u_i = -d_i + p(d_i) \sum_j b_{ij} d_j, b_{ii} = 0 \tag{9}$$

The first term is P_i's cost to join the system and it increases linearly as peer contributes more disk/bandwidth to the system. P_i's benefit depends on how

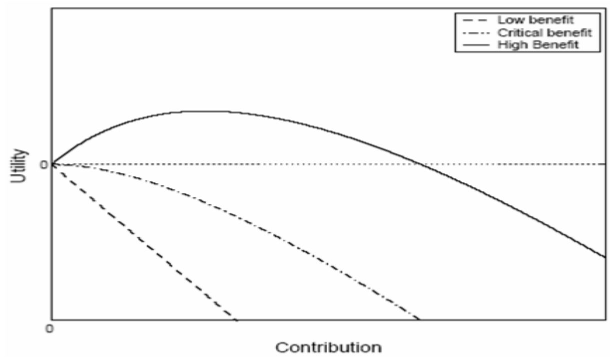

Figure 3: Possible utility function for different levels of benefit b_c.

much the other peers are contributing to the system (d_j), how that contribution is worth to him (b_{ij}), and the probability that he is able to download that content ($p(d_i)$). Or in other words, whether the other would like to accept his request.

Because $p(0) = 0$ and $p(\infty) = 1$, two limits of the utiliy function are:

$$\lim_{d_i \to 0} u_i = 0, \lim_{d_i \to \infty} = -\infty$$

Figure 3 shows possible utility function for different levels of benefit b_c. It shows that two dot-lines are below 0(peers will definitely not join the system). And only when benefit larger than some threshold b_c, utility value will be greater than 0 and peers will join the system.

4 Find Nash equilibrium in Homogeneous system

Every research starts with ideal phenomenon. This would give some insight into the problem and keep away from unimportant issue. The same idea applies to complex P2P reality, we simplify the system: each peers gets equal benefit from everyone else, which is called homogeneous system. In mathematical form, $b_{ij} = b, j \neq i$.

So the equation 9 becomes

$$u = -d + (N-1)bdp(d) \tag{10}$$

4.1 Two players game

We simplify the system again and due with only two player's game. The utility function of two players becomes:

$$\begin{aligned} u_1 &= -d_1 + b_{12} d_2 p(d_1) \\ u_2 &= -d_2 + b_{21} d_1 p(d_2) \end{aligned} \tag{11}$$

If $\alpha = 0$, we can get the best response functions for two peers. This is done by differentiating equation 11.

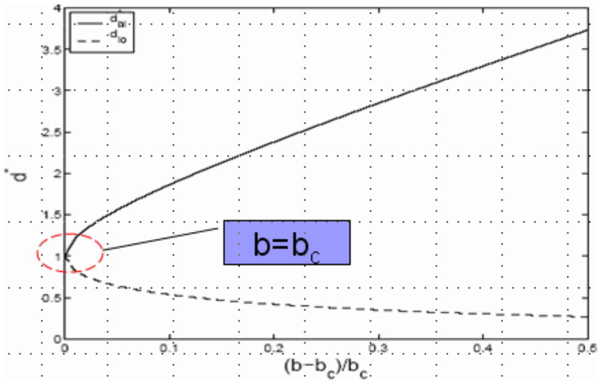

Figure 4: Nash equilibrium contributions for the two peer system plotted as a function of scaled benefit$(b - b_c)/b_c$. For $b < b_c$, there are no equilibria.For all $b > b_c$ there are two possible equilibria.

$$r_1(d_2) = d_1 = \sqrt{b_{12}d_2} - 1$$
$$r_2(d_1) = d_2 = \sqrt{b_{21}d_1} - 1 \qquad (12)$$

Nash equilibrium exists if and only there exists solutions (d_1^*, d_2^*) to the equation 12, such that:

$$d_1^* = \sqrt{b_{12}d_2^*} - 1$$
$$d_2^* = \sqrt{b_{21}d_1^*} - 1 \qquad (13)$$

Because $b_{12} = b_{21} = b$ for homogeneous system, the above equations are easily to solve.

$$d^* = (\frac{b}{2} - 1) \pm \sqrt{(\frac{b}{2} - 1)^2 - 1} \qquad (14)$$

Now we can see that the solution exists if and only if $b \geq 4 \equiv b_c$. Thus, $b_c = 4$ is the critical value of benefit illustrated in Figure 4 below which it is not profitable for a peer to join the system. For $b = b_c$, the only solution is $d_1^* = d_2^* = 1$. For $b > b_c$, there are two solutions:

$$d_1^* = d_2^* = d_{low}^* < 1$$
$$d_1^* = d_2^* = d_{high}^* > 1 \qquad (15)$$

4.2 N players game

In analog to the equation 11 and 12, we can get best response function for N peer as following:

$$d^* = \sqrt{b(N-1)d^*} - 1 \qquad (16)$$
$$d^* = (\frac{b(N-1)}{2} - 1) \pm \sqrt{(\frac{b(N-1)}{2} - 1)^2 - 1} \qquad (17)$$

So replace $b(N - 1)$ to b, this formula is exactly two peers game.

```
Algorithms: iterative learning model
di = random contribution
While (converge  == false){
   new_di = computeContribution (d, b);
  if (new_di == di) {
       converge = true;
   }
   di = new_di;
}
```

Figure 5: Iterative learning algorithms.

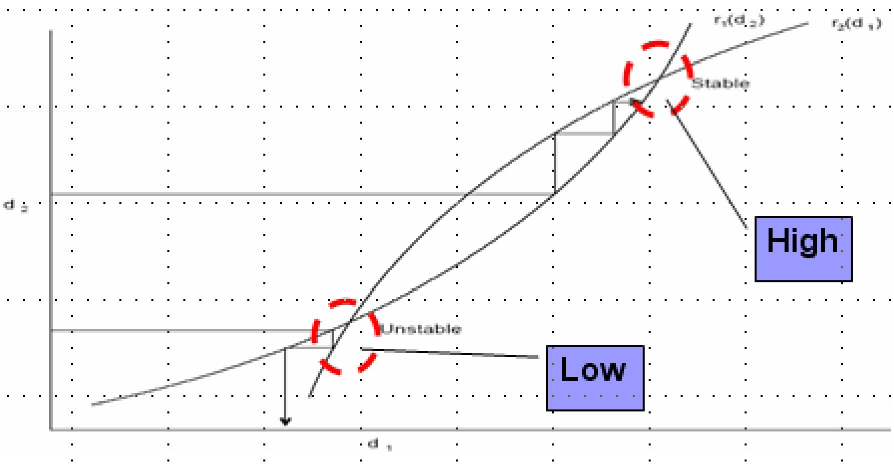

Figure 6: Learning process near the vicinity of the two fixed points.

5 Find Nash equilibrium in Heterogeneous system

Now we start to due with Heterogeneous system, we study Heterogeneous system based on Homogeneous system we study before. We can get fix point equation by analogy of homogeneous system of equation 13.

$$d_i^* = \sqrt{\left[\sum_{j\neq i} b_{ij} d_j^*\right]} - 1 \tag{18}$$

It seems not easy to solve this equation. So we can use iterative learning algorithms to solve this problem.

5.1 Iterative learning algorithms

Let us consider the interaction of peers of P2P system in reality. Any particular peer P_i interacts only with a fraction of all possible peers. These are the peers who have files that are interest to P_i. As it interacts with these peers, P_i learns of the contributions made by them and to maximize its utility adjusts its own contribution. Obviously this contribution that P_i makes is not globally optimal because it is based only on information from a limited set of peers. But after P_i has set its own contributions, this information will be propagated to the peers it interacts with and those peers will adjust their own contribution. In this way the actions of any peer P_i will eventually reach all possible peers. The reaction of the other peers to P_i will again affect P_i and P_i might change his strategy(contribution) once more. In this way, every peer will go through an iterative process of setting its contribution. If and when this process converges, the resulting contributions will constitute a Nash equilibrium.

The iterative learning algorithm showed in figure 5 solves the equation 18. At the beginning, all the peers have some random set of contributions(line 1). In a single iteration of the algorithm, every peer P_i determines the optimal value of d_i that it should contribute given the values of d for other peers and the values of b_{ij}(line 3). At the end of the iteration the peers update their contribution to their new optimal values(line 7). Since now the contributions d_i are all different, the peers need to recompute their optimal values of d_i and a new iteration starts. When this iterative process converges to a stable point(line 5), we reach a Nash equilibrium.

5.2 Stability of Nash equilibrium

The learning process and convergence is showed in Figure 6. Under this learning process, either the peers will quit the game (zero utility) or they will converge to the equilibrium d^*_{high}.

For any starting value of $d_2 > d^*_{low}$(d^*_{low} is the unstable fixed point), the learning process converges to the stable fixed point. If the starting point is too close to the origin, then the iteration moves away from the unstable fixed point and eventually ends up to 0. The fixed point d^*_{high}(d^*_{low}) is locally stable(unstable), i.e. if the two peers start near the fixed point, under iteration of the mappings, they will move closer to (away from) the fixed point.

5.3 Simulation results

Figure 7 shows the equilibrium average contribution by the peers as a function of scaled benefit. The equilibrium contribution increases monotonically with increasing benefit. For average benefit $b_{av} < b_c$, the iterative algorithm converges to $d_i = 0$. From the figure we see that two sets of results for 500 and 1000 peers almost coincide with each other. The simulation results shows the system's behavior is independent of peers' size.

Figure 8 shows convergence speed to Nash equilibrium by different scaled benefit. The two data sets(two in top of the figure, another two in lower of the figure) correspond to different values of average b_{av}. Higher the average value of b_{av}, faster is the convergence to equilibrium. As the value of b_{av} approach the critical value b_c, approach to equilibrium becomes slower and slower. It have been observed that for a wide set of initial conditions for d_i, the process always converges to a unique Nash equilibrium. For very small initial values of d_i, we are close to the unstable Nash equilibrium and the iteration converges to zero, i.e. the contribution by all peers vanish and the system collapses.

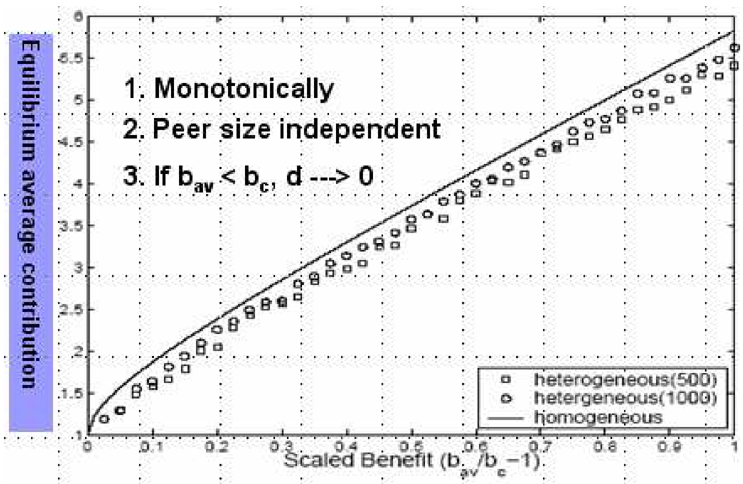

Figure 7: Average contribution at Nash equilibrium.The solid line is the solution from the homogeneous system.

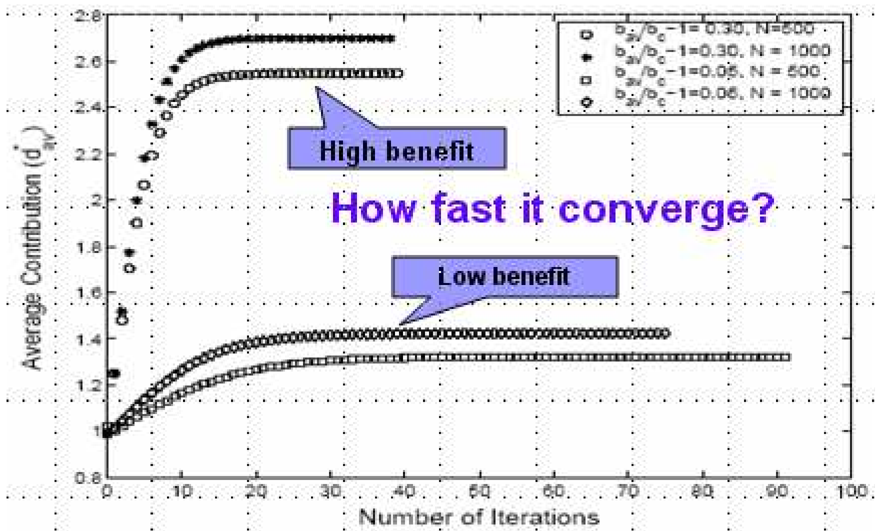

Figure 8: Convergence speed to Nash equilibrium by different scaled benefit.

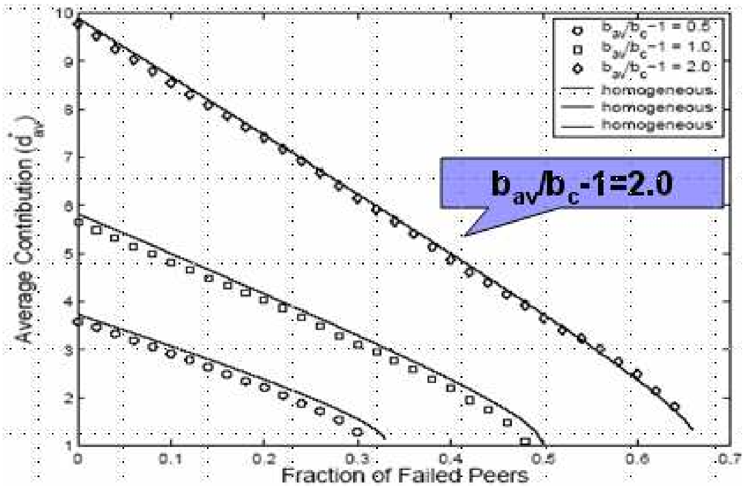

Figure 9: The effect of some peers leaving the system.

Figure 9 shows the effect of some peers leaving the system. If some peers leave the system, the benefit per peer would be reduced. The simulation result confirm this intuition. As the fraction of active peers dwindle, the contribution from each of the peers decrease and at some point, the benefits are too low for the peers and the whole system collapses. The system can be pretty robust for high benefits : for a benefit level of $(b_{av} - b_c)/b_c = 2.0$, the system can survive until 2/3 of the peers leave the system. This shows a big advantage for P2P systems in contrast to traditional distributed systems. As the system grows bigger and bigger, benefits for each peer increases and the system becomes more robust.

6 Summary and Discussion

This paper proposed a differential service model based on Game Theory. P2P system that implements this model can eliminate free riding and provide predictable level of service. Such predictable level of service is the following:

1. System that implements this model will eventually operate on Nash equilibrium.

2. There existing a critical benefit value b_c. When b_i, which is the benefit that a peer P_i can get from the system, is larger than b_c, P_i would like to join the system, then operate at the Nash equilibrium value of contribution. If $b_i < b_c$, the peer is better leaving or not joining the system. When $b_i = b_c$, the peer is indifferent between these two options.

Some current systems restrict download by only enforcing queues and maximum number of possible open connections. To implement the above described incentive model, one can tag a probability $p(d_i)$ for every request from peer P_i as meta data. Whether the other peer will accept this request, depends on this probability value $p(d_i)$.

In this report, we use the general term "contribution". In real implementation, such "contribution" could be measured in terms of uptime and disk space. New user can be given a default value of contribution for some limited point of time so that they can immediately start to use the system at a reasonable level.

How the benefit matrix that described in section 3.2 could be implemented in real P2P system is still an unresolved issue. One simple implementation could measure the number of uploads or disk-space over time provided by peer. But such simple implementation does not discriminate against low bandwidth peers or peers which provide less popular files. Another unresolved issue is that the above model is based on peers being rational and trustworthy. But trust is not easy to enforce in reality. Malicious peers could contribute fake file or peers could fake contribution for themselves. Such unresolved issues are left as open questions for future research.

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