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Criticality-based Analysis and Design of Peer-to-Peer Networks as "Complex Systems"

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1 INTRODUCTION

The core operation in most peer-to-peer systems is to locate the shared objects which are distributed among the nodes efficiently. In unstructured Peer-to-Peer Networks it is quite difficult to rout queries in an efficient way towards the node that holds the desired object. Therefore, in such systems the queries are flooded over the entire network till the node holding the object receives the query and responds. Obviously, this search approach is not scalable. Many different techniques were suggested to address this scalability problem with flooding search, nevertheless no one of them could provide an efficient model that characterizes the unstructured peer-to-peer networks in both accurate and applicable way.

1.1 Motivation

The motivation of this paper was to improve the scalability of flooding search used in unstructured peer-to-peer networks, such like Gnutella.

1.2 Flooding Search

In normal flooding search there is no hints about the right direction for the search, therefore the query is simply flooded overall the network till either the object is found or the TTL value expired. Here is a simple algorithm of flooding search:

- a node looking for an object initiates a query,
- it sets TTL value,
- sends the query to all of its neighbors
- each receiver of the query decrements TTL by one and,
- forwards the query to all of its neighbors in turn
- the search continues till the object is found or TTL = 0

Due to this approach a particular node may receive the same query from different nodes. The first time a node receives a query this is not considered as overhead cost. The main source of overhead cost is duplicated queries. Another important reason for poor scalability is setting the TTL value regardless of the actual size of (actual number of active nodes in) the Peer-to-peer network.

2 PROBABILISTIC FLOODING

2.1 Introduction to Probabilistic flooding

To address those problems it was suggested to replace the normal flooding search with a so-called *probabilistic flooding* search. In Probabilistic flooding search a node forwards the query into the link to one of its neighbors with probability $\boldsymbol{p}$ and drops this query with probability 1- $\boldsymbol{p}$. The normal flooding search is an extreme case of probabilistic flooding with $\boldsymbol{p}$ =1 where the resulting overlaying network covers the whole of the underlying peer-to-peer network with a numerous number of redundant paths between nodes, which represent duplicated queries.
By decreasing p towards 0 the probabilistic flooding cuts many of the redundant paths (and probably some of the essential ones) in the overlaying network.
Decreasing the value of $\boldsymbol{p}$ furthermore results in a large number of nodes being not covered at the end of this search, which in turn yields to low reachability, hence low efficiency.

To find out the ideal case, in which all (or most) of the redundant paths are cut while full reachability is preserved, it is required to compute the optimal value of $\boldsymbol{p}$ that should be implemented in probabilistic flooding algorithm. Obviously, This goal cannot be achieved without a formal and efficient modeling for the unstructured peer-to-peer networks.

2.2 Complex Systems

It was proposed in this paper to recognize the unstructured peer-to-peer networks as "*Complex Systems*" due to the common characteristics between both of them, and to exploit the statistical models applied effectively on Complex Systems to formally model and analyze peer-to-peer systems.

Complex Systems are large-scale, dynamic, self-configure systems such like thermodynamic systems, biological systems, social networks and so on. In this sense, unstructured peer-to-peer networks can be considered as Complex Systems where they are built of a large number of nodes connected randomly and each node can join and leave the network freely without any restrictions.

3 CRITICALITY-BASED ANALYSIS

3.1 Introduction

Some of the global properties or behaviors of the Complex Systems change extremely at a certain point under certain conditions. This phenomenon is known as the phase transition phenomenon. Studying and finding out those *critical* points at which this phenomenon does appear is known as criticality-based analysis. In context of peer-to-peer networks, it is intended to find out the critical optimal probability value p_c at which the probabilistic flooding search works effectively.

3.2 Percolation Theory

One of the most important theories applied on Complex Systems that can help to compute the critical value p_c is "Percolation Theory". To introduce the idea beyond this theory, we consider a 2-D lattice with many dots (known as sites) and lines (known as bonds) between them as shown in **figure 1**.

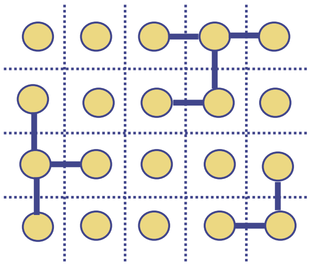

Figure 1

Given that a bond between two sites is open with probability p and closed with probability $(1 - p)$, depending on the particular value of p some clusters of connected sites appear on the lattice. The larger the p, the larger the size of these clusters is. Due to Percolation Theory, above a threshold all clusters are unified and a giant cluster spanning the whole lattice starts to appear **figure 2**.

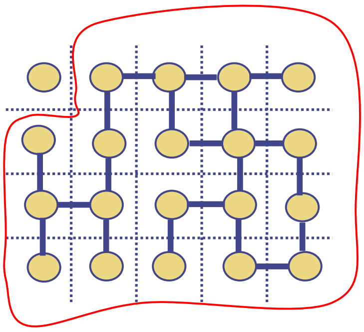

Figure 2

In terms of peer-to-peer networks, nodes and links between them can be thought as sites and bonds respectively. Thus, according to Percolation Theory there should be a threshold p_c above which a giant cluster starts to appear spanning the entire network for first time with minimum number of redundant paths between nodes. It is required to compute this p_c.

3.3 Analysis

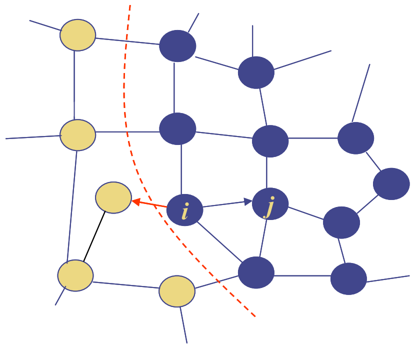

Figure 3

The formal analysis in the paper is based upon the following assumption:
"percolation threshold takes place when each node ***i*** connected to a node ***j*** in the spanning cluster, is also connected to at least one other node. Otherwise the cluster is fragmented".

i.e. the spanning cluster appears for first time when all nodes in the network have an average degree ***k*** of 2.

This can be written as follows:

$$\langle k_i \,|\, i \leftrightarrow j \rangle = 2$$

where $\boldsymbol{k_i}$ is the degree of node *i*, and the angular brackets denote the expected value which can be computed as follows:

$$\langle k_i \,|\, i \leftrightarrow j \rangle = \sum_{k_i} k_i \, P(k_i \,|\, i \leftrightarrow j) = 2 \qquad (1)$$

where $P(k_i \,|\, i \leftrightarrow j)$ is the conditional probability for a node *i* having degree ***k*** given that it is connected to node *j*.

due to Bayes rule,

$$P(k_i \,|\, i \leftrightarrow j) = \frac{P(k_i, i \leftrightarrow j)}{P(i \leftrightarrow j)} = \frac{P(i \leftrightarrow j \,|\, k_i) P(k_i)}{P(i \leftrightarrow j)}$$

where, $P(i \leftrightarrow j) = \dfrac{\langle k \rangle}{N-1}$ and $P(i \leftrightarrow j \,|\, k_i) = \dfrac{k_i}{N-1}$

and **N** is the total number of nodes.

substituting in equation (1) yields that at criticality: $\dfrac{\langle k^2 \rangle}{\langle k \rangle} = 2$ (2)

which is the ratio between the second and first moment of ***k***.

Given the connectivity distribution **P(k)** of the underlying network, the effective connectivity distribution after using probabilistic flooding **P$_e$(k)**, i.e. of the spanning cluster, can be computed as follows:

$$P_e(k) \;=\; \sum_{n=k}^{\infty}\binom{n}{k} p^k (1-p)^{n-k} P(n) \qquad (3)$$

using equation (3), the ratio given in (2) can be computed as follows:

$$\begin{aligned}
\langle k \rangle_e \;&=\; \sum_{k=0}^{\infty} k \sum_{n=k}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} P(n) \\
&= \sum_{n=0}^{\infty} P(n) \sum_{k=0}^{n} k \binom{n}{k} p^k (1-p)^{n-k} \\
&= p \sum_{n=0}^{\infty} nP(n) \\
&= p\,\langle k \rangle \qquad (4)
\end{aligned}$$

and,

$$\begin{aligned}
\langle k^2 \rangle_e \;&=\; \sum_{k=0}^{\infty} k^2 \sum_{n=k}^{\infty} \binom{n}{k} p^k (1-p)^{n-k} P(n) \\
&= \sum_{n=0}^{\infty} P(n) \sum_{k=0}^{n} k^2 \binom{n}{k} p^k (1-p)^{n-k} \\
&= \sum_{n=0}^{\infty} P(n)\,(np(1-p) + n^2 p^2) \\
&= p^2 \langle k^2 \rangle \;+\; p(1-p)\,\langle k \rangle \qquad (5)
\end{aligned}$$

from (4) and (5) the ratio of the second to first moment is:

$$\frac{\langle k^2 \rangle_e}{\langle k \rangle_e} \;=\; p_c \frac{\langle k^2 \rangle}{\langle k \rangle} + (1-p_c) \;=\; 2 \qquad \Rightarrow \qquad p_c \;=\; \frac{1}{\alpha - 1} \qquad (6)$$

where $\alpha = \dfrac{\langle k^2 \rangle}{\langle k \rangle}$ is the ratio of the second to first moment of the actual graph, i.e. the underlying network..

Equation (6) shows that the larger α the smaller the critical value of $\boldsymbol{p}$ needed to be applied in probabilistic flooding, which in turn yields to better results.

This indicates that for those real-world networks having their ratio α less than or equal 2 because of their actual connectivity distribution, it does not make sense to use probabilistic flooding search. Considering the completely unstructured large-scale peer-to-peer networks this ratio is mostly larger than 2.

For instance, the connectivity distribution of Gnutella follows the power-law distribution,

i.e. in the form : $P(k) = Ck^{-\tau} e^{-k/v}$ (7)

where C, $\boldsymbol{\tau}$, and $\boldsymbol{v}$ are constants and,

C : a normalization factor

$e^{-k/v}$: exponential cutoff factor required for real-world networks

the ratio $\boldsymbol{\alpha}$ is computed from equation (7),

$$\Rightarrow \quad \alpha = \frac{\mathrm{Li}_{\tau-2}(e^{-1/v})}{\mathrm{Li}_{\tau-1}(e^{-1/v})}$$

$$\Rightarrow \quad p_c = \frac{1}{\alpha - 1} = \frac{\mathrm{Li}_{\tau-1}(e^{-1/v})}{\mathrm{Li}_{\tau-2}(e^{-1/v}) - \mathrm{Li}_{\tau-1}(e^{-1/v})} \qquad (8)$$

where Li_τ x is the τ-th polylogarithm of x and can be computed as follows:

$$Li_\tau(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^\tau}$$

Equation (8) shows that α, thus $\boldsymbol{p_c}$ are factors of cutoff-index $\boldsymbol{v}$ and $\boldsymbol{\tau}$.

For Gnutella, the power-law exponent $\boldsymbol{\tau}$ is estimated as low as 1.4 and as high as 2.3 in different times and $\boldsymbol{v}$ is in the range of 100 to 1000.

A plot of p_c as a function of v as in figure 4 shows that for Gnutella the value of p_c is mostly below 0.1, which indicates that the communicating cost of probabilistic flooding search can be less than 1% of that with normal flooding without losing reachability, which in turn improves the scalability.

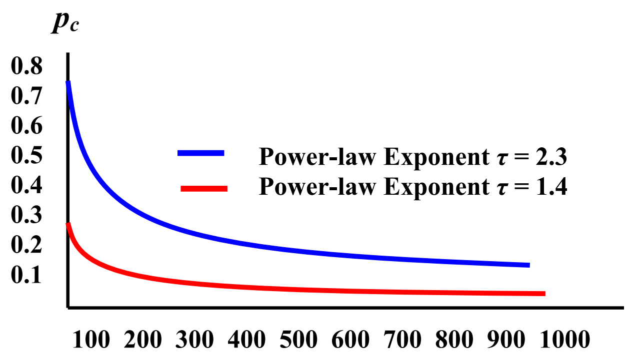

Figure 4

4 TTL SELECTION POLICY

One of the problems with normal flooding discussed earlier is the restriction of TTL value to the value set by the search originator node, which is normally fixed and chosen without taking in account the actual number of active nodes at the beginning of the search. This results in a not-scalable search. To solve this problem it was proposed by the authors of this paper to give each node the opportunity to estimate the appropriate value for TTL based on local information. This is accomplished by exploiting the results provided by Newman which gives a formal manner to estimate the typical length λ of the shortest path between two randomly chosen nodes on any random graph.
This is given by the following equation:

$$\lambda = \frac{\ln\left[(N-1)(z_2 - z_1) + z_1^2\right] - \ln(z_1^2)}{\ln(z_2/z_1)}$$

where,

N the average number of active nodes is not heavily variant in short time-intervals
z_1 number of neighbors which are one hop away
z_2 number of neighbors which are two hops away

Each node can estimate the number of its (active) neighbors in the first and second level at the beginning of its query by sending some local packets with TTL=1 and TTL=2 respectively. These values can be used to compute the value of λ at the start of the search using the equation above. Thus each node can set the value of TTL based on λ. It is obvious that this proposed TTL selection policy improves the scalability of the normal flooding search.

5 CONCLUSIONS

It was shown in this paper that due to the similarities between Complex Systems and unstructured peer-to-peer networks, it is very useful and helpful to exploit the rich theory of criticality in Complex Systems for formal modeling and analyzing of peer-to-peer networks. Some proposed techniques were introduced to improve the scalability of normal flooding search used in unstructured peer-to-peer networks like Gnutella. Criticality based analysis was employed to compute the appropriate value of p for an effective probabilistic flooding.

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