Reference Tables on Probability Distributions and Statistics (1)

Appendix A

Statistical Tables

A.1 Discrete Random Variables

Table 1A. Properties of Some Common Discrete Random Variables[1]

Random Variable	Parameters	$p(\cdot)$
Bernoulli	$0 < p < 1$	$p(k) = p^k q^{1-k}$ $k = 0, 1$
Binomial	n $0 < p < 1$	$p(k) = \binom{n}{k} p^k q^{n-k}$, $k = 0, 1, \ldots, n$
Multinomial	n, r, p_i, k_i $\sum_{i=1}^{r} p_i = 1$ $\sum_{i=1}^{r} k_i = n$,	$p(\overline{k}) = \frac{n!}{k_1! k_2! \cdots k_r!} p_1^{k_1} p_2^{k_2} \cdots p_r^{k_r}$ where $\overline{k} = (k_1, k_2, \ldots, k_r)$

[1] $q = 1 - p$.

Table 1A. (continued)

Random Variable	Parameters	$p(\cdot)$
Hypergeometric	$N > 0$ $n, k \geq 0$	$p(k) = \frac{\binom{r}{k}\binom{N-r}{n-k}}{\binom{N}{n}}$, $k = 0, 1, \ldots, n$, where $k \leq r$ and $n - k \leq N - r$.
Multivariate Hypergeometric	$\sum_{i=1}^{l} r_i = N$	$p(k_1, k_2, \ldots, k_l) = \frac{\binom{r_1}{k_1}\binom{r_2}{k_2} \cdots \binom{r_l}{k_l}}{\binom{N}{n}}$, for $k_i \in \{0, 1, \ldots, n\}$, $k_i \leq r_i \ \forall i$ and $\sum_{i=1}^{l} k_i = n$.
Geometric	$0 < p < 1$	$p(k) = q^k p, \quad k = 0, 1, \ldots.$
Pascal (negative binomial)	$0 < p < 1$ r positive integer	$p(k) = \binom{r+k-1}{k} p^r q^k$, $k = 0, 1, \cdots$
Poisson	$\alpha > 0$	$p(k) = e^{-\alpha} \frac{\alpha^k}{k!}, \quad k = 0, 1, \cdots$

Source: Arnold O. Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, Academic Press, 1990

Reference Tables on Probability Distributions and Statistics (2)

Table 1B. Properties of Some Common Discrete Random Variables[2]

Random Variable	z-transform $g[z]$	$E[X]$	$\mathrm{Var}[X]$
Bernoulli	$q+pz$	p	pq
Binomial	$(q+pz)^n$	np	npq
Multinomial	$(p_1z_1+p_2z_2+\cdots+p_rz_r)^n$	$E[X_i]=np_i$	$\mathrm{Var}[X_i]=np_iq_i$
Hypergeometric	—	$\frac{nr}{N}$	$\frac{nr(N-r)(N-n)}{N^2(N-1)}$
Multivariate Hypergeometric	—	—	—
Geometric	$\frac{p}{1-qz}$	$\frac{q}{p}$	$\frac{q}{p^2}$
Pascal (negative binomial)	$p^r(1-qz)^{-r}$	$\frac{rq}{p}$	$\frac{rq}{p^2}$
Poisson	$e^{\alpha(z-1)}$	α	α

[2] $q_i = 1 - p_i$.

Table 2A. Properties of Some Common Continuous Random Variables

Random Variable	Parameters	Density $f(\cdot)$
Uniform	$a<b$	$\frac{1}{b-a}$, $a \le x \le b$, 0 otherwise
Exponential	$\alpha > 0$	$f(x)=\alpha e^{-\alpha x}$, $x>0$, 0 if $x \le 0$
Gamma	$\beta, \alpha > 0$	$f(x)=\frac{\alpha(\alpha x)^{\beta-1}}{\Gamma(\beta)}e^{-\alpha x}$, $x>0$; 0, $x \le 0$
Erlang-k	$k>0$, $\mu>0$	$f(x)=\frac{\mu k(\mu k x)^{k-1}}{(k-1)!}e^{-\mu k x}$, $x>0$; 0, $x \le 0$
H_k [3]	$q_i, \mu_i > 0$, $\sum_{i=1}^{k}\frac{q_i}{\mu_i}=\frac{1}{\mu}$	$f(x)=\sum_{i=1}^{k} q_i\mu_i e^{-\mu_i x}$, $x>0$; 0, $x \le 0$
Chi-square	$n>0$	$f(x)=\frac{x^{((n/2)-1)}e^{-x/2}}{2^{n/2}\Gamma(n/2)}$, $x>0$, 0 if $x \le 0$
Normal	$\sigma > 0$	$f(x)=\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{1}{2}\frac{(x-\mu)^2}{\sigma^2}\right)$
Student's t	n	$f(x)=\frac{\Gamma[(n+1)/2]}{\sqrt{n\pi}\Gamma(n/2)}\left(1+\frac{x^2}{n}\right)^{-(n+1)/2}$
F	n, m	$f(x)=\frac{(n/m)^{n/2}\Gamma[(n+m)/2]x^{((n/2)-1)}}{\Gamma(n/2)\Gamma(m/2)(1+(n/m)x)^{(n+m)/2}}$, $x>0$

[3] Hyperexponential with k stages.

Source: Arnold O. Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, Academic Press, 1990

Reference Tables on Probability Distributions and Statistics (3)

Table 2B. Properties of Some Common Continuous Random Variables

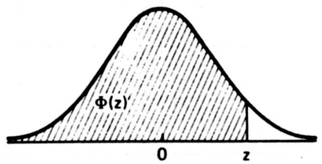
Random Variable	$E[X]$	$\mathrm{Var}[X]$	Laplace–Stieltjes Transform $X^*[\theta]$
Uniform	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{-b\theta} - e^{-a\theta}}{\theta(a-b)}$
Exponential	$\frac{1}{\alpha}$	$\frac{1}{\alpha^2}$	$\frac{\alpha}{\alpha+\theta}$
Gamma	$\frac{\beta}{\alpha}$	$\frac{\beta}{\alpha^2}$	$\left(\frac{\alpha}{\alpha+\theta}\right)^{\beta}$
Erlang-k	$\frac{1}{\mu}$	$\frac{1}{k\mu^2}$	$\left(\frac{k\mu}{k\mu+\theta}\right)^{k}$
H_k [4]	$\frac{1}{\mu}$	$\left(2\sum_{i=1}^{k}\frac{q_i}{\mu_i^2}\right) - \frac{1}{\mu^2}$	$\sum_{i=1}^{k}\frac{q_i\mu_i}{\mu_i+\theta}$
Chi-square	n	$2n$	$\left(\frac{1}{1+2\theta}\right)^{n/2}$
Normal	μ	σ^2	$\exp\left(-\theta\mu - \frac{1}{2}\theta^2\sigma^2\right)$
Student's t	0 for $n > 1$	$\frac{n}{n-2}$ for $n > 2$	does not exist
F	$\frac{m}{m-2}$ if $m > 2$	$\frac{m^2(2n+2m-4)}{n(m-2)^2(m-4)}$ if $m > 4$	does not exist

[4]Hyperexponential with k stages.

A.3 Statistical Tables

Table 3

The Normal Distribution Functions $\Phi(z) = \int_{-\infty}^{z} \frac{e^{-t^2/2}}{\sqrt{2\pi}} dt$



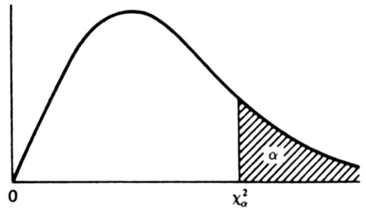
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.50000	.50399	.50798	.51197	.51595	.51994	.52392	.52790	.53188	.53586
0.1	.53983	.54380	.54776	.55172	.55567	.55962	.56356	.56749	.57142	.57535
0.2	.57926	.58317	.58706	.59095	.59483	.59871	.60257	.60642	.61026	.61409
0.3	.61791	.62172	.62552	.62930	.63307	.63683	.64058	.64431	.64803	.65173
0.4	.65542	.65910	.66276	.66640	.67003	.67364	.67724	.68082	.68439	.68793
0.5	.69146	.69497	.69847	.70194	.70540	.70884	.71226	.71566	.71904	.72240
0.6	.72575	.72907	.73237	.73565	.73891	.74215	.74537	.74857	.75175	.75490
0.7	.75804	.76115	.76424	.76730	.77035	.77337	.77637	.77935	.78230	.78524
0.8	.78814	.79103	.79389	.79673	.79955	.80234	.80511	.80785	.81057	.81327
0.9	.81594	.81859	.82121	.82381	.82639	.82894	.83147	.83398	.83646	.83891
1.0	.84134	.84375	.84614	.84849	.85083	.85314	.85543	.85769	.85993	.86214
1.1	.86433	.86650	.86864	.87076	.87286	.87493	.87698	.87900	.88100	.88298
1.2	.88493	.88686	.88877	.89065	.89251	.89435	.89617	.89796	.89973	.90147
1.3	.90320	.90490	.90658	.90824	.90988	.91149	.91308	.91466	.91621	.91774
1.4	.91924	.92073	.92220	.92364	.92507	.92647	.92785	.92922	.93056	.93189
1.5	.93319	.93448	.93574	.93699	.93822	.93943	.94062	.94179	.94295	.94408
1.6	.94520	.94630	.94738	.94845	.94950	.95053	.95154	.95254	.95352	.95449
1.7	.95543	.95637	.95728	.95818	.95907	.95994	.96080	.96164	.96246	.96327
1.8	.96407	.96485	.96562	.96638	.96712	.96784	.96856	.96926	.96995	.97062
1.9	.97128	.97193	.97257	.97320	.97381	.97441	.97500	.97558	.97615	.97670
2.0	.97725	.97778	.97831	.97882	.97932	.97982	.98030	.98077	.98124	.98169
2.1	.98214	.98257	.98300	.98341	.98382	.98422	.98461	.98500	.98537	.98574
2.2	.98610	.98645	.98679	.98713	.98745	.98778	.98809	.98840	.98870	.98899
2.3	.98928	.98956	.98983	.99010	.99036	.99061	.99086	.99111	.99134	.99158
2.4	.99180	.99202	.99224	.99245	.99266	.99286	.99305	.99324	.99343	.99361
2.5	.99379	.99396	.99413	.99430	.99446	.99461	.99477	.99492	.99506	.99520
2.6	.99534	.99547	.99560	.99573	.99585	.99598	.99609	.99621	.99632	.99643
2.7	.99653	.99664	.99674	.99683	.99693	.99702	.99711	.99720	.99728	.99736
2.8	.99744	.99752	.99760	.99767	.99774	.99781	.99788	.99795	.99801	.99807
2.9	.99813	.99819	.99825	.99831	.99836	.99841	.99846	.99851	.99856	.99861
3.0	.99865	.99869	.99874	.99878	.99882	.99886	.99889	.99893	.99896	.99900
3.1	.99903	.99906	.99910	.99913	.99916	.99918	.99921	.99924	.99926	.99929
3.2	.99931	.99934	.99936	.99938	.99940	.99942	.99944	.99946	.99948	.99950
3.3	.99952	.99953	.99955	.99957	.99958	.99960	.99961	.99962	.99964	.99965
3.4	.99966	.99968	.99969	.99970	.99971	.99972	.99973	.99974	.99975	.99976
3.5	.99977	.99978	.99978	.99979	.99980	.99981	.99981	.99982	.99983	.99983
3.6	.99984	.99985	.99985	.99986	.99986	.99987	.99987	.99988	.99988	.99989
3.7	.99989	.99990	.99990	.99990	.99991	.99991	.99992	.99992	.99992	.99992
3.8	.99993	.99993	.99993	.99994	.99994	.99994	.99994	.99995	.99995	.99995

Source: Arnold O. Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, Academic Press, 1990

Reference Tables on Probability Distributions and Statistics (4)

Table 4

Critical Values of the Chi-Square Distribution[a]



n^b \ α	0.995	0.990	0.975	0.950	0.05	0.025	0.010	0.005
1	**0.0^4393^c**	**0.0^3157^c**	**0.0^3982^c**	**0.0^2393^c**	3.8415	5.0239	6.6349	7.8794
2	0.0100	0.0201	0.0506	0.1026	5.9915	7.3778	9.2103	10.597
3	0.0717	0.1148	0.2158	0.3518	7.8147	9.3484	11.345	12.838
4	0.2070	0.2971	0.4844	0.7107	9.4877	11.143	13.277	14.860
5	0.4117	0.5543	0.8312	1.1455	11.071	12.833	15.086	16.750
6	0.6757	0.8721	1.2373	1.6354	12.592	14.449	16.812	18.548
7	0.9893	1.2390	1.6899	2.1674	14.067	16.013	18.475	20.278
8	1.3444	1.6465	2.1797	2.7326	15.507	17.535	20.090	21.955
9	1.7350	2.0879	2.7004	3.3251	16.920	19.023	21.666	23.589
10	2.1559	2.5582	3.2470	3.9403	18.307	20.483	23.209	25.188
11	2.6032	3.0535	3.8158	4.5748	19.675	21.920	24.725	26.757
12	3.0738	3.5706	4.4038	5.2260	21.026	23.337	26.217	28.300
13	3.5650	4.1069	5.0087	5.8919	22.362	24.736	27.688	29.819
14	4.0747	4.6604	5.6287	6.5706	23.685	26.119	29.141	31.319
15	4.6009	5.2294	6.2621	7.2609	24.996	27.488	30.578	32.801
16	5.1422	5.8122	6.9077	7.9616	26.296	28.845	32.000	34.267
17	5.6972	6.4078	7.5642	8.6718	27.587	30.191	33.409	35.719
18	6.2648	7.0149	8.2308	9.3905	28.869	31.526	34.805	37.156
19	6.8440	7.6327	8.9066	10.117	30.144	32.852	36.191	38.582
20	7.4339	8.2604	9.5908	10.851	31.410	34.170	37.566	39.997
21	8.0337	8.8972	10.283	11.591	32.671	35.479	38.932	41.401
22	8.6427	9.5425	10.982	12.338	33.924	36.781	40.289	42.796
23	9.2604	10.196	11.689	13.091	35.173	38.076	41.638	44.181
24	9.8862	10.856	12.401	13.848	36.415	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	37.653	40.647	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	40.113	43.194	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.993
29	13.121	14.257	16.047	17.708	42.557	45.722	49.588	52.336
30	13.787	14.954	16.791	18.493	43.773	46.980	50.892	53.672
40	20.707	22.164	24.433	26.509	55.759	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	67.505	71.420	76.154	79.490
60	35.535	37.485	40.482	43.188	79.082	83.298	88.380	91.952
70	43.275	45.442	48.758	51.739	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.392	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.930	124.342	129.561	135.807	140.169
z_α	−2.5758	−2.3263	−1.9600	−1.6449	+1.6449	+1.9600	+2.3263	+2.5758

[a] Adapted from *Biometrika Tables for Statisticians*, (E. S. Pearson and H. O. Hartley, eds.), Vol. 1, 4th ed. Cambridge University Press, Cambridge, 1966, by permission of Biometrika Trustees.

[b] For $n > 100$ use

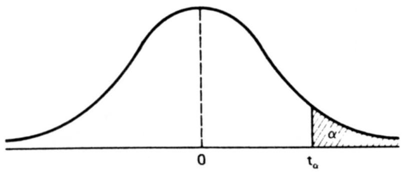
$$\chi_\alpha^2 = n\left\{1 - \frac{2}{9n} + z_\alpha\sqrt{\frac{2}{9n}}\right\}^3$$

where z_α is given on the bottom line of the table.

[c] **The expression 0.0^4393 means 0.0000393, etc.**

Table 5

Critical Values of the Student-t Distribution[a]



n \ α	0.10	0.05	0.025	0.01	0.005
1	3.078	6.314	12.706	31.821	63.657
2	1.886	2.920	4.303	6.965	9.925
3	1.638	2.353	3.182	4.541	5.841
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355
9	1.383	1.833	2.262	2.821	3.250
10	1.372	1.812	2.228	2.764	3.169
11	1.363	1.796	2.201	2.718	3.106
12	1.356	1.782	2.179	2.681	3.055
13	1.350	1.771	2.160	2.650	3.012
14	1.345	1.761	2.145	2.624	2.977
15	1.341	1.753	2.131	2.602	2.947
16	1.337	1.746	2.120	2.583	2.921
17	1.333	1.740	2.110	2.567	2.898
18	1.330	1.734	2.101	2.552	2.878
19	1.328	1.729	2.093	2.539	2.861
20	1.325	1.725	2.086	2.528	2.845
21	1.323	1.721	2.080	2.518	2.831
22	1.321	1.717	2.074	2.508	2.819
23	1.319	1.714	2.069	2.500	2.807
24	1.318	1.711	2.064	2.492	2.797
25	1.316	1.708	2.060	2.485	2.787
26	1.315	1.706	2.056	2.479	2.779
27	1.314	1.703	2.052	2.473	2.771
28	1.313	1.701	2.048	2.467	2.763
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	2.021	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617
∞	1.282	1.645	1.960	2.326	2.576

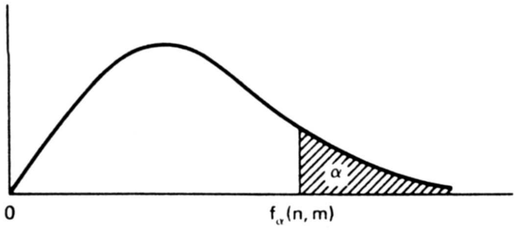
[a] Adapted from *Biometrika Tables for Statisticians* (E. S. Pearson and H. O. Hartley, eds.), Vol. 1, 4th ed. Cambridge University Press, Cambridge, 1966, by permission of Biometrika Trustees.

Source: Arnold O. Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, Academic Press, 1990

Reference Tables on Probability Distributions and Statistics (5)

Table 6

Critical Values of the *F* Distribution*



Denominator m	Numerator n	$f_{0.05}(n, m)$ 1	2	3	4	5	6	7	8	9
1		161.40	199.50	215.70	224.60	230.20	234.00	236.80	238.90	240.50
2		18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.35	19.38
3		10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81
4		7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00
5		6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77
6		5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10
7		5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68
8		5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39
9		5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18
10		4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02
11		4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90
12		4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80
13		4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71
14		4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65
15		4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59
16		4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54
17		4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49
18		4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46
19		4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42
20		4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39
21		4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37
22		4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34
23		4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32
24		4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30
25		4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28
26		4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27
27		4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25
28		4.20	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24
29		4.18	3.33	2.93	2.70	2.55	2.43	2.35	2.28	2.22
30		4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21
40		4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12
60		4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04
120		3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96
∞		3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88

* Adapted from *Biometrika Tables for Statisticians* (E. S. Pearson and H. O. Hartley, eds.), Vol. 1, 4th ed. Cambridge University Press, Cambridge, 1966, by permission of Biometrika Trustees.

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Reference Tables on Probability Distributions and Statistics (6)

A.4 The Laplace–Stieltjes Transform

Table 10. Laplace Transform Properties and Identities[5]

	Function	Transform
1.	$f(t)$	$f^*[\theta] = \int_{0-}^{\infty} e^{-\theta t} f(t)\,dt$
2.	$af(t) + bg(t)$	$af^*[\theta] + bg^*[\theta]$
3.	$f\left(\frac{t}{a}\right), \quad a > 0$	$af^*[a\theta]$
4.	$f(t-a)$ for $t \geq a$	$e^{-a\theta} f^*[\theta]$
5.	$e^{-at} f(t)$	$f^*[\theta + a]$
6.	$tf(t)$	$-\frac{df^*[\theta]}{d\theta}$
7.	$t^n f(t)$	$(-1)^n \frac{d^n f^*[\theta]}{d\theta^n}$
8.	$\int_0^t f(u)g(t-u)\,du$	$f^*[\theta]g^*[\theta]$
9.	$\frac{df(t)}{dt}$	$\theta f^*[\theta] - f(0)$
10.	$\frac{d^n f(t)}{dt^n}$	$\theta^n f^*[\theta] - \sum_{i=1}^{n} \theta^{n-i} f^{(i-1)}(0)$
11.	$\int_0^t f(x)\,dx$	$\frac{f^*[\theta]}{\theta}$
12.	$\frac{\partial f(t)}{\partial a}$ a a parameter	$\frac{\partial f^*[\theta]}{\partial a}$

[5]All functions f are assumed to be piecewise continuous and of exponential order. That is, there exist positive constants M and a such that $|f(t)| \leq Me^{at}$ for $t \geq 0$.

Table 11. Laplace Transform Pairs

	Function	Transform
1.	$f(t)$	$f^*[\theta] = \int_{0-}^{\infty} e^{-\theta t} f(t)\,dt$
2.	$f(t) = c$	$\frac{c}{\theta}$
3.	$t^n, \quad n = 1, 2, 3, \cdots$	$\frac{n!}{\theta^{n+1}}$
4.	$t^a, \quad a > 0$	$\frac{\Gamma(a+1)}{\theta^a + 1}$
5.	e^{at}	$\frac{1}{\theta - a}, \quad \theta > a$
6.	te^{at}	$\frac{1}{(\theta - a)^2}, \quad \theta > a$
7.	$t^n e^{at}$	$\frac{n!}{(\theta - a)^{n+1}}, \quad \theta > a$
8.[6]	$\delta(t)$	1
9.	$\delta(t-a)$	$e^{-a\theta}$
10.[7]	$U(t-a)$	$\frac{e^{-a\theta}}{\theta}$
11.	$f(t-a)U(t-a)$	$e^{-a\theta} f^*[\theta]$

[6]The Dirac delta function $\delta(\cdot)$ is defined by $\delta(t) = 0$ for $t \neq 0$ but $\int_{a-\epsilon}^{a+\epsilon} \delta(t-a) f(t)\,dt = f(a)$ for each f and each $\epsilon > 0$.

[7]The unit step function $U(\cdot)$ is defined by

$$U(t-a) = \begin{cases} 0 & \text{for } t < a \\ 1 & \text{for } t \geq a. \end{cases}$$

Source: Arnold O. Allen, Probability, Statistics, and Queueing Theory with Computer Science Applications, Academic Press, 1990