Geometric Registration for Deformable Shapes

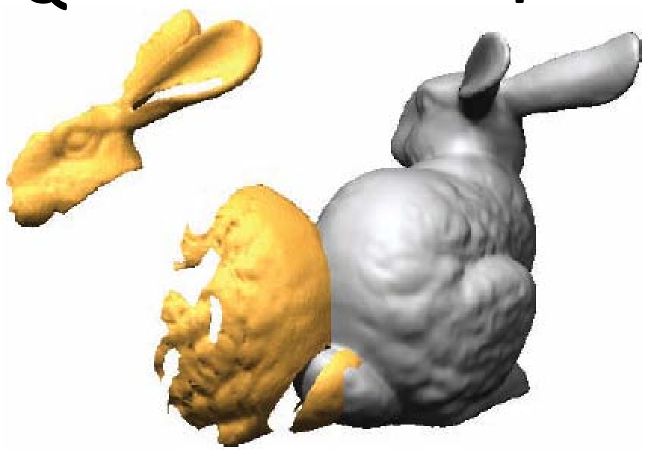
2.1 ICP + Tangent Space optimization for Rigid Motions

31st Annual Conference of the European Association for Computer Graphics
eurographics 2010

Registration Problem

Given

Two point cloud data sets **P** (*model*) and **Q** (*data*) sampled from surfaces $\Phi_{\mathbf{P}}$ and $\Phi_{\mathbf{Q}}$ respectively.



Assume $\Phi_{\mathbf{Q}}$ is a part of $\Phi_{\mathbf{P}}$.

Registration Problem

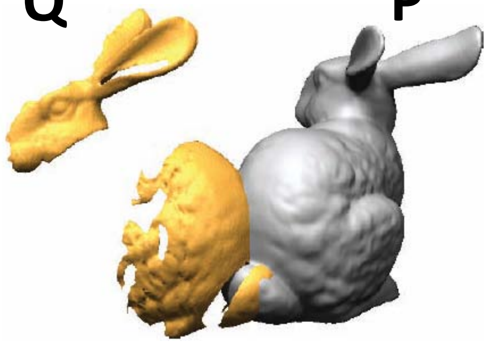
Given

Two point cloud data sets **P** and **Q**.

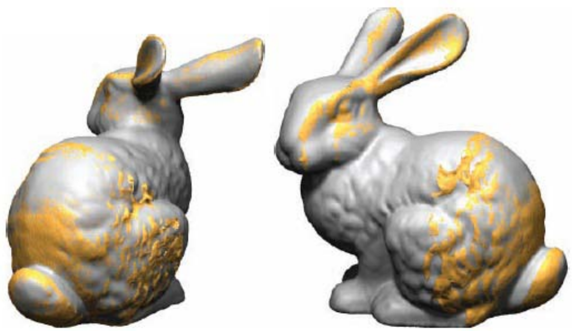
Goal

Register **Q** against **P** by minimizing the squared distance between the underlying surfaces using only *rigid transforms.*

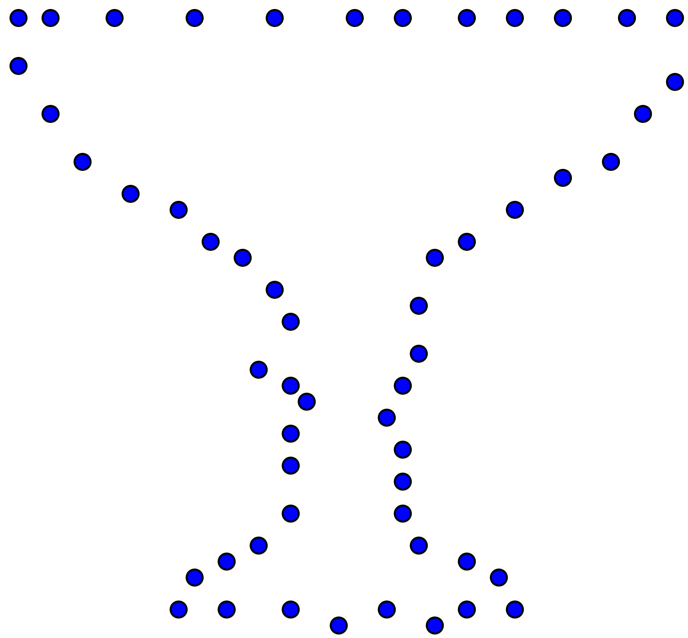
Q **P**



data model



Notations



$$P = \{p_i\}$$

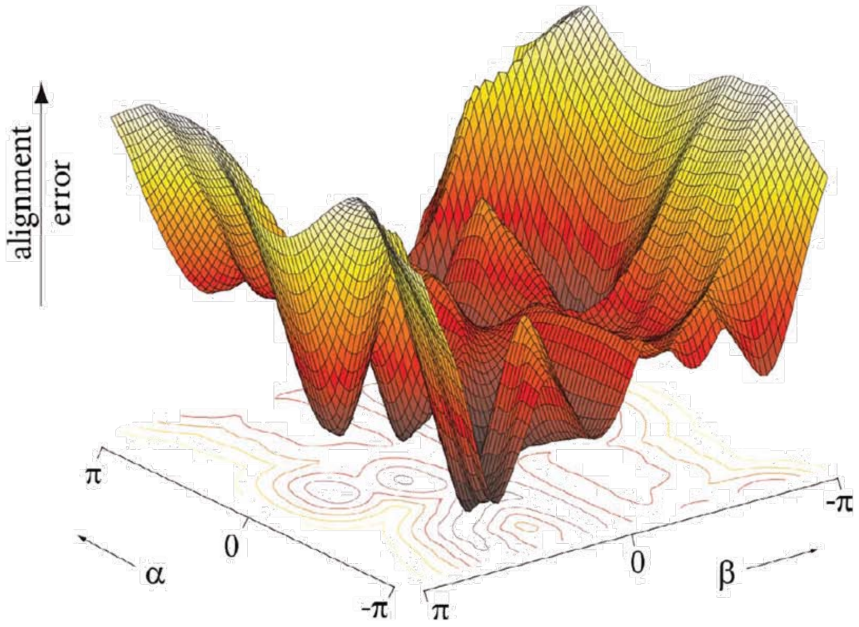
Registration with known Correspondence

$$\{p_i\} \text{ and } \{q_i\} \text{ such that } p_i \rightarrow q_i$$

Registration with known Correspondence

$$\{p_i\} \text{ and } \{q_i\} \text{ such that } p_i \rightarrow q_i$$

$$p_i \rightarrow Rp_i + t \quad \Rightarrow \quad \min_{R,t} \sum_{\mathrm{i}} \left\| Rp_i + t - q_i \right\|^2$$

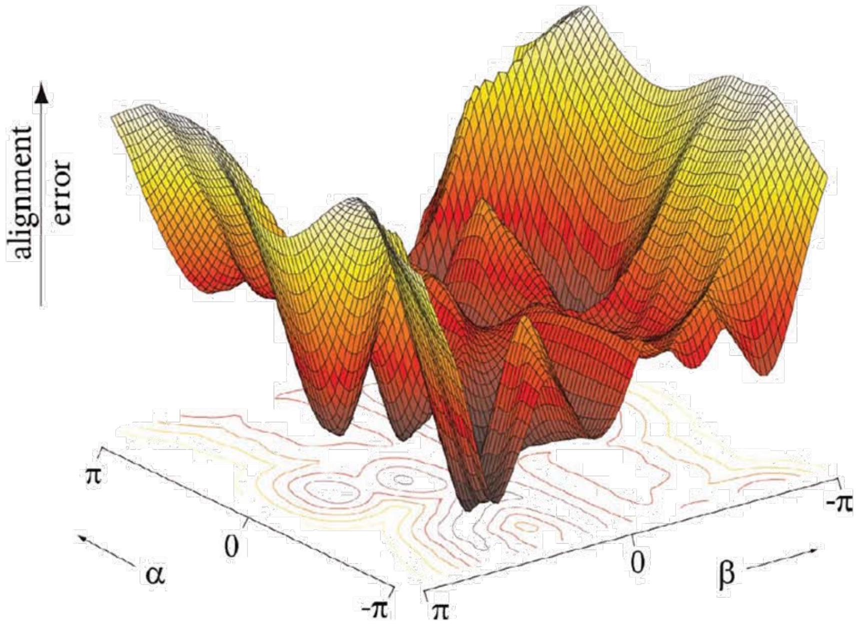


R obtained using SVD of covariance matrix.

Registration with known Correspondence

$$\{p_i\} \text{ and } \{q_i\} \text{ such that } p_i \to q_i$$

$$p_i \to Rp_i + t \;\; \Rightarrow \;\; \min_{R,t} \sum_{\mathrm{i}} \left\| Rp_i + t - q_i \right\|^2$$



R obtained using SVD of covariance matrix.

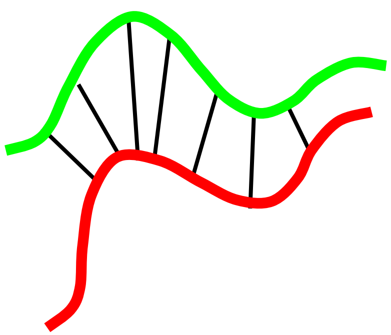
$$t = \overline{\mathrm{q}} - R\overline{p}$$

ICP (Iterated Closest Point)

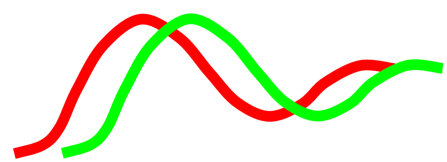
Iterative minimization algorithms (ICP)

[Besl 92, Chen 92]

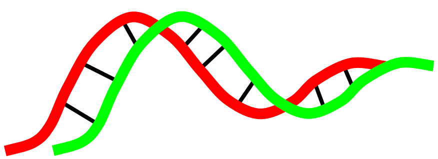
1. Build a set of corresponding points



2. Align corresponding points



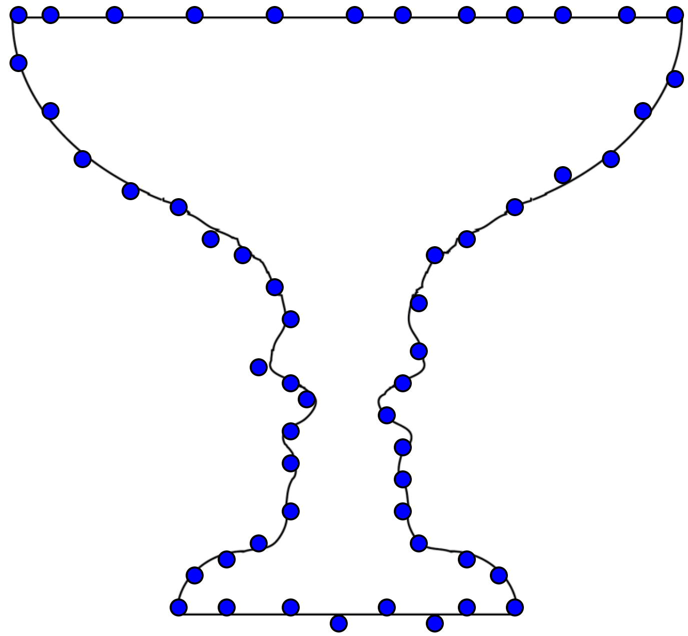
3. Iterate



Properties

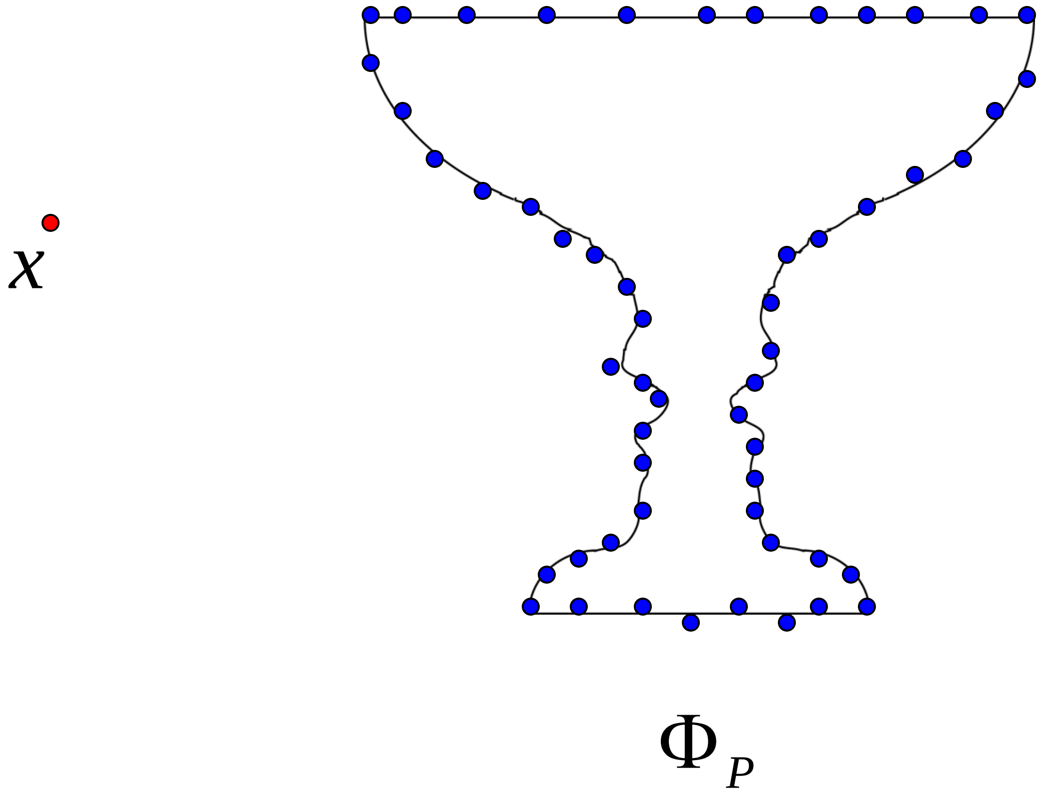
- Dense correspondence sets
- Converges if starting positions are “close”

No (explicit) Correspondence

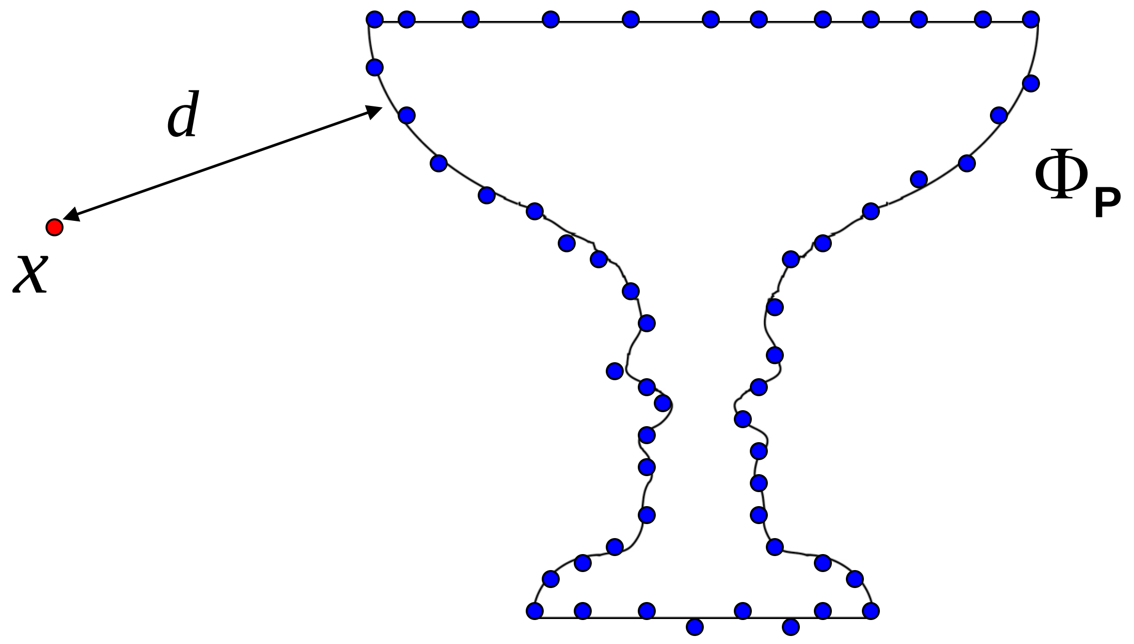


Φ_P

Squared Distance Function (F)



Squared Distance Function (F)



$$F(x, \Phi_P) = d^2$$

Registration Problem

Rigid transform α that takes points $q_i \rightarrow \alpha(q_i)$

Our goal is to solve for,

$$\min_{\alpha} \sum_{q_i \in Q} F(\alpha(q_i), \Phi_P)$$

An optimization problem in the squared distance field of **P**, the model PCD.

Registration Problem

$$\alpha = \text{rotation}\,(R) + \text{translation}(t)$$

Our goal is to solve for,

$$\min_{R,t} \sum_{q_i \in Q} F(Rq_i + t, \Phi_P)$$

Optimize for **R** and **t**.

Registration in 2D

- Minimize residual error $\varepsilon(\theta, \mathrm{t_x}, \mathrm{t_y})$

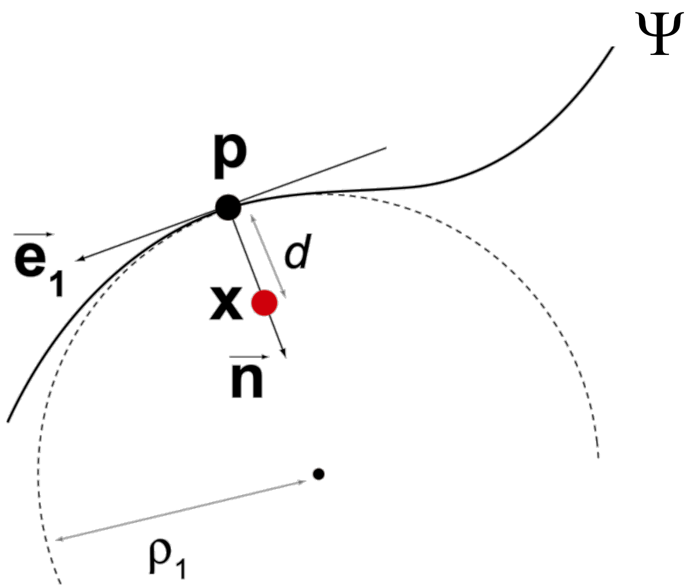
$$\begin{bmatrix} & & \\ & \mathbf{M}_1 & \\ & & \end{bmatrix} \begin{bmatrix} \theta \\ \mathrm{t_x} \\ \mathrm{t_y} \end{bmatrix} = \begin{bmatrix} \\ \mathbf{M}_2 \\ \\ \end{bmatrix}$$

depends on $\mathbf{F}^+$

data PCD (**Q**).

Approximate Squared Distance

For a curve Ψ,



$$\mathbf{F}(\mathbf{x},\ \Psi) = \frac{d}{d-\rho_1} x_1^2 + x_2^2 = \delta_1 x_1^2 + x_2^2$$

[Pottmann and Hofer 2003]

ICP in Our Framework

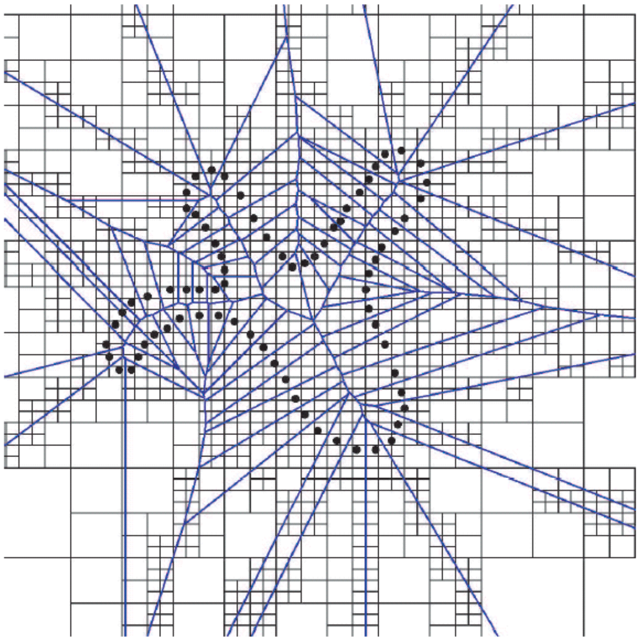
- Point-to-point ICP (good for large *d*)

$$\mathrm{F}(\mathbf{x}, \Phi_{\mathbf{P}}) = (\mathbf{x} - \mathbf{p})^2 \quad \Rightarrow \quad \delta_{\mathrm{j}} = 1$$

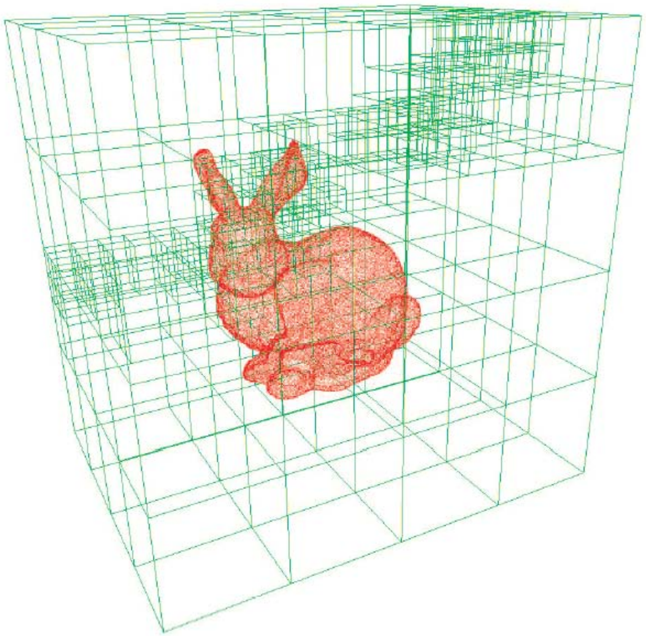
- Point-to-plane ICP (good for small *d*)

$$\mathrm{F}(\mathbf{x}, \Phi_{\mathbf{P}}) = (\vec{\mathrm{n}} \cdot (\mathbf{x} - \mathbf{p}))^2 \quad \Rightarrow \quad \delta_{\mathrm{j}} = 0$$

Example d2trees

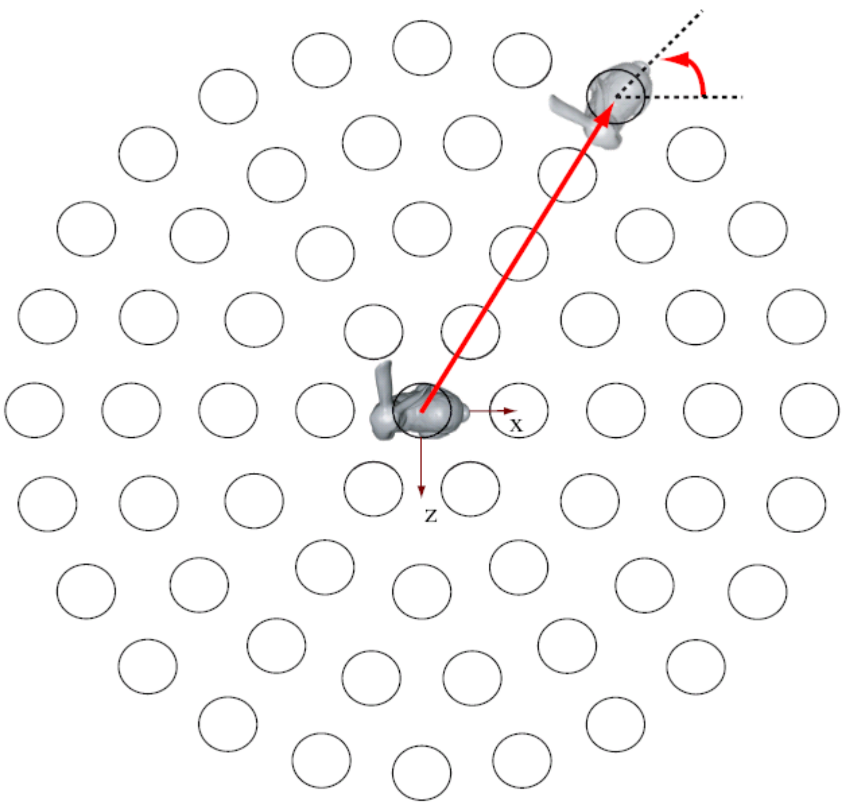


2D

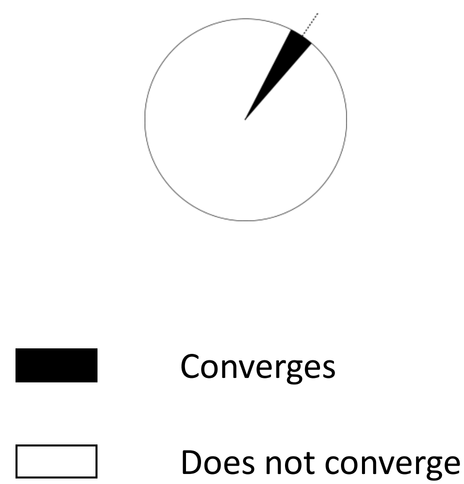


3D

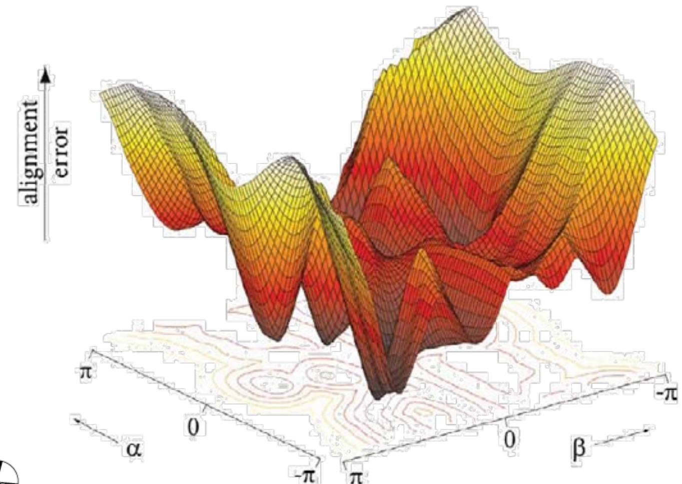
Convergence Funnel

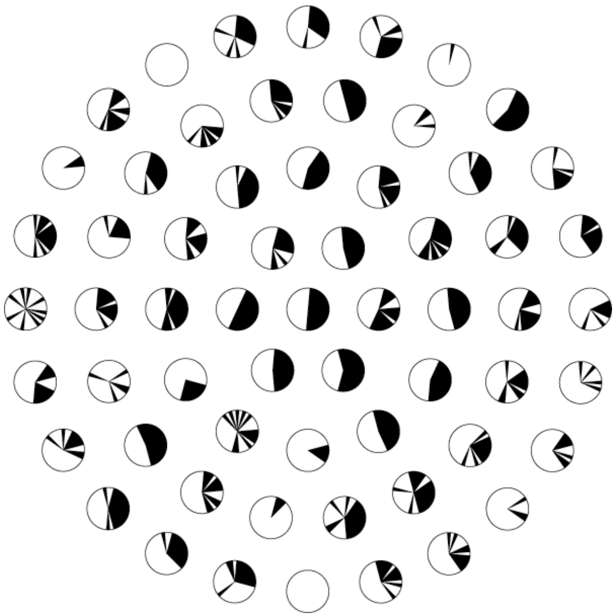


Translation in x-z plane.
Rotation about y-axis.

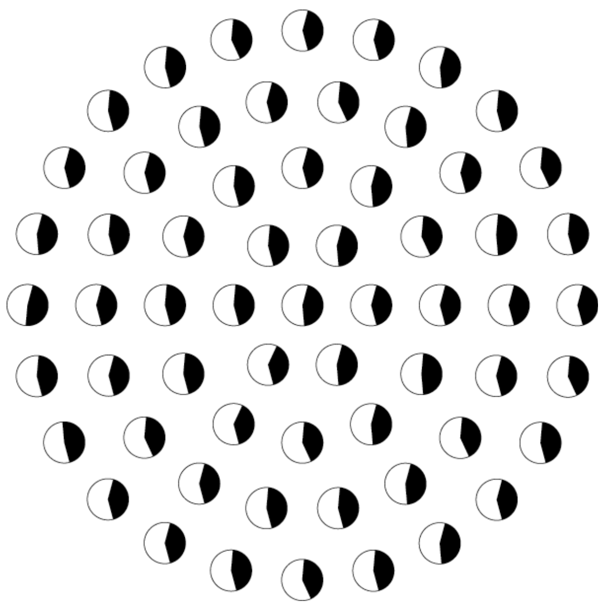


Convergence Funnel





Plane-to-plane ICP



distance-field formulation

Descriptors

$$P = \{ p_i \}$$

- closest point $\rightarrow$ based on Euclidean distance

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$$P = \{p_i\}$$

- closest point $\rightarrow$ based on Euclidean distance

$$P = \{p_i, a_i, b_i, \ldots\}$$

- closest point $\rightarrow$ based on Euclidean distance between point + descriptors (attributes)

(Invariant) Descriptors

$$P = \{p_i\}$$

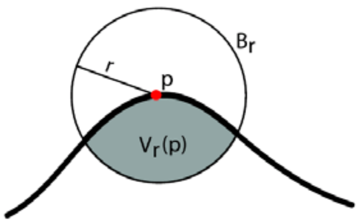
• closest point $\rightarrow$ based on Euclidean distance

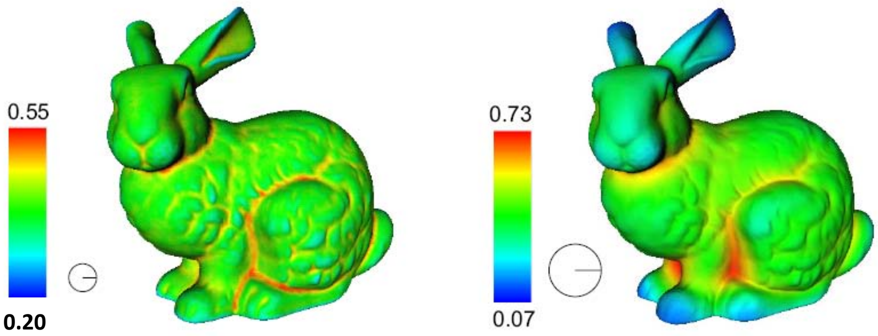
$$P = \{p_i, a_i, b_i, \ldots\}$$

• closest point $\rightarrow$ based on Euclidean distance between point + descriptors (attributes)

Integral Volume Descriptor

$$V_r(p) = \int_{B_r(p)\cap S} dx$$





Relation to mean curvature

$$V_r(\mathbf{p}) = \frac{2\pi}{3}r^3 - \frac{\pi H}{4}r^4 + O(r^5)$$

When Objects are *Poorly* Aligned

- **Use descriptors for global registrations**

global alignment → refinement with local (e.g., ICP)