Geometric Registration for Deformable Shapes

2.2 Deformable Registration

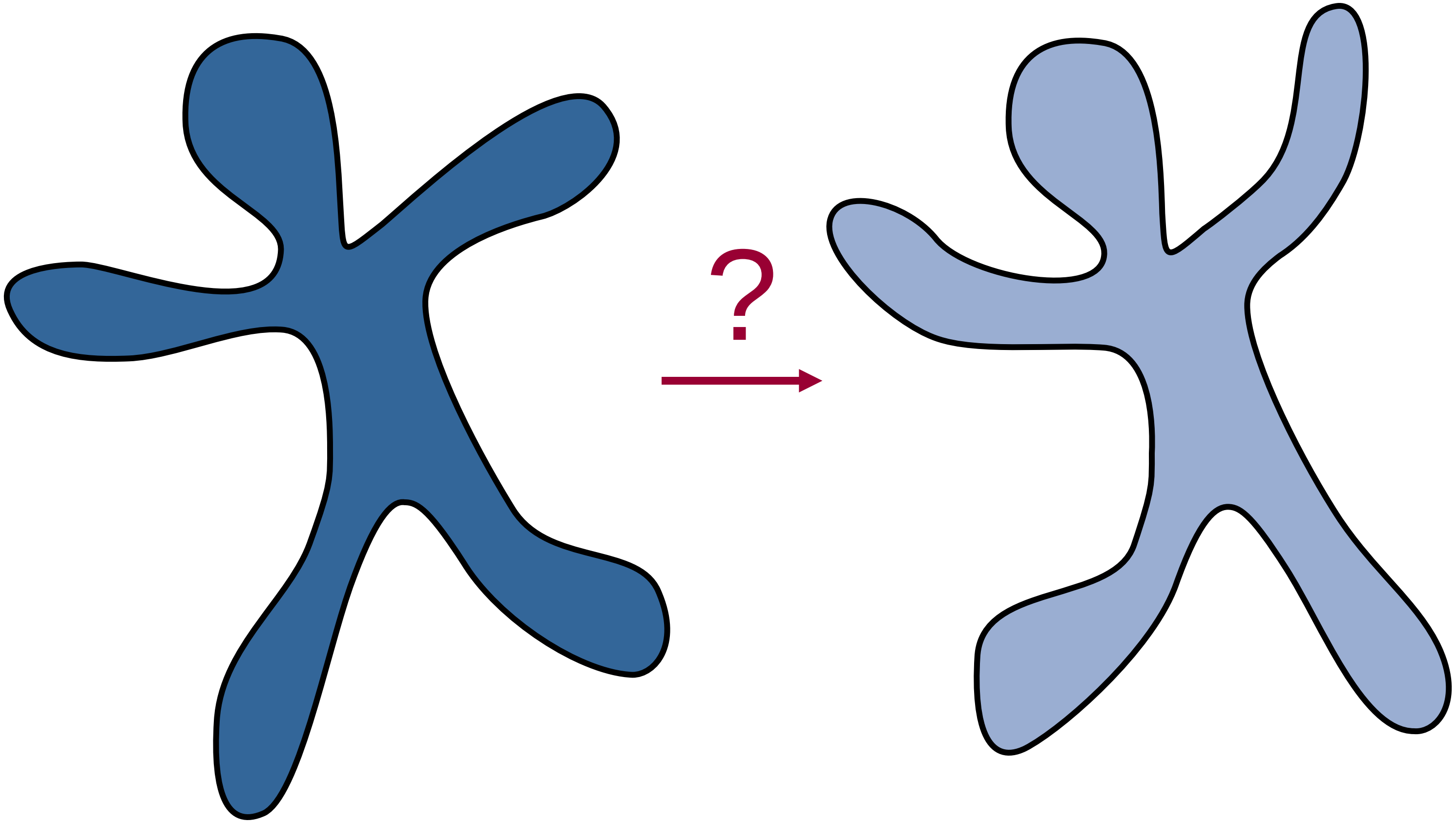
Variational Model · Deformable ICP

31st Annual Conference of the European Association for Computer Graphics
eurographics 2010

Variational Model

What is deformable shape matching?

Example



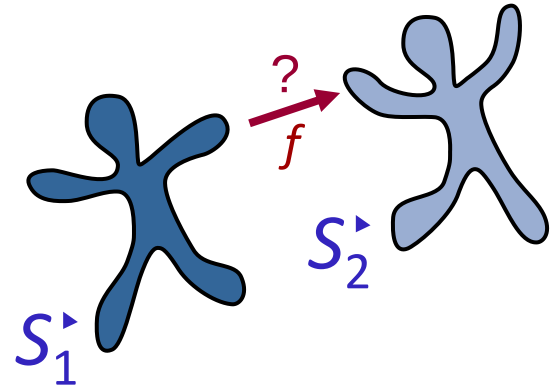
What are the Correspondences?

What are we looking for?

Problem Statement:

Given:

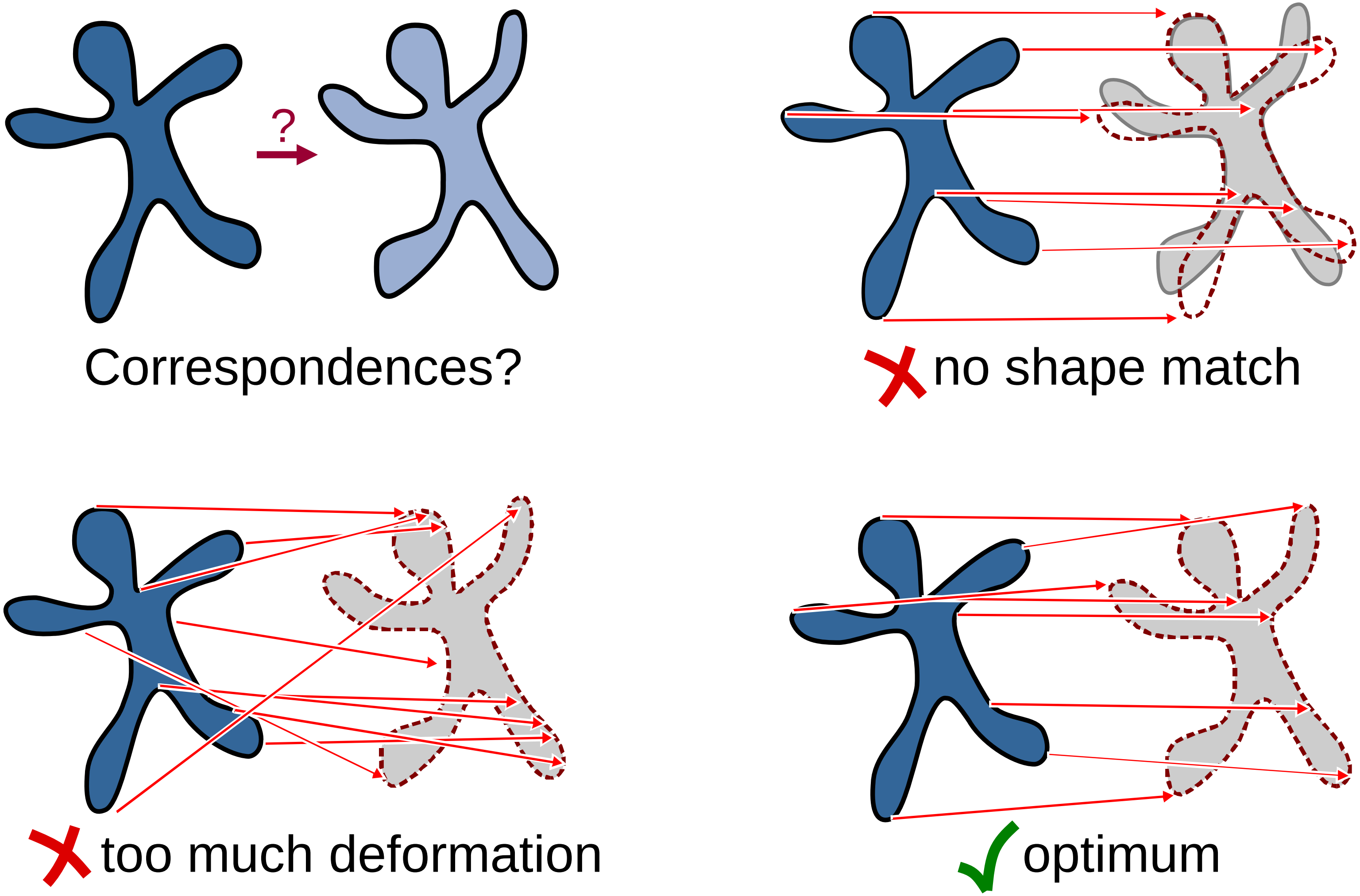
- Two surfaces $S_1, S_2 \subseteq \mathbb{R}^3$



We are looking for:

- A *reasonable* deformation function $f: S_1 \rightarrow \mathbb{R}^3$ that brings S_1 close to S_2

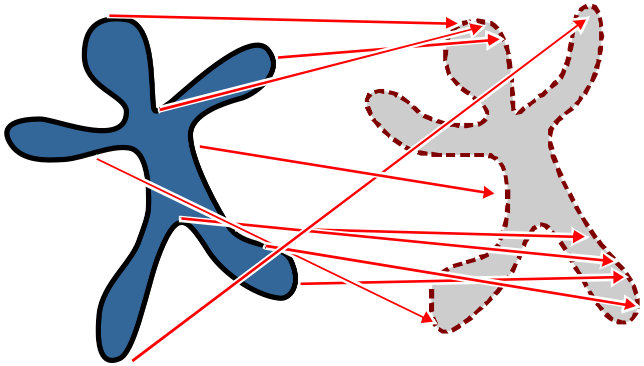
Example


?
Correspondences?
no shape match
too much deformation
optimum

This is a Trade-Off

Deformable Shape Matching is a Trade-Off:

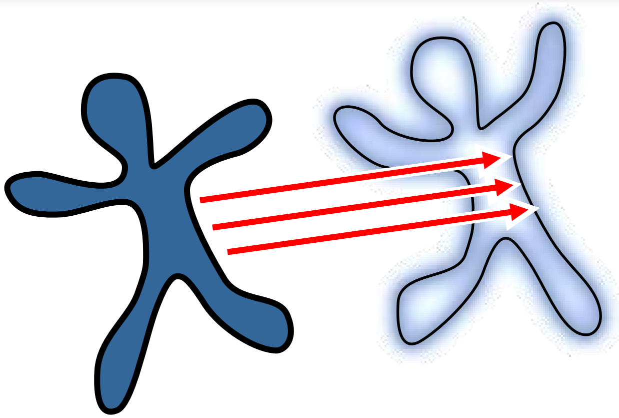
- We can match any two shapes using a weird deformation field



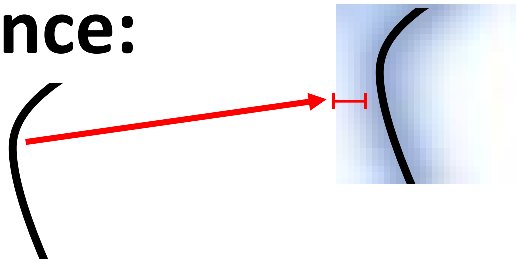
- We need to trade-off:
  - Shape matching (close to data)
  - Regularity of the deformation field (reasonable match)

Variational Model

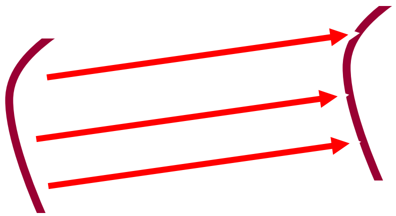
Components:



Matching Distance:



Deformation / rigidity:

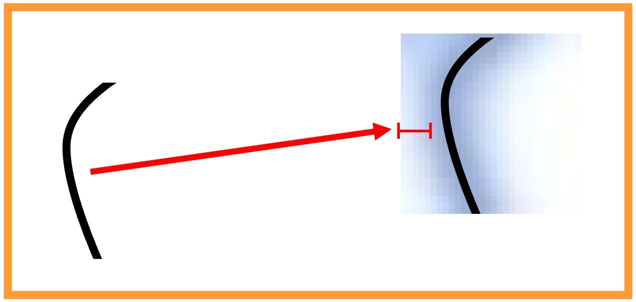


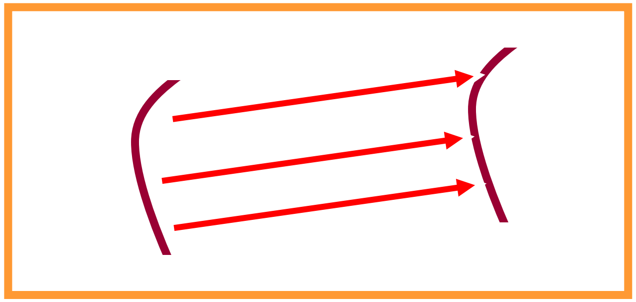
Variational Model

Variational Problem:

- Formulate as an energy minimization problem:

$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$



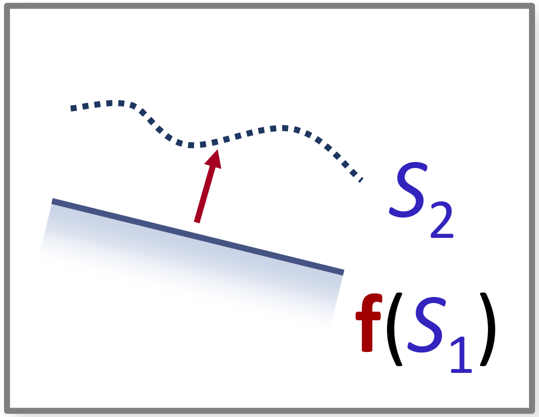


Part 1: Shape Matching

Assume:

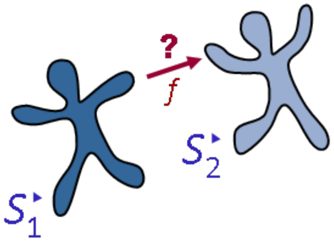
- Objective Function:

$$E^{(match)}(f) = dist\left(f_{1,2}(S_1), S_2\right)$$



- Example: least squares distance

$$E^{(match)}(f) = \int_{x_1 \in S_1} dist(\mathbf{x}_1, S_2)^2 \, d\mathbf{x}_1$$



- Other distance measures:
  Hausdorf distance, L_p-distances, etc.
- L_2 measure is frequently used (models Gaussian noise)

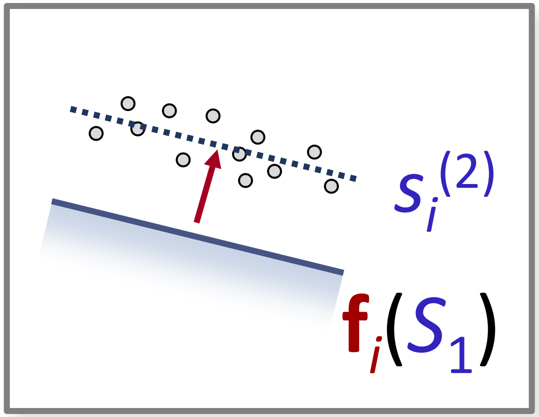
Point Cloud Matching

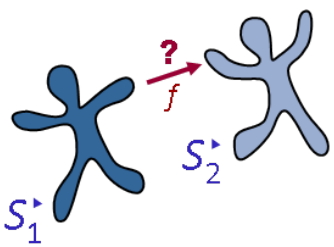
Implementation example: Scan matching

- Given: S_1, S_2 as point clouds
  - $S_1 = \{\mathbf{s}_1^{(1)}, \ldots, \mathbf{s}_n^{(1)}\}$
  - $S_2 = \{\mathbf{s}_1^{(2)}, \ldots, \mathbf{s}_m^{(2)}\}$
- Energy function:

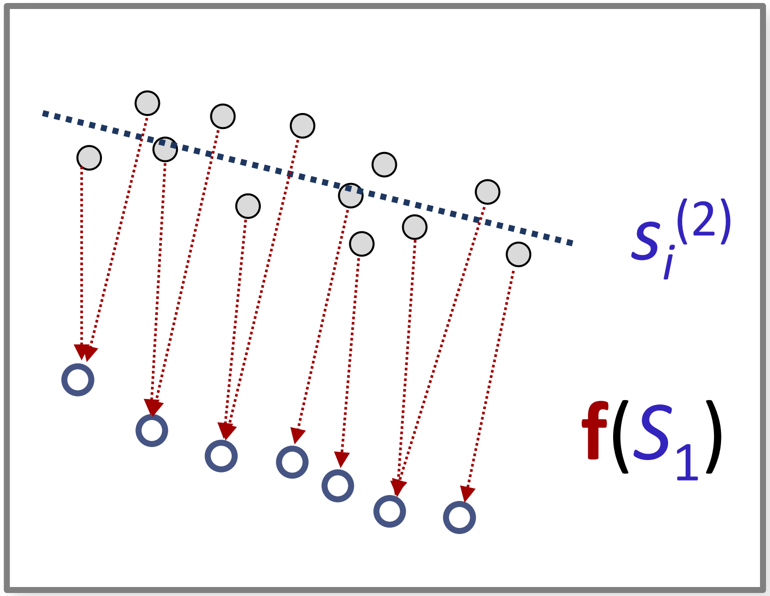
$$E^{(match)}(f) = \frac{|S_1|}{m} \sum_{i=1}^{m} dist\left(S_1, \mathbf{s}_i^{(2)}\right)^2$$

- How to measure $dist\left(S_1, \mathbf{x}\right)$?
  - Estimate distance to a point sampled surface





Surface approximation

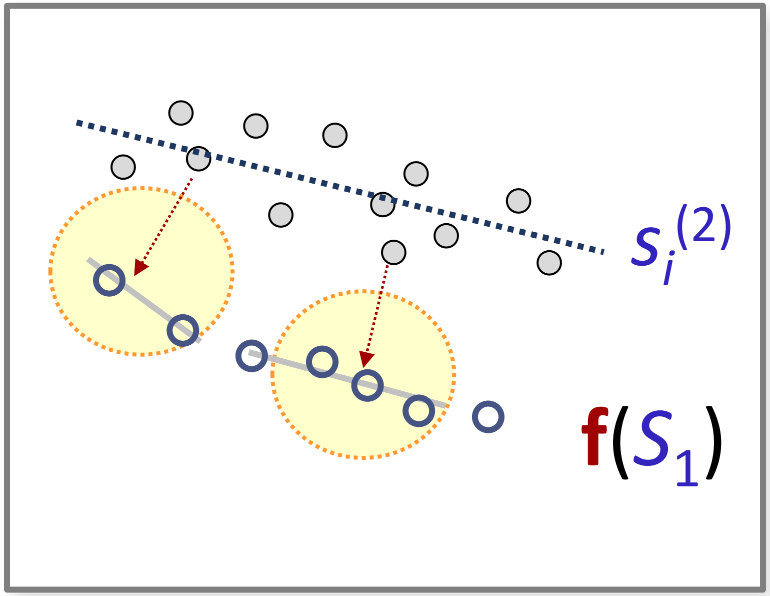


Solution #1: Closest point matching

- "Point-to-point" energy

$$E^{(match)}(f) = \frac{|S_1|}{m} \sum_{i=1}^{m} dist\left(s_i^{(2)}, NN_{in\ S_1}(s_i^{(2)})\right)^2$$

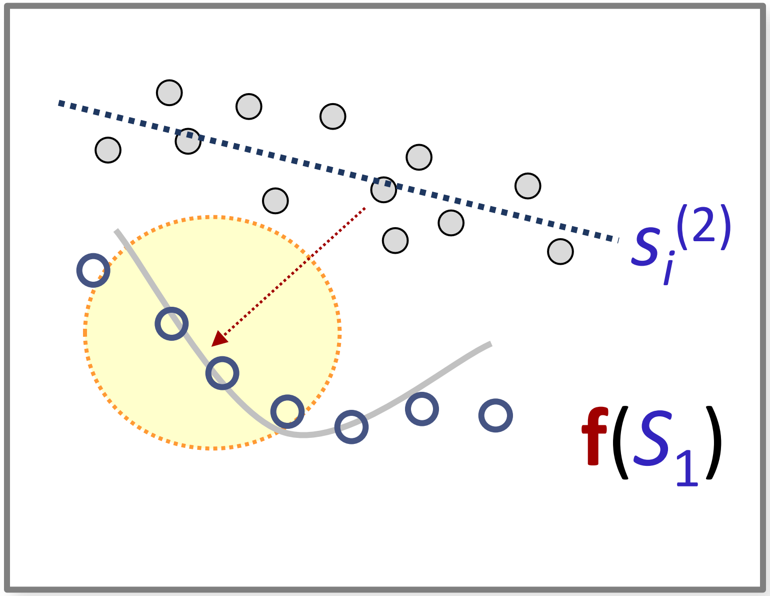
Surface approximation



Solution #2: Linear approximation

- "Point-to-plane" energy
- Fit plane to *k*-nearest neighbors
- *k* proportional to noise level, typically $k \approx 6 \dots 20$

Surface approximation



Solution #3: Higher order approximation

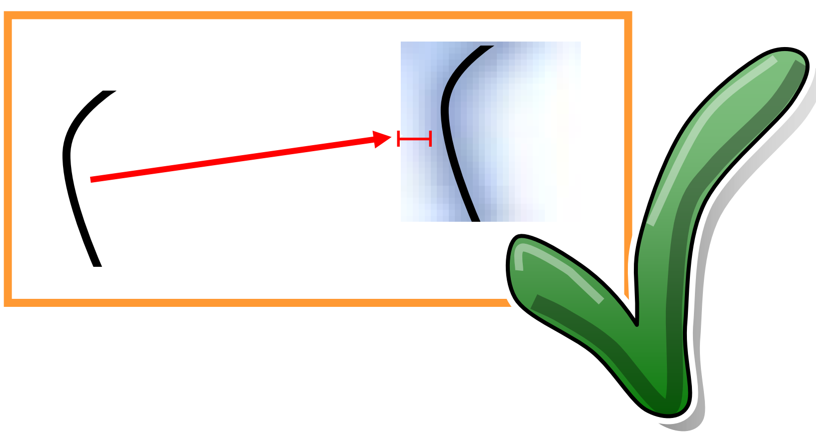
- Higher order fitting (e.g. quadratic)
  - Moving least squares

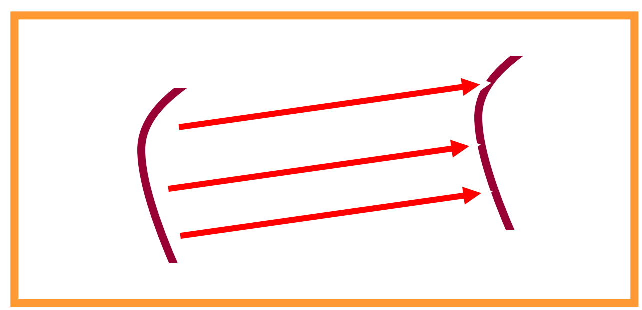
Variational Model

Variational Problem:

- Formulate as an energy minimization problem:

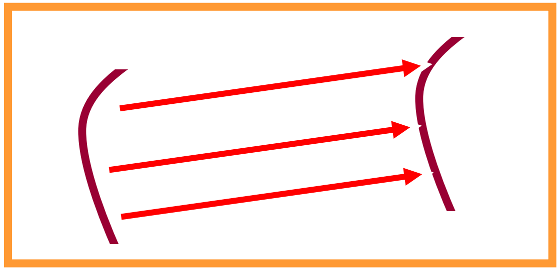
$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$





Part II: Deformation Model

$$E^{(regularizer)}(f)$$



What is a "nice" deformation field?

- Isometric "elastic" energies
  - Extrinsic ("volumetric deformation")
  - Intrinsic ("as-isometric-as possible embedding")
- Thin shell model
  - Preserves shape (metric *plus curvature*)
- Thin-plate splines
  - Allowing strong deformations, but keep shape

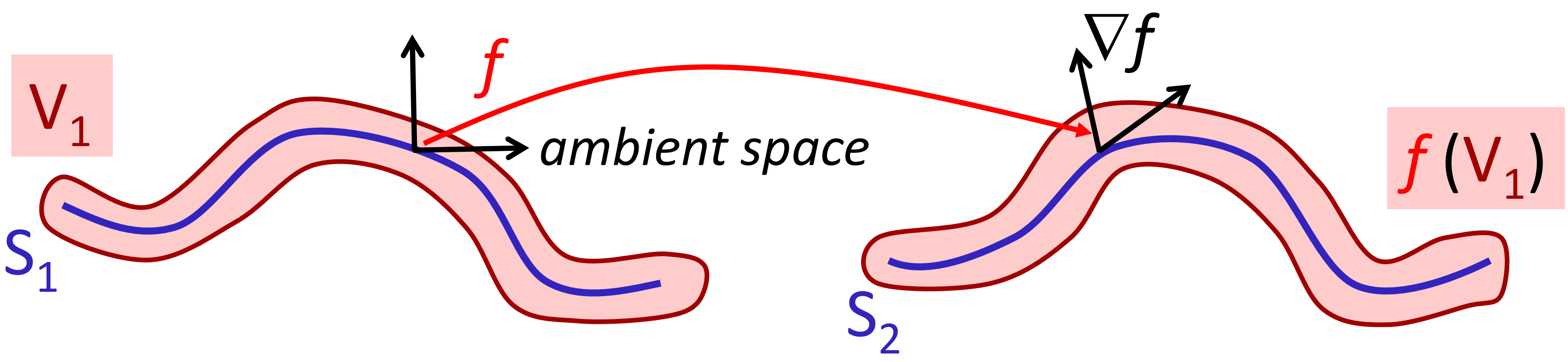
Elastic Volume Model

Extrinsic Volumetric "As-Rigid-As Possible"

- Embed source surface S_1 in volume
- f should preserve 3×3 metric tensor (least squares)

$$E^{(regularizer)}(f) = \int_{V_1} \left[\nabla f \, \nabla f^{\mathrm{T}} - \mathbf{I}\right]^2 dx$$

first fundamental form $\mathbf{I}$ ($\mathbb{R}^{3\times3}$)



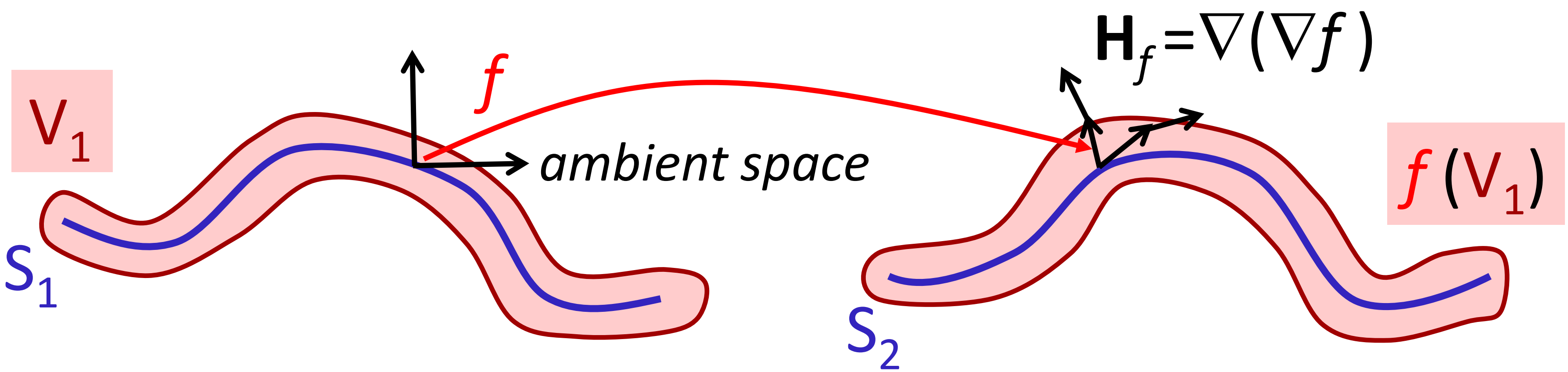
Volume Model

Variant: Thin-Plate-Splines

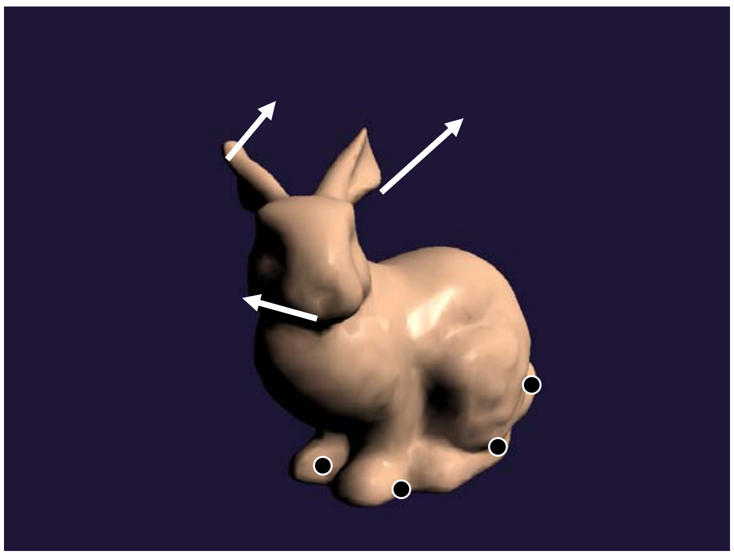
- Use regularizer that penalizes curved deformation

$$E^{(regularizer)}(f) = \int_{V_1} H_f(x)^2 \, dx$$

second derivative ($\mathbb{R}^{3\times3}$)

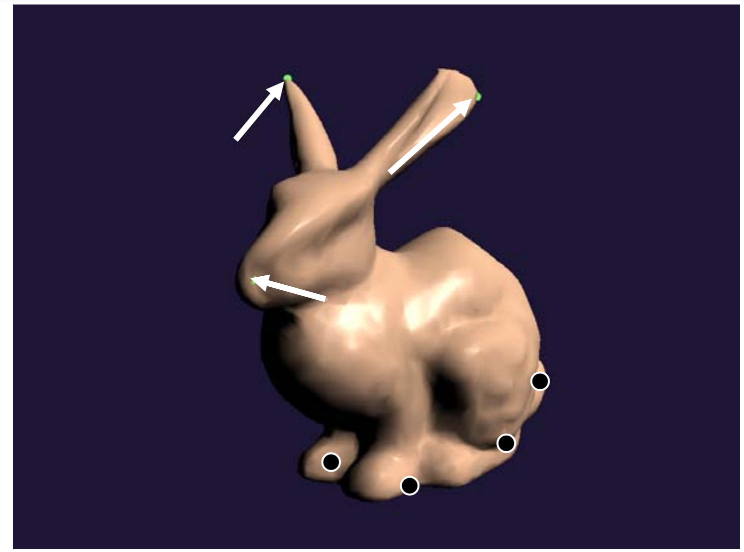


How does the deformation look like?

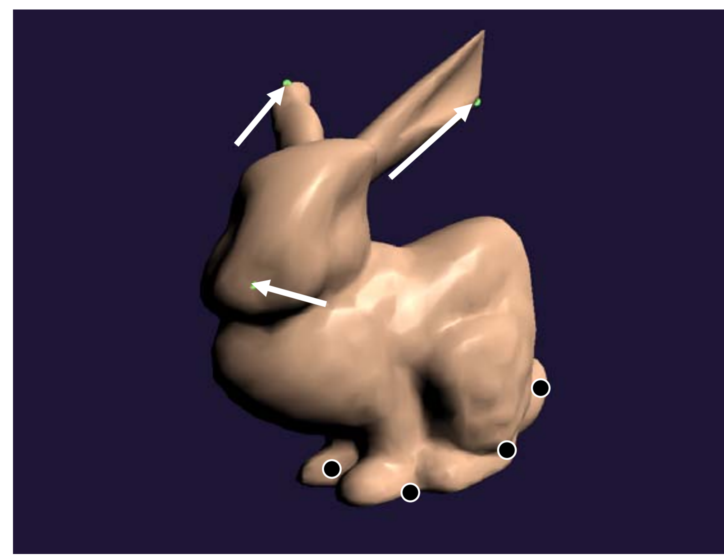


original

as-rigid-as possible volume



thin plate splines



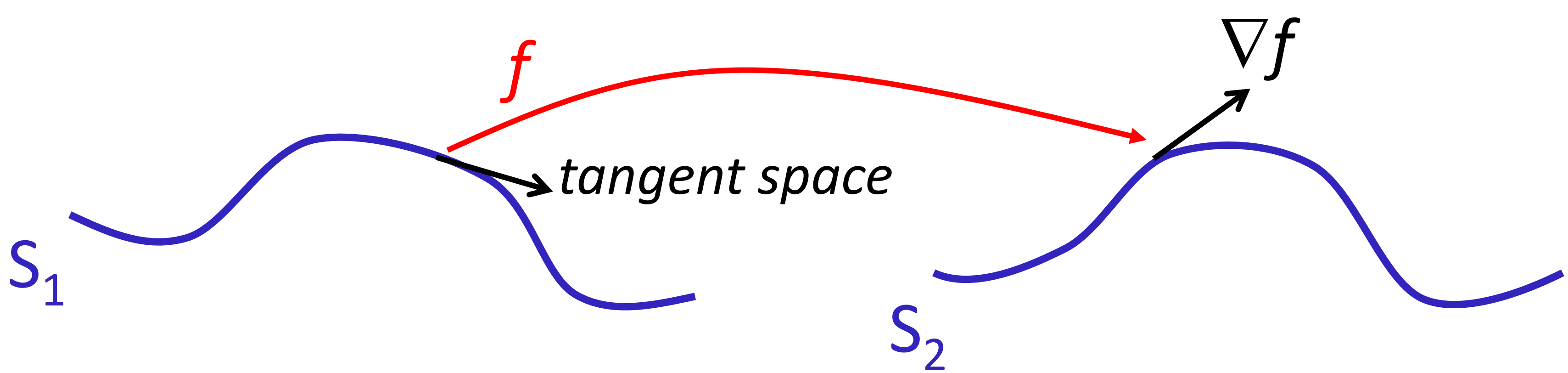
Isometric Regularizer

Intrinsic Matching (2-Manifold)

- Target shape is given and *complete*
- Isometric embedding

$$E^{(regularizer)}(f) = \int_{S_1} \left[\nabla f \nabla f^{\mathrm{T}} - \mathbf{I} \right]^2 dx$$

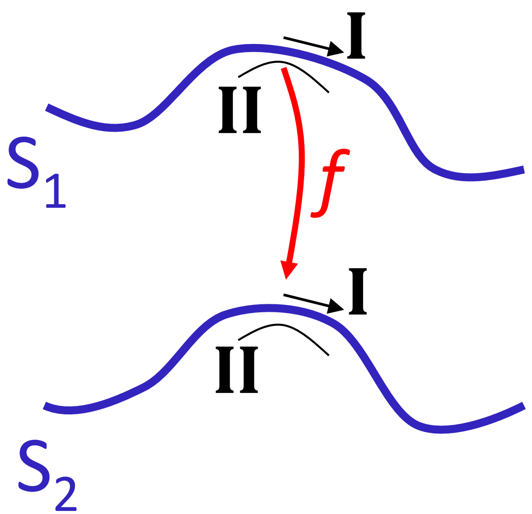
first fund. form (S_1, intrinsic)



Elastic "Thin Shell" Regularizer

"Thin Shell" Energy

- Differential geometry point of view
  - Preserve first fundamental form **I**
  - Preserve second fundamental form **II**
  - …in a least least squares sense
- Complicated to implement
- Usually approximated
  - Volumetric shells (as shown before)
  - Other approximation (next slide)



Example Implementation

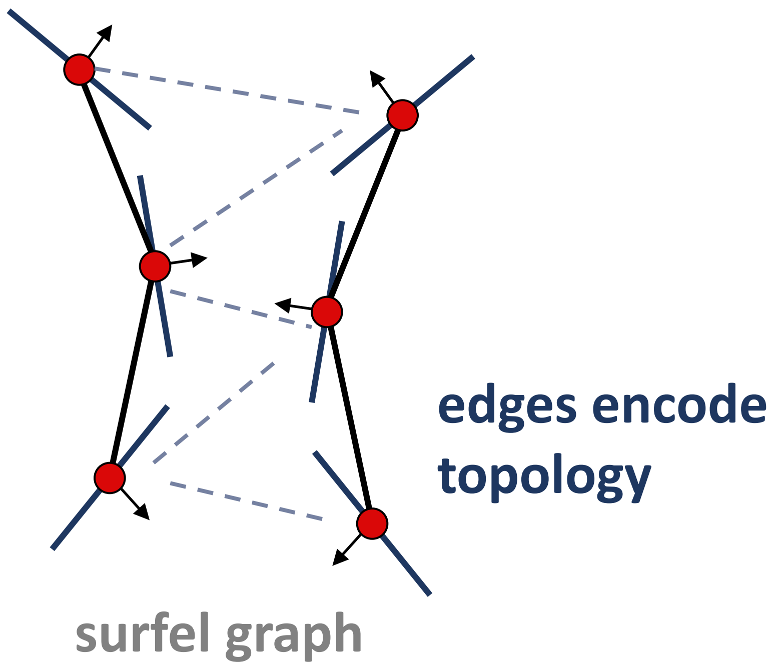
Example: approximate thin shell model

- Keep locally rigid
  - Will preserve metric & curvature implicitly
- Idea
  - Associate local *rigid* transformation with surface points
  - Keep as similar as possible
  - Optimize simultaneously with deformed surface
- Transformation is *implicitly defined* by deformed surface (*and vice versa*)

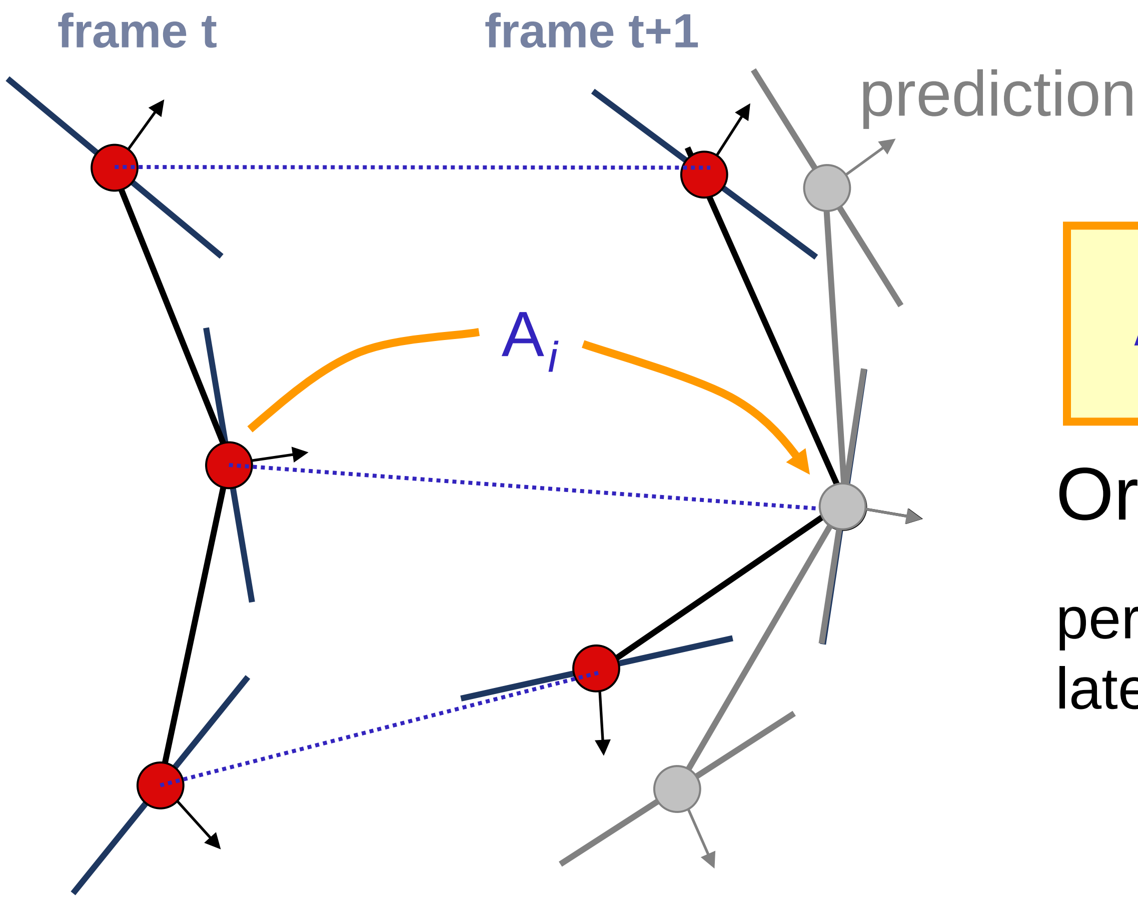
Parameterization

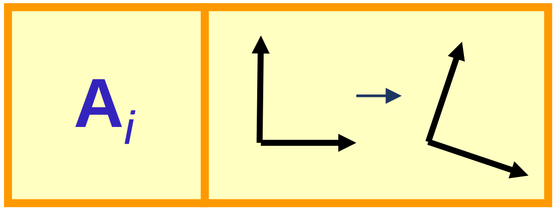
Parameterization of S_1

- Surfel graph
- This could be a mesh, but does not need to



Deformation

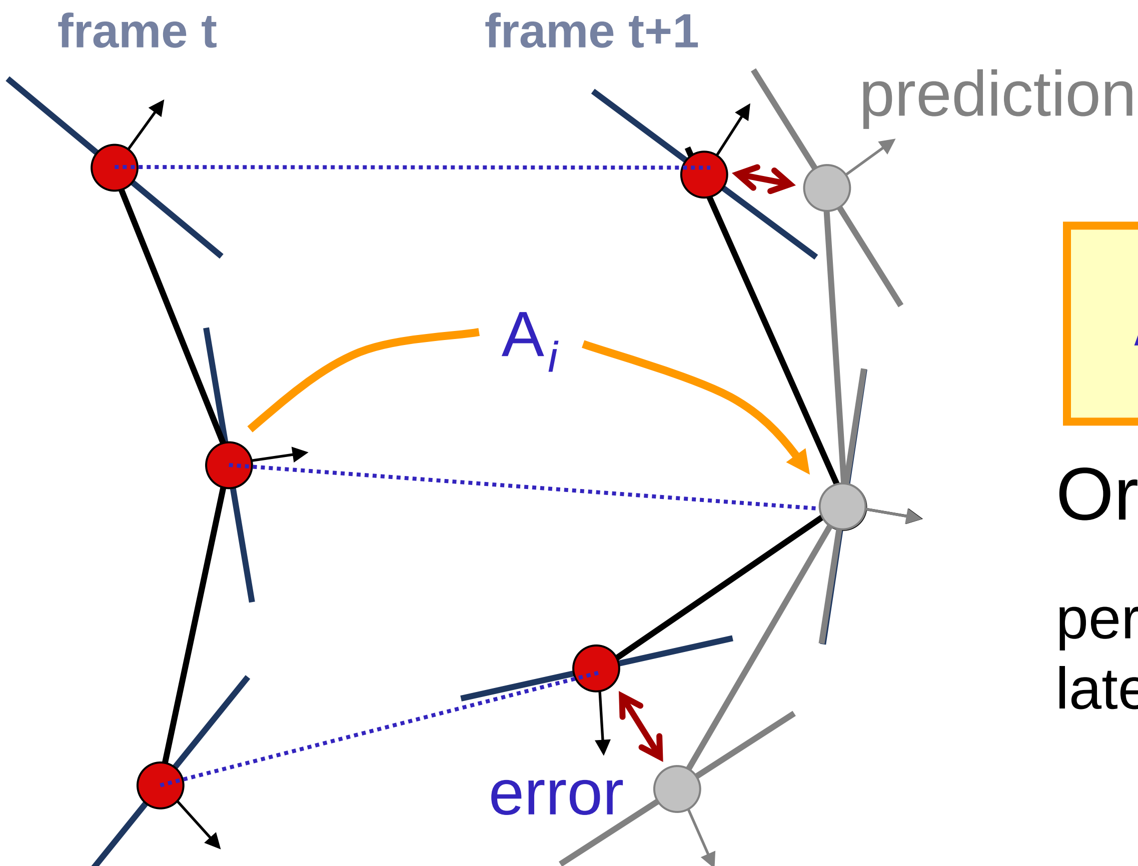


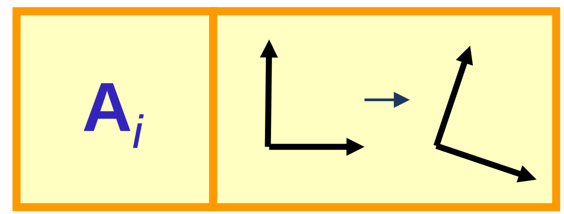


Orthonormal Matrix $\mathbf{A}_i$

per surfel (neighborhood), latent variable

Deformation





Orthonormal Matrix $\mathbf{A}_i$

per surfel (neighborhood), latent variable

$$E^{(regularizer)} = \sum_{surfels} \sum_{neighbors} \left[\mathbf{A}_i^t \left(\mathbf{s}_i^{(t)} - \mathbf{s}_{i_j}^{(t)} \right) - \left(\mathbf{s}_i^{(t+1)} - \mathbf{s}_{i_j}^{(t+1)} \right) \right]^2$$

Unconstrained Optimization

Orthonormal matrices

- Local, 1st order, non-degenerate parametrization:

$$\mathbf{C}_{\times i}^{(t)} = \begin{pmatrix} 0 & \alpha & \beta \\ -\alpha & 0 & \gamma \\ -\beta & -\gamma & 0 \end{pmatrix} \qquad \begin{aligned} \mathbf{A}_i &= \mathbf{A}_0 \exp(\mathbf{C}_{\times i}) \\ &\doteq \mathbf{A}_0 (I + \mathbf{C}_{\times i}^{(t)}) \end{aligned}$$

- Optimize parameters α, β, γ, then recompute $\mathbf{A}_0$
- Compute initial estimate using [*Horn 87*]

Variational Model

Variational Problem:

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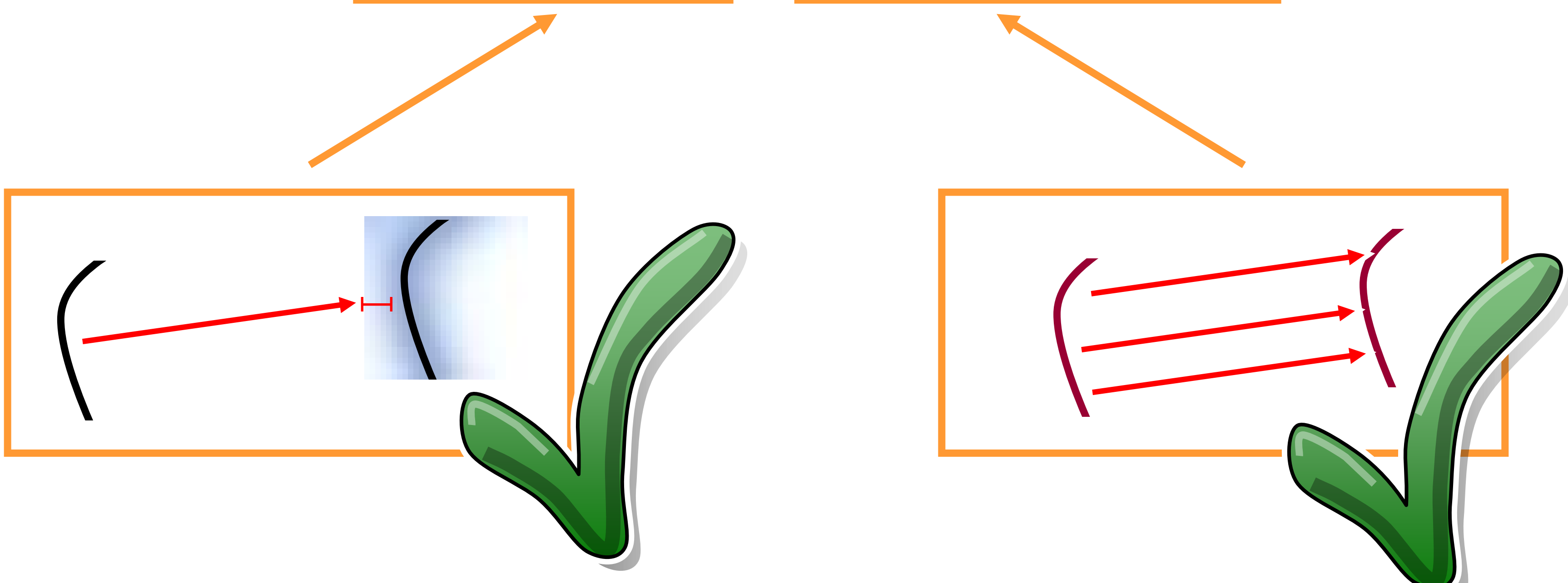


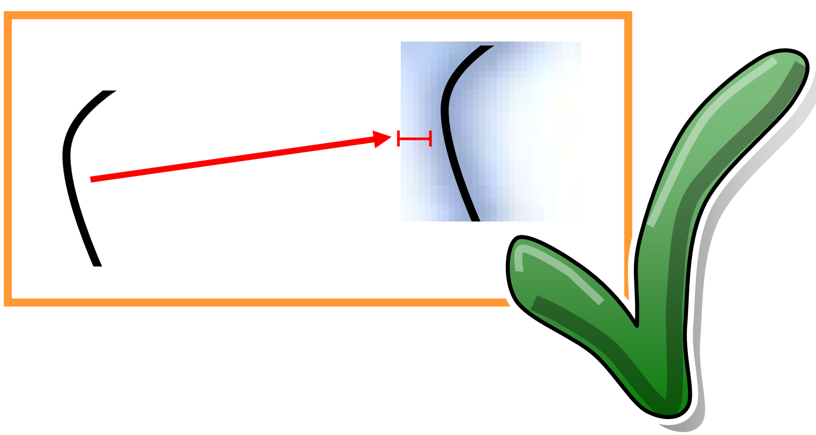
Deformable ICP

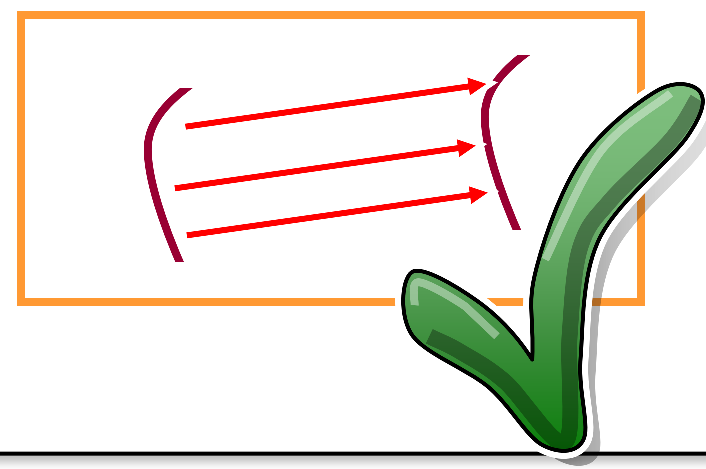
Deformable ICP

How to build a deformable ICP algorithm

- Pick a surface distance measure
- Pick an deformation model / regularizer

$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$





Deformable ICP

How to build a deformable ICP algorithm

- Pick a surface distance measure
- Pick an deformation model / regularizer
- Initialize $f(S_1)$ with S_1 (i.e., f = id)
- Pick a non-linear optimization algorithm
  - Gradient decent (easy, but bad performance)
  - Preconditioned conjugate gradients (better)
  - Newton or Gauss Newton (recommended, but more work)
  - Always use analytical derivatives!
- Run optimization

Example

Example

- Elastic model
- Local rigid coordinate frames
- Align A→B, B→A