Geometric Registration for Deformable Shapes

3.2 Isometric Matching and Quadratic Assignment

Quadratic Assignment · Spectral Matching · MRF Model

31st Annual Conference of the European Association for Computer Graphics
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Overview and Motivation

Global Isometric Matching

Goal

- We want to compute correspondences between deformable shape
- *Global algorithm*, no initilization

Global Isometric Matching

Approach & Problems

- Consistency criterion: global isometry

Problem

- How to find globally consistent matches?

Model

- Quadratic assignment problem
  - General QA-problem is NP-hard
  - But it turns out: solution can usually be computed in polynomial time (more later)

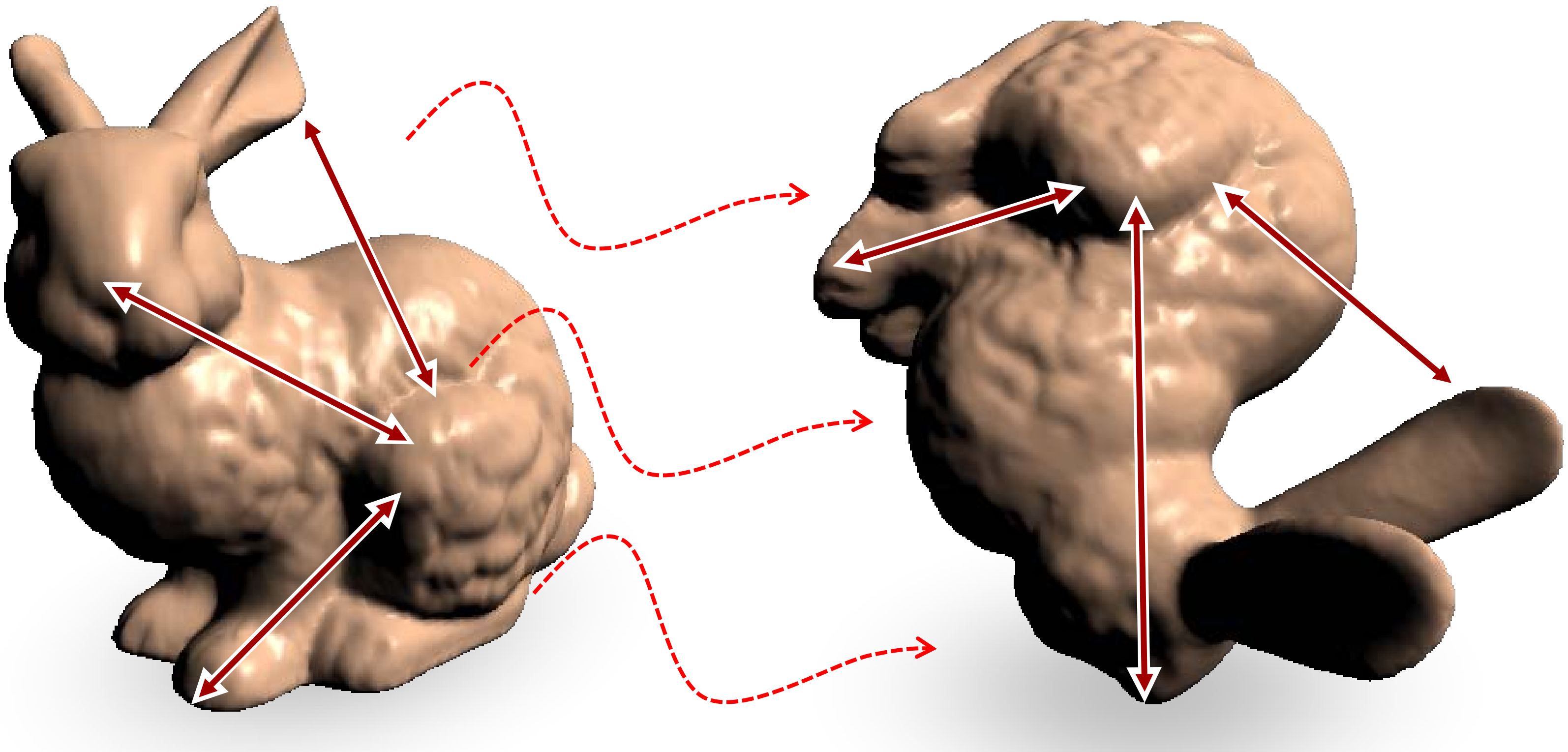
Isometric Matching

(vs. extrinsic matching)

Invariants

Rigid Matching

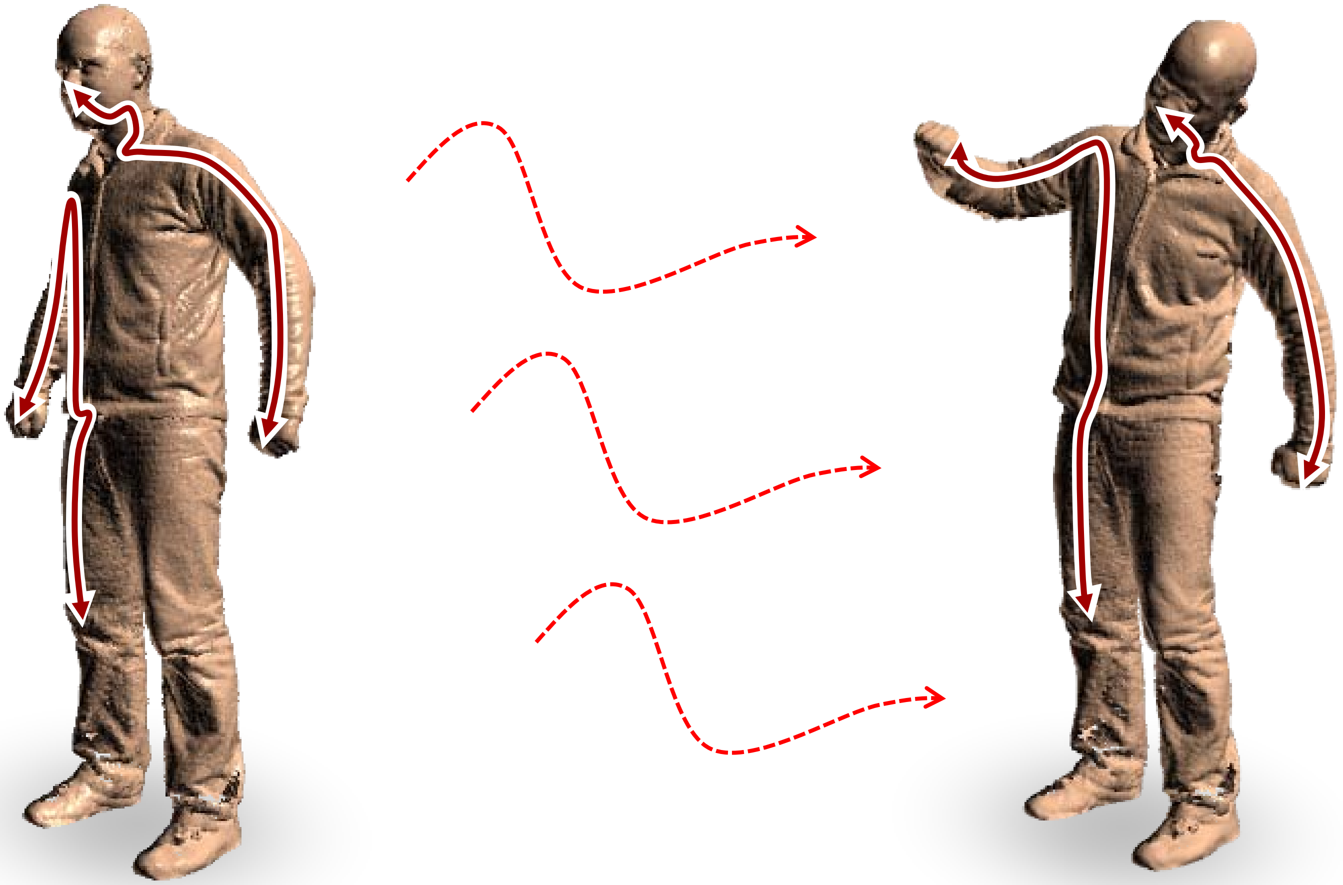
- Invariants: All Euclidean distances are preserved



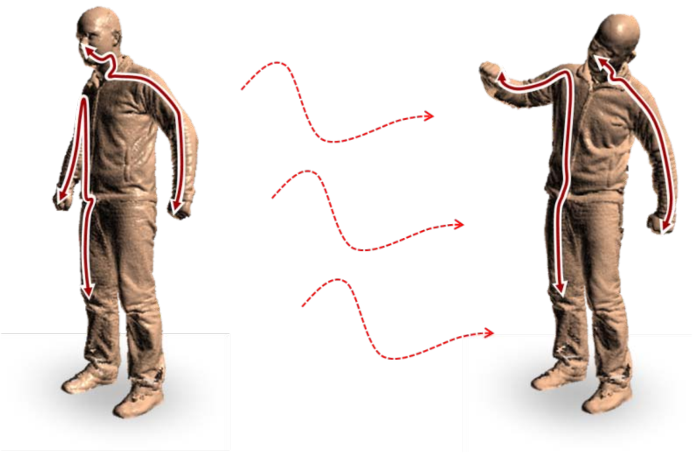
Invariants

Intrinsisc Matching

- Invariants: All geodesic distances are preserved



Invariants



Intrinsisc Matching

- Presevation of geodesic distances („intrinsic distances“)
- Approximation
  - Cloth is almost unstretchable
  - Skin does not stretch a lot
  - Most live objects show approximately isometric surfaces
- Accepted model for deformable shape matching
  - In cases where one subject is presented in different poses
  - Accross different subjects: Other assumptions necessary
  - Then: global matching is an open problem

Feature Based Matching

Quadratic Assignment Model

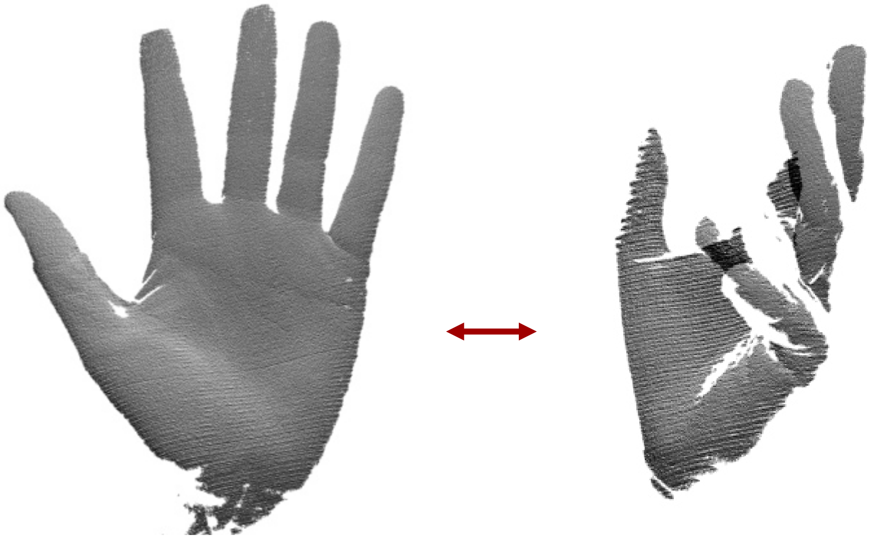
Problem Statement

Deformable Matching

- Two shapes: original, deformed
- How to establish correspondences?
- Looking for global optimum
  - Arbitrary pose

Assumption

- Approximately isometric deformation

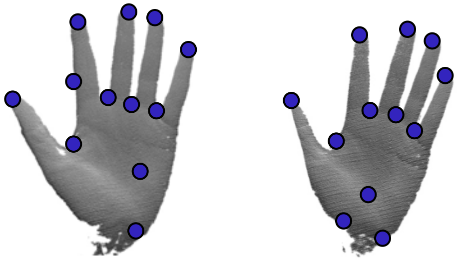


[data set: S. König, TU Dresden]

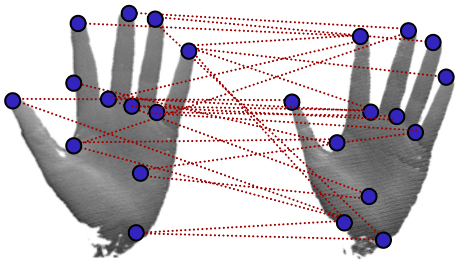
Algorithm

Feature-Matching

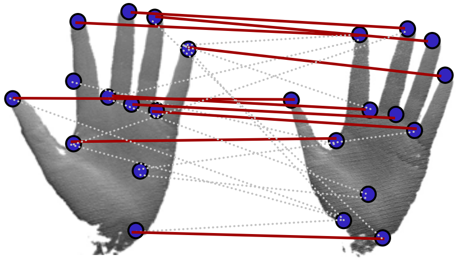
- Detect feature points



- Local matching: potential correspondences



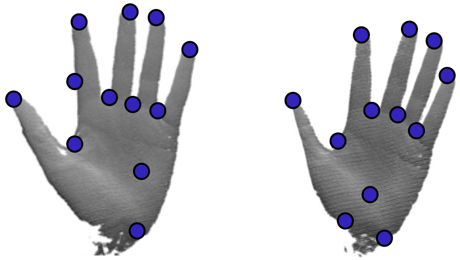
- Global filtering: correct subset



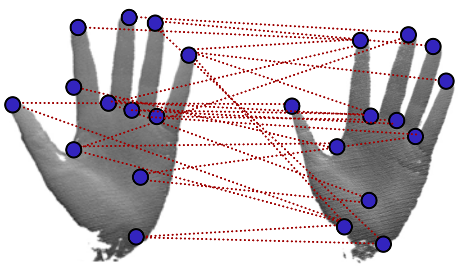
Algorithm

Feature-Matching

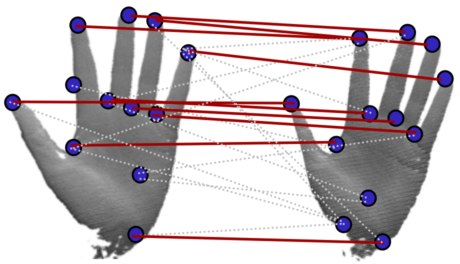
- Detect feature points
  - Locally unique points
  - Such as: maxima of Gaussian curvature
  - E.g.: Geometric MLS-SIFT Features



- Local matching: potential correspondences

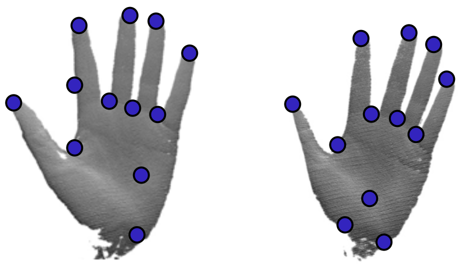


- Global filtering: correct subset



Algorithm

Feature-Matching

- Detect feature points
  
  - Locally unique points
  - Such as: maxima of Gaussian curvature
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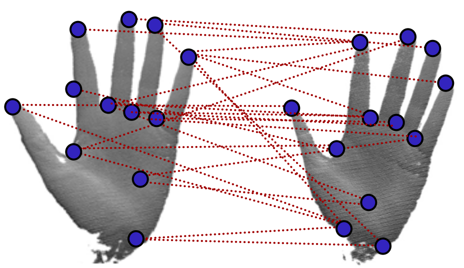
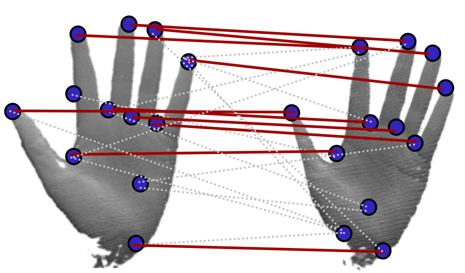
- Local matching: potential correspondences
  
  - Descriptors
  - E.g. curvature histograms
- Global filtering: correct subset
  

Algorithm

Feature-Matching

- Detect feature points
  - Locally unique points
  - Such as: maxima of Gaussian curvature
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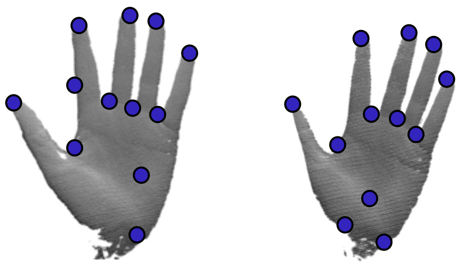
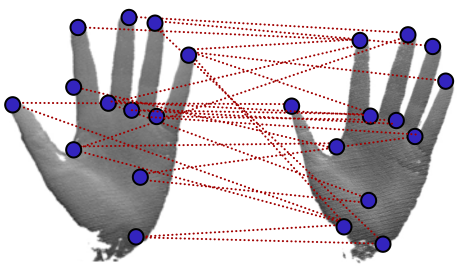
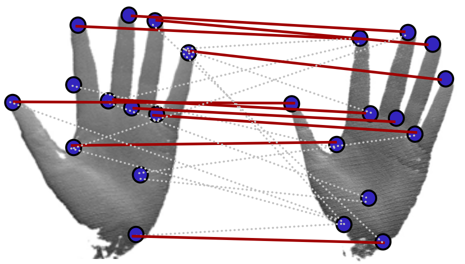


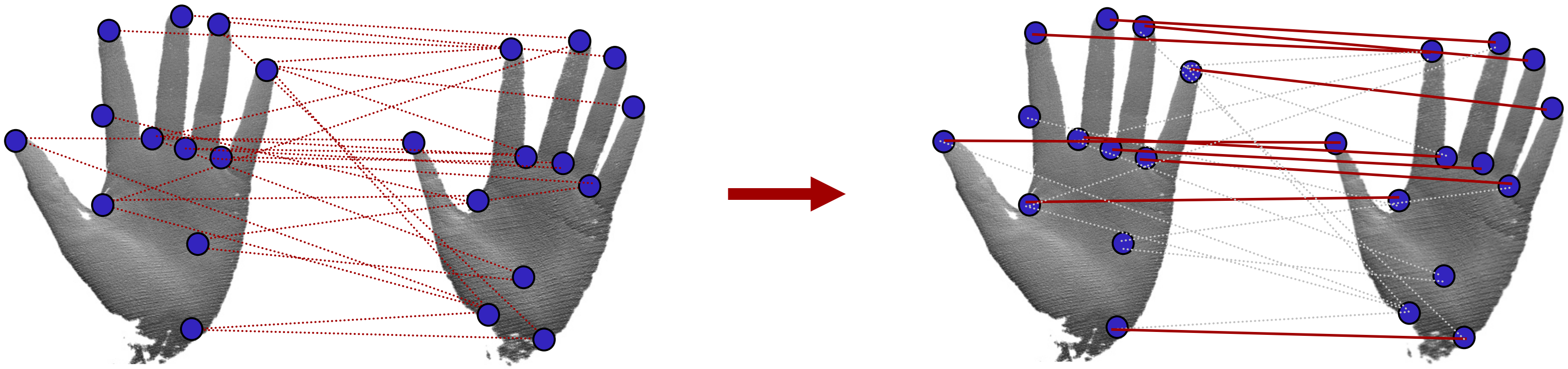
- Local matching: potential correspondences
  - Descriptors
  - E.g. curvature histograms



- Global filtering: correct subset
  - Quadratic assignment
  - Spectral relaxation [Leordeanu et al. 05]
  - RANSAC



Quadratic Assignment



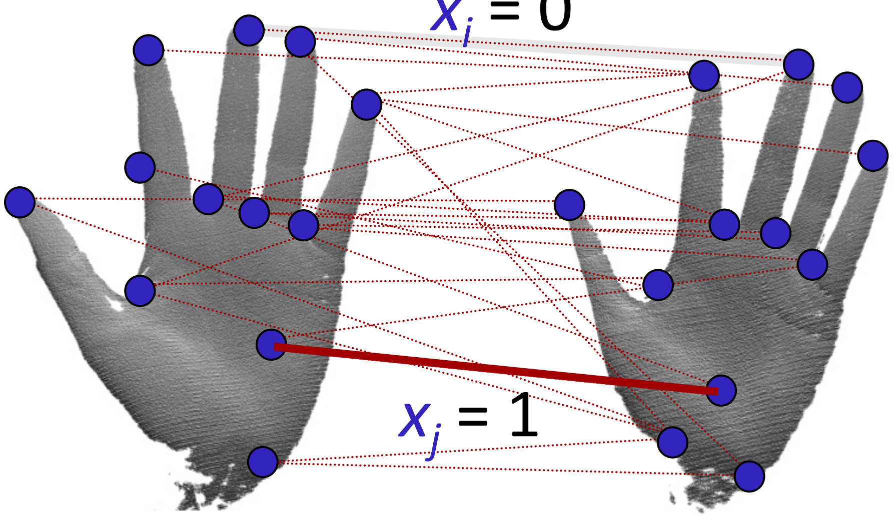
Most difficult part: Global filtering

- Find a consistent subset
- Pairwise consistency:
  - Correspondence pair must preserve intrinsic distance
- Maximize number of pairwise consistent pairs
  - Quadratic assignment (in general: NP-hard)

Quadratic Assignment Model

Quadratic Assignment

- *n* potential correspondences
- Each one can be turned on or off
- Label with variables x_i
- Compatibility score:

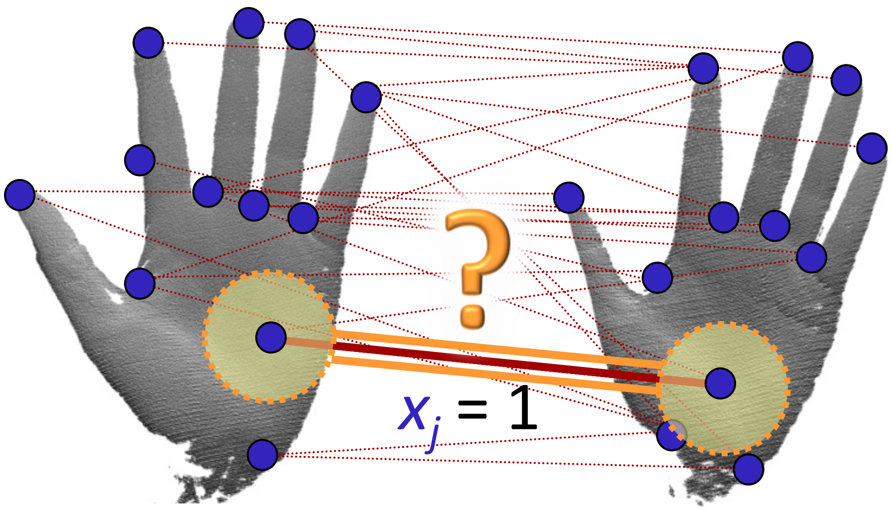


$$P^{(match)}(x_1,\dots,x_n) = \prod_{i=1}^{n} P_i^{(single)} \prod_{i,j=1}^{n} P_{i,j}^{(compatible)}, x_i \in \{0,1\}$$

Quadratic Assignment Model

Quadratic Assignment

- Compatibility score:
  - **Singeltons:**
    Descriptor match

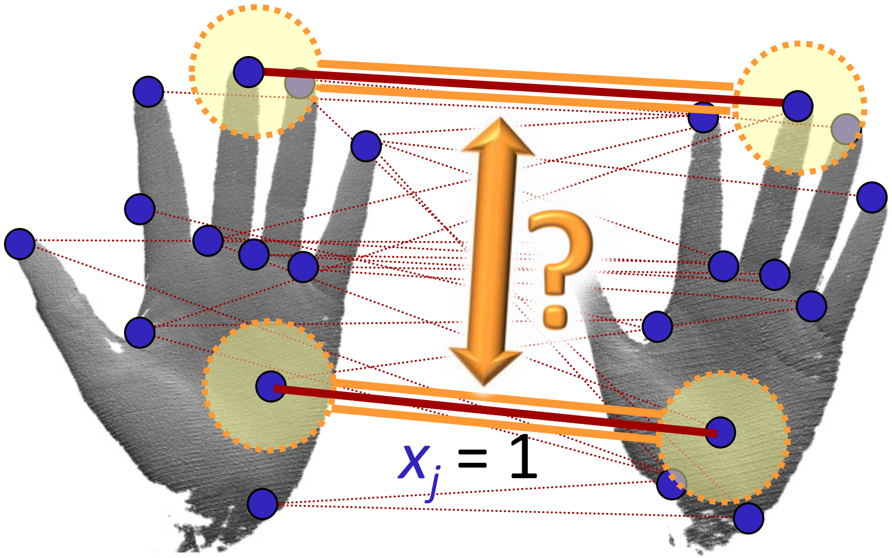


$$P^{(match)}(x_1,\dots,x_n) = \prod_{i=1}^{n} P_i^{(single)} \prod_{i,j=1}^{n} P_{i,j}^{(compatible)}, x_i \in \{0,1\}$$

Quadratic Assignment Model

Quadratic Assignment

- Compatibility score:
  - **Singeltons:** Descriptor match
  - **Doubles:** Compatibility



$$P^{(match)}(x_1,\dots,x_n) = \prod_{i=1}^{n} P_i^{(single)} \prod_{i,j=1}^{n} P_{i,j}^{(compatible)}, x_i \in \{0,1\}$$

Quadratic Assignment Model

Quadratic Assignment

- Matrix notation:

$$P^{(match)}(x_1,...,x_n) = \prod_{i=1}^{n} P_i^{(single)} \prod_{i,j=1}^{n} P_{i,j}^{(compatible)}$$

$$\log P^{(match)}(x_1,...,x_n) = \sum_{i=1}^{n} \log P_i^{(single)} + \sum_{i,j=1}^{n} \log P_{i,j}^{(compatible)}$$

$$= \quad \mathbf{xs} \quad + \quad \mathbf{x}^T \mathbf{D} \mathbf{x}$$

- Quadratic scores are encoded in Matrix **D**
- Linear scores are encoded in Vector **s**
- Task: find optimal binary vector **x**

Spectral Matching

Approximate Quadratic Assignment

Spectral Matching

Simple & Effective Approximation:

- Spectral matching [Leordeanu & Hebert 05]
- Form compatibility matrix:

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{21} & a_{31} & \\ a_{12} & a_{22} & & \\ a_{13} & & \ddots & \\ & & & \ddots \end{pmatrix}$$

Diagonal:
Descriptor match

Off-Diagonal:
Pairwise compatibility

All entries within [0..1]
= [no match...perfect match]

Spectral Matching

Approximate largest clique:

- Compute eigenvector with largest eigenvalue
- Maximizes Rayleigh quotient:

$$\arg\max \frac{\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}}{\|\mathbf{x}\|^2}$$

- "Best yield" for bounded norm
  - The more consistent pairs (rows of 1s), the better
  - Approximates largest clique
- Implementation
  - For example: power iteration

Spectral Matching

Postprocessing

- Greedy quantization
  - Select largest remaining entry, set it to 1
  - Set all entries to 0 that are not pairwise consistent with current set
  - Iterate until all entries are quantized

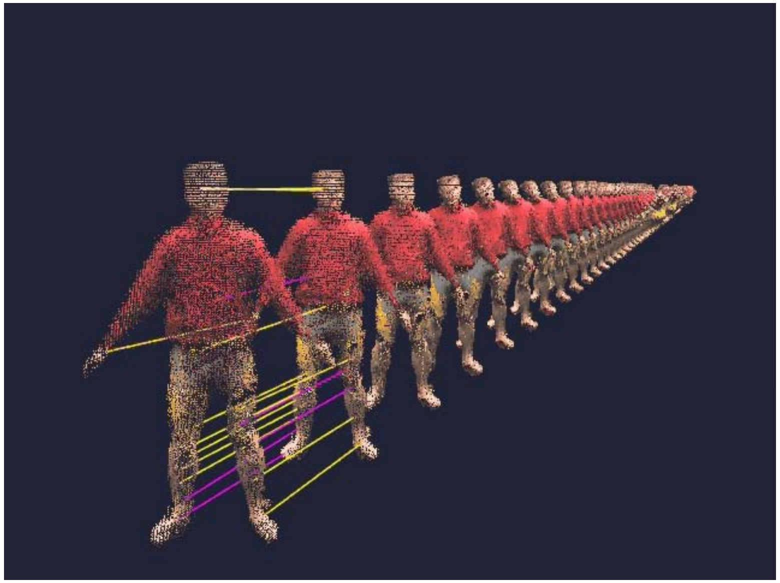
In practice...

- This algorithm turns out to work quite well.
- Very easy to implement
- Limited to (approx.) quadratic assignment model

Spectral Matching Example

Application to Animations

- **Feature points:**
  Geometric MLS-SIFT features [Li et al. 2005]
- **Descriptors:**
  Curvature & color ring histograms
- **Global Filtering:**
  Spectral matching
- **Pairwise animation matching:**
  Low precision passive stereo data



Data courtesy of C. Theobald, MPI Informatik

Markov Random Field Model

Probabilistic Interpretation

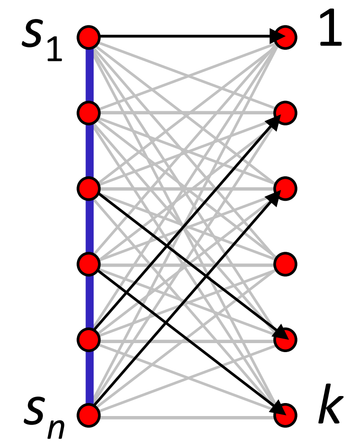
Direct MRF Approach

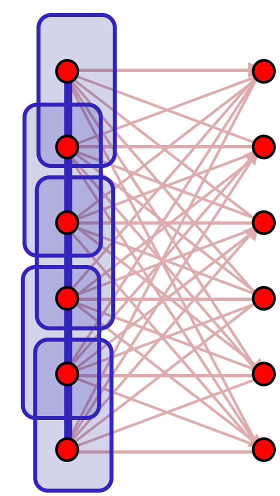
Bayesian interpretation

- Probability Space
  - $\Omega = \left\{ f : (s_1 .. s_n) \to \{1, \ldots, k\}^n \right\}$
  - Exponential size!
- Markov-Random Field / graphical model
- Distribution:

$$P(f) = \frac{1}{Z} \left[\prod_{i=1}^{n} P^{(D)}(\mathbf{s}_i, f(\mathbf{s}_i)) \right] \left[\prod_{(i,j) \in G} P^{(S)}(\mathbf{s}_i, \mathbf{s}_j, f(\mathbf{s}_i), f(\mathbf{s}_j)) \right]$$

match local shape — preserve local distance

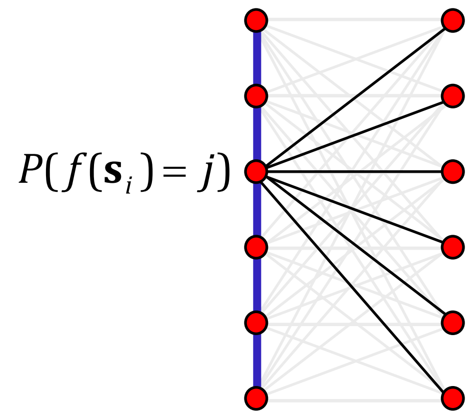




Direct MRF Approach

Solution

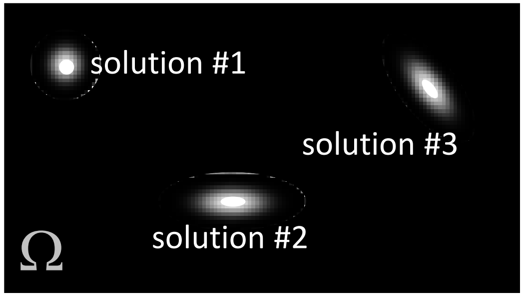
- Posterior distribution is *exponential*
- Instead, we compute marginals: "Average" of all solutions



$$P(f(\mathbf{s}_i)=j)=\sum_{i_1=1}^{k}\cdots\sum_{i_n=1}^{k}P\big(f=(i_1,\ldots,j,\ldots,i_n)\big)$$

Postprocessing:

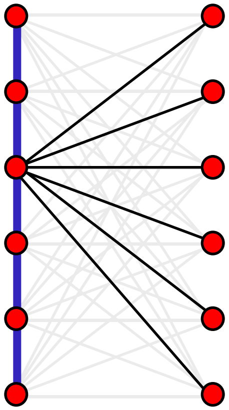
- Extract solutions
- Few solutions in a very large space



Direct MRF Approach

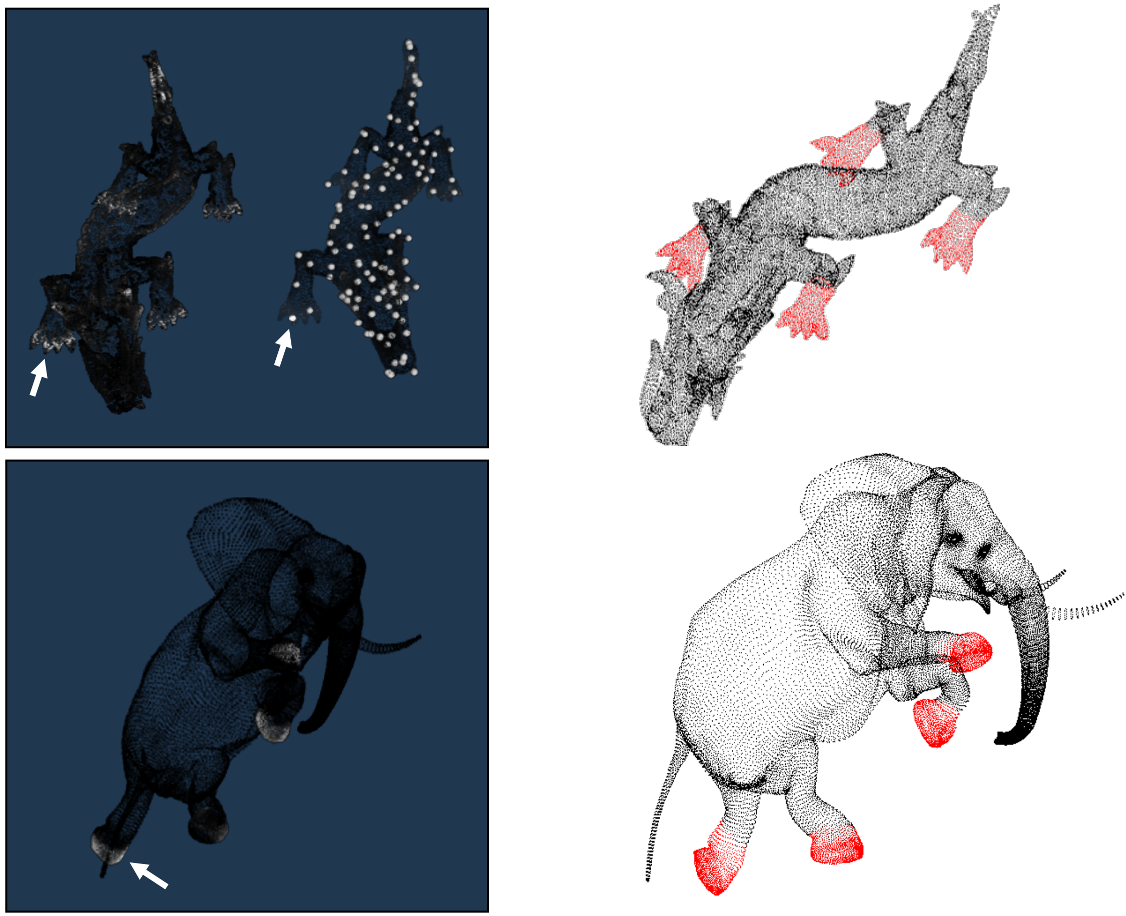
Inference

$$P(f(\mathbf{s}_i) = j) = \sum_{i_1=1}^{k} \cdots \sum_{i_n=1}^{k} P\big(f = (i_1, \ldots, j, \ldots, i_n)\big)$$



- Representation is polynomial, but computation is still NP hard
- Heuristic approximation: *Loopy belief propagation*
- Works well in practice

Example Result



Self-matching: Deformable Symmetries