

# Non-Rigid Registration

*Hao Li*



Local Shape Matching: Non-Rigid Registration



# Correspondence Problem Classification

How many meshes?

**Two:** Pairwise registration

Initial registration available?

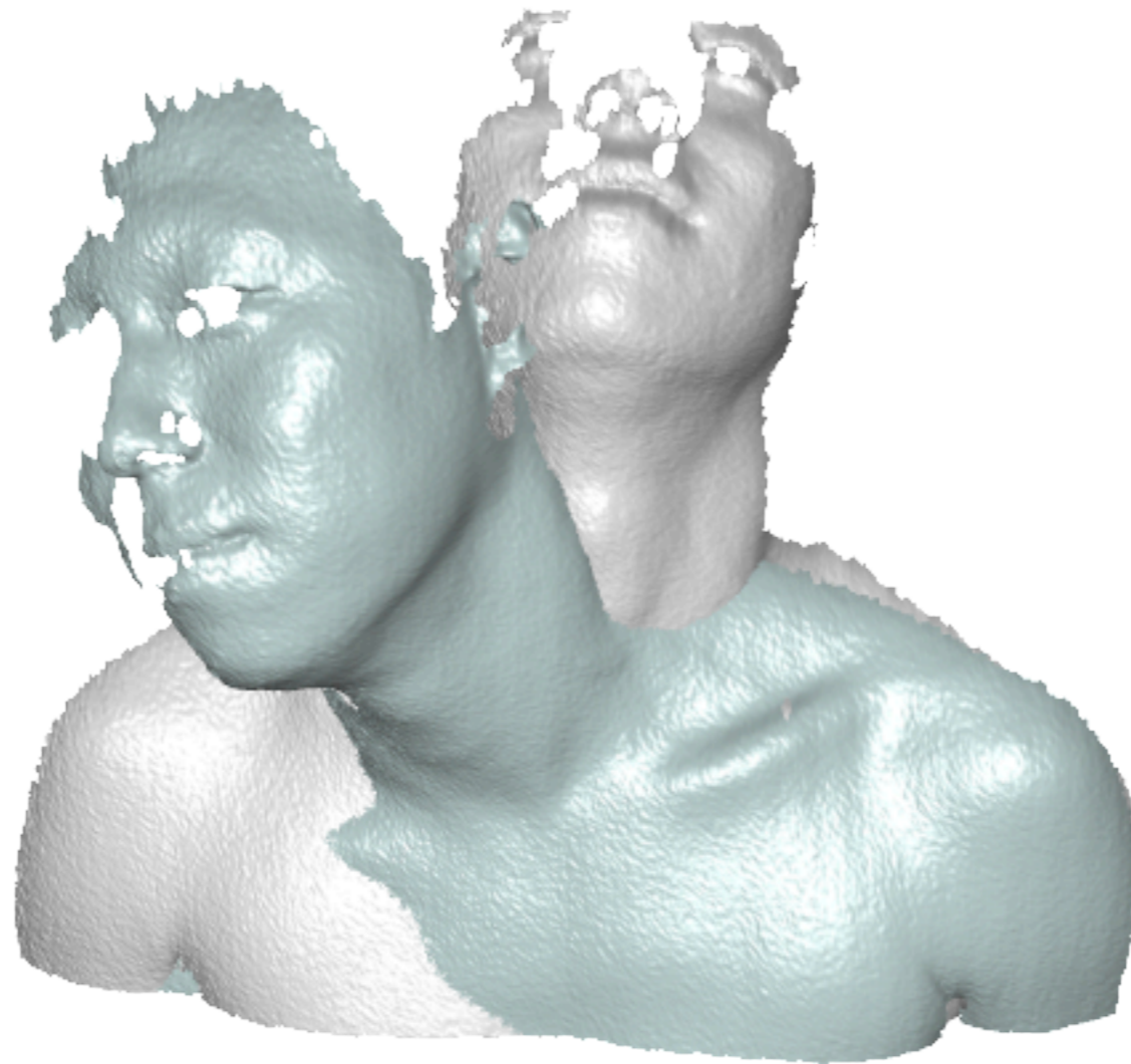
**Yes:** Local optimization methods

Class of transformations?

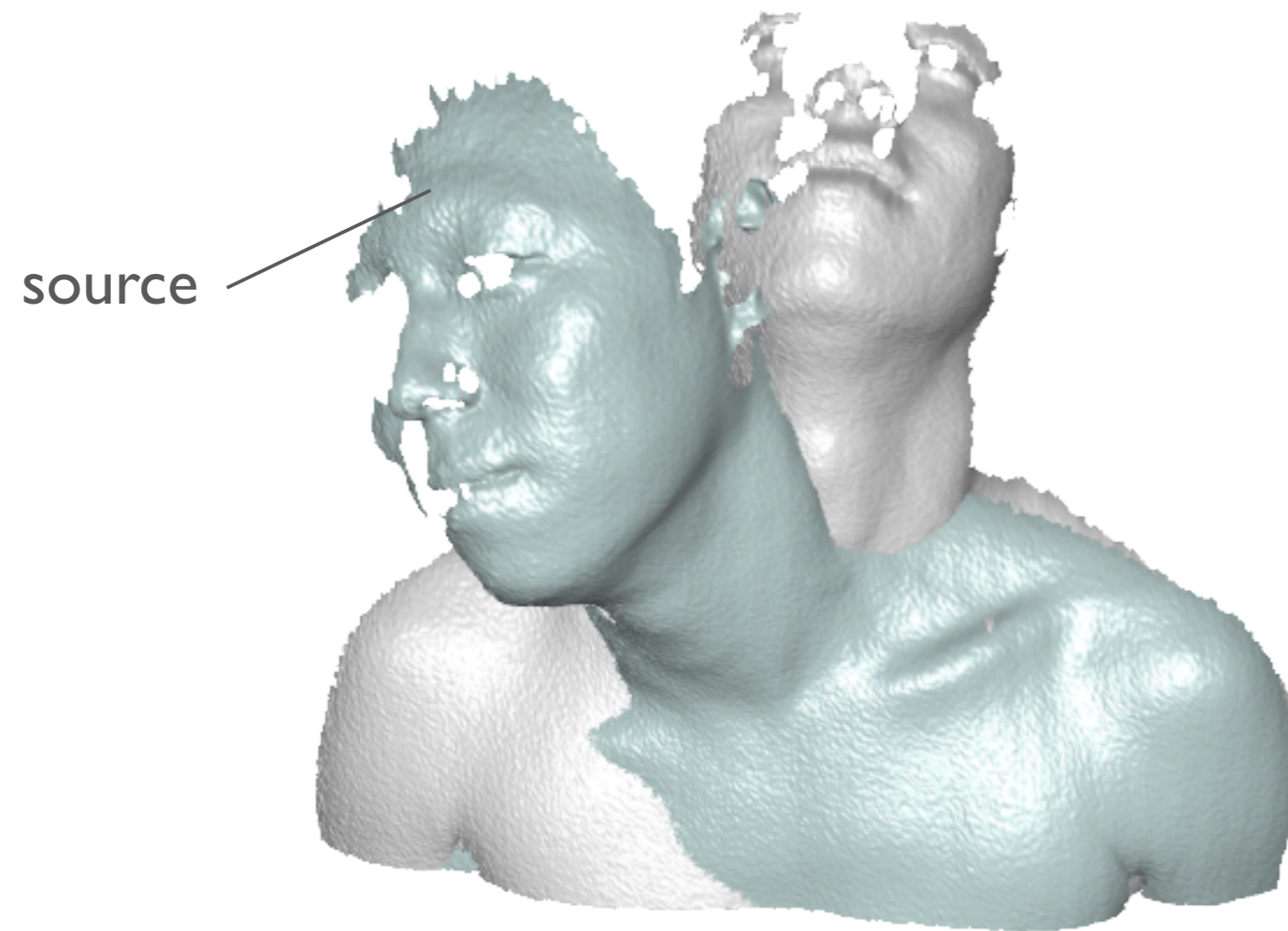
**Non-rigid deformations**



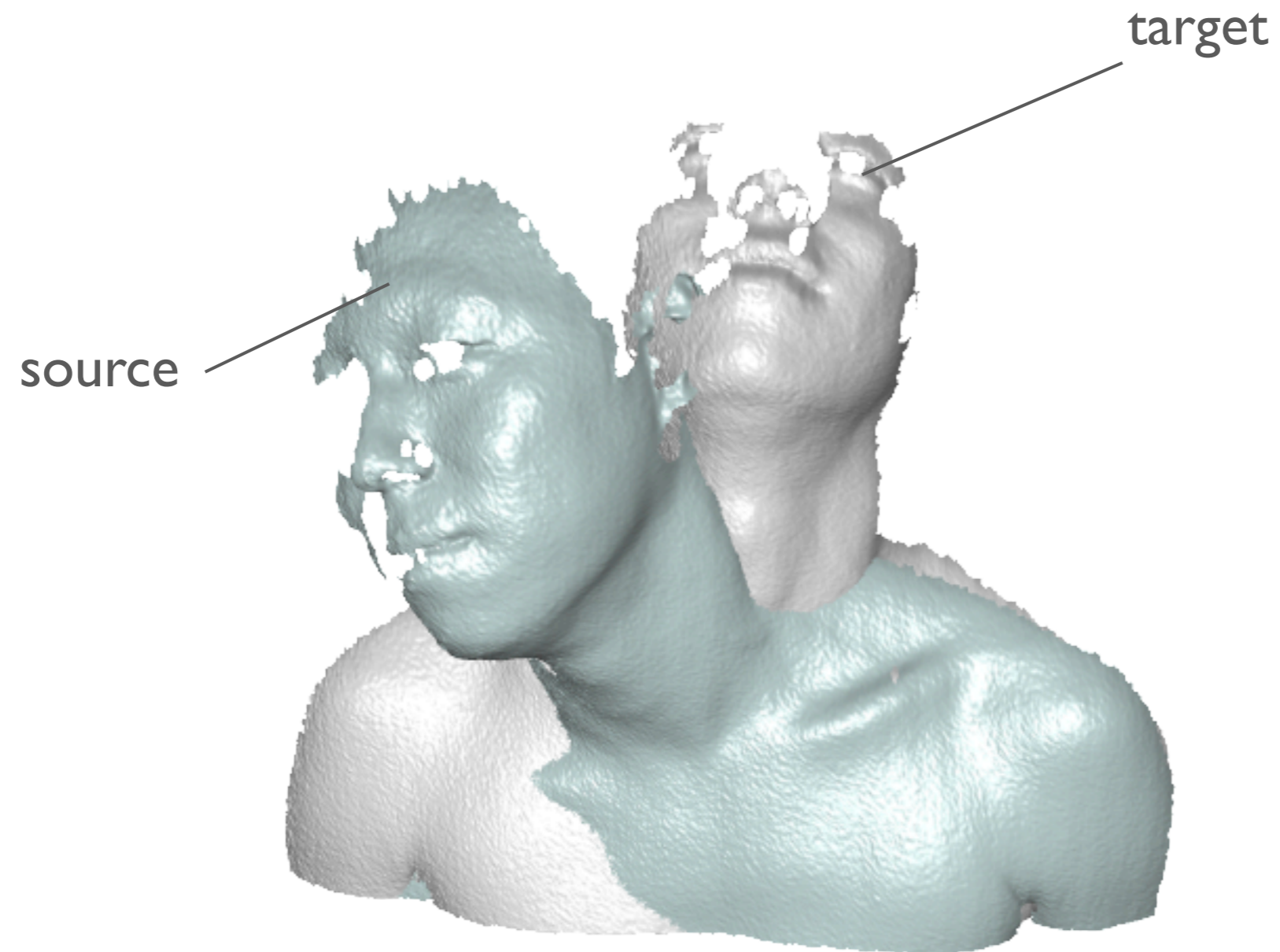
# Pair of Scans



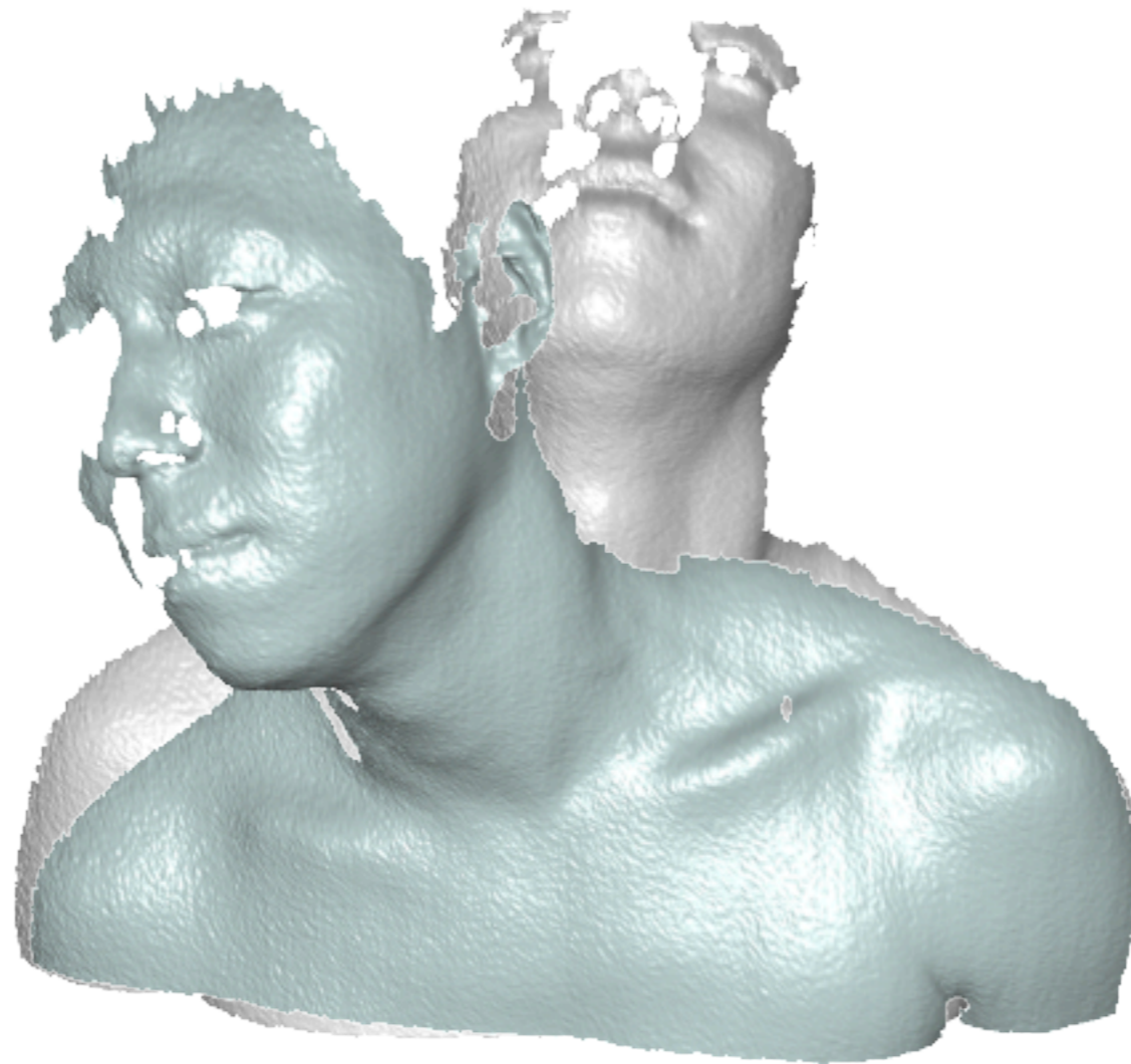
# Pair of Scans



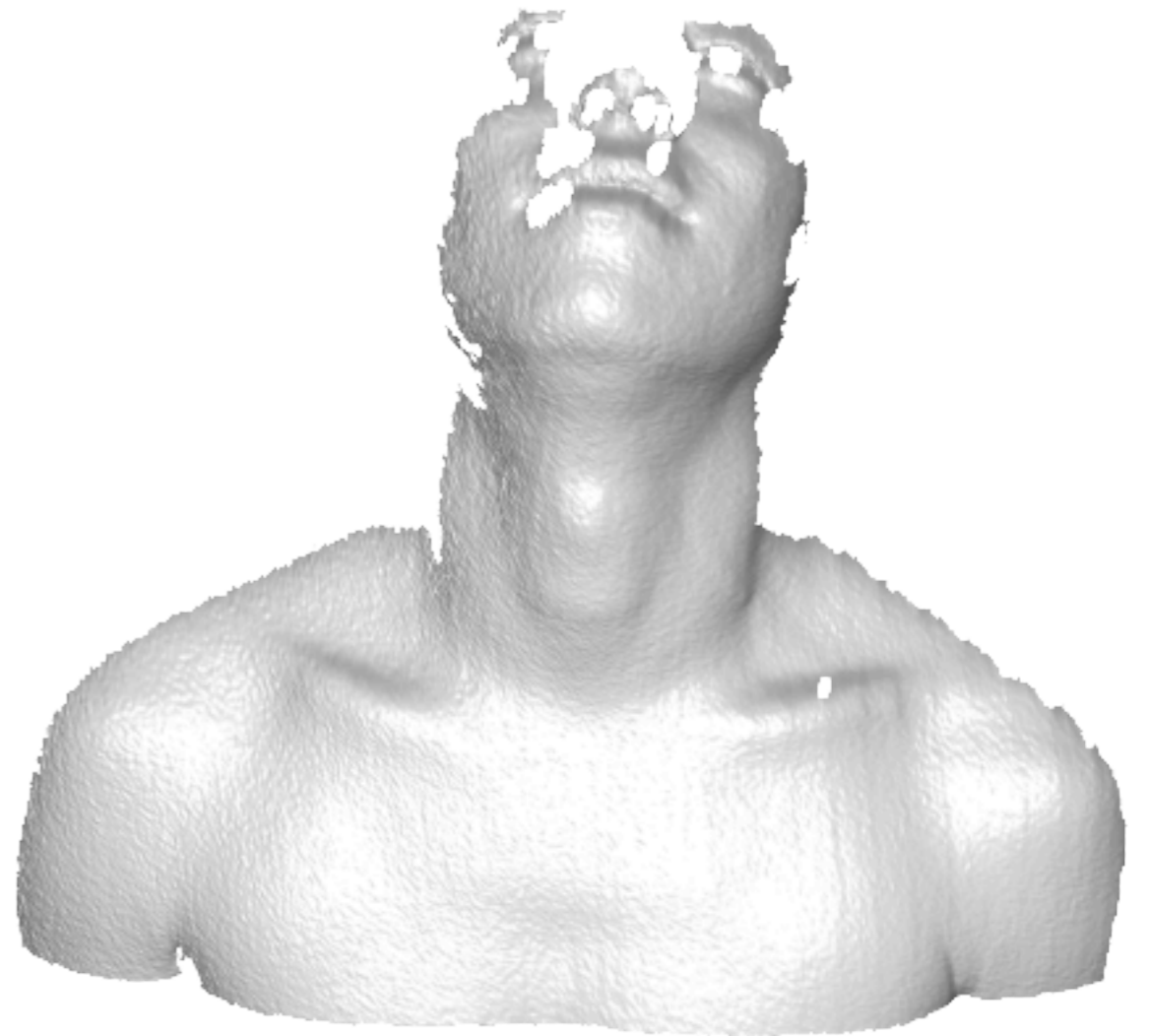
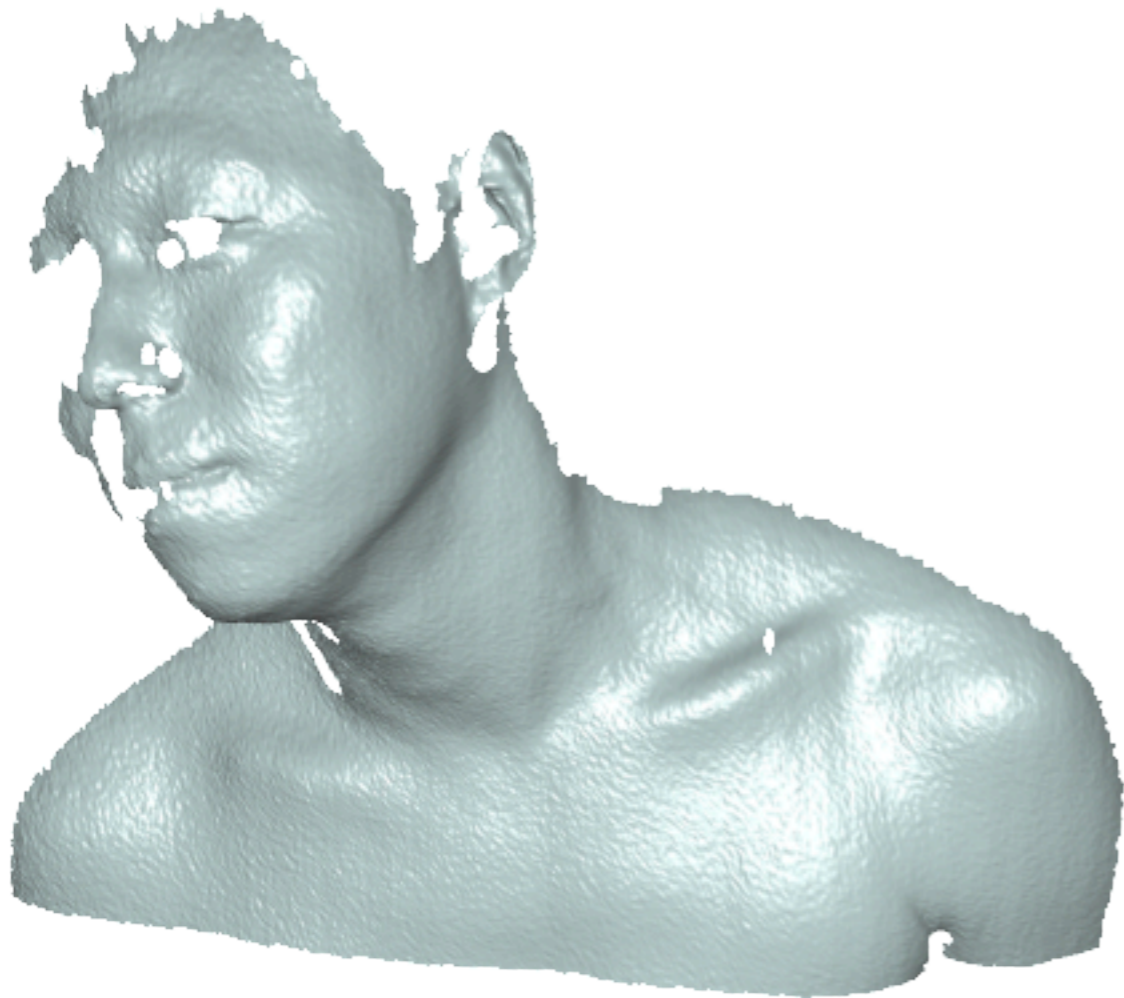
# Pair of Scans



# Correspondences are lost

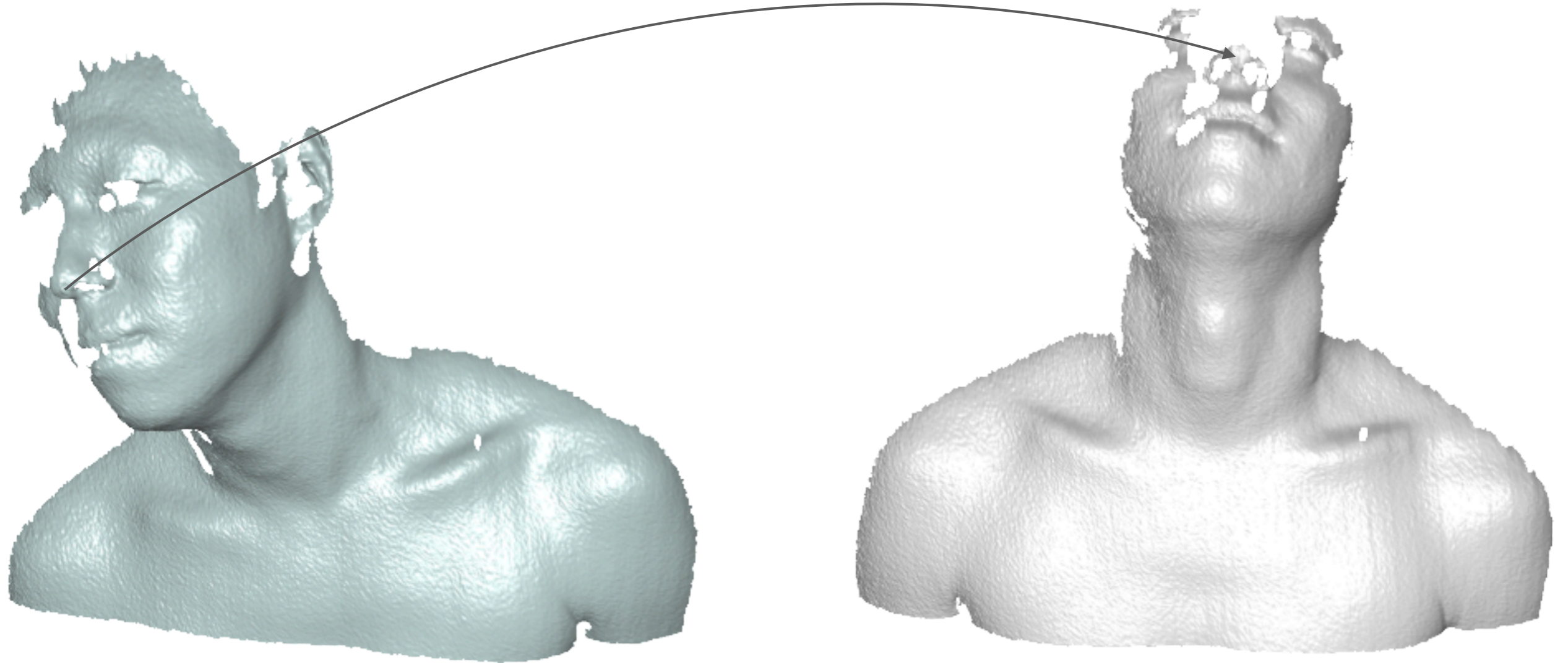


# Correspondences are lost

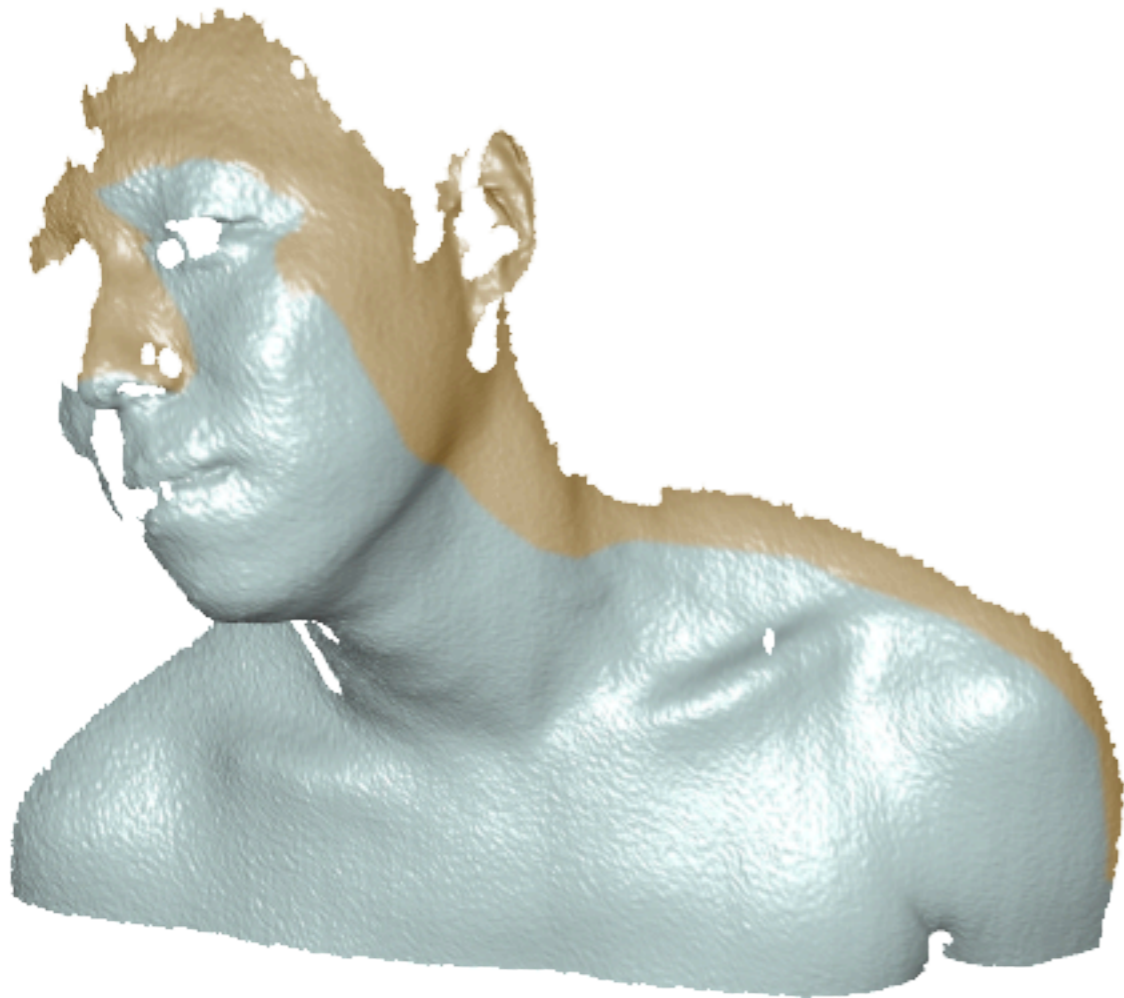


# Correspondences are lost

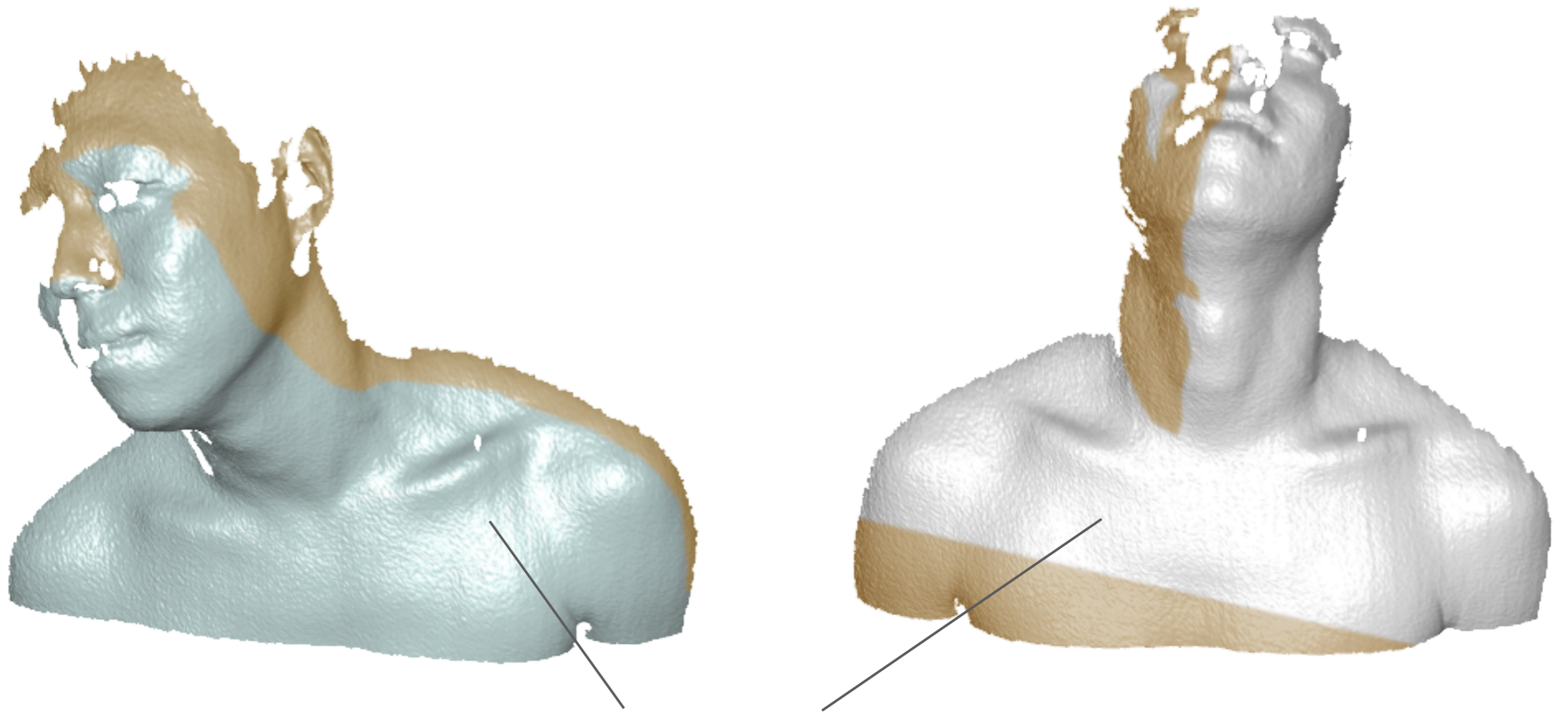
?



# What are the regions of overlap?

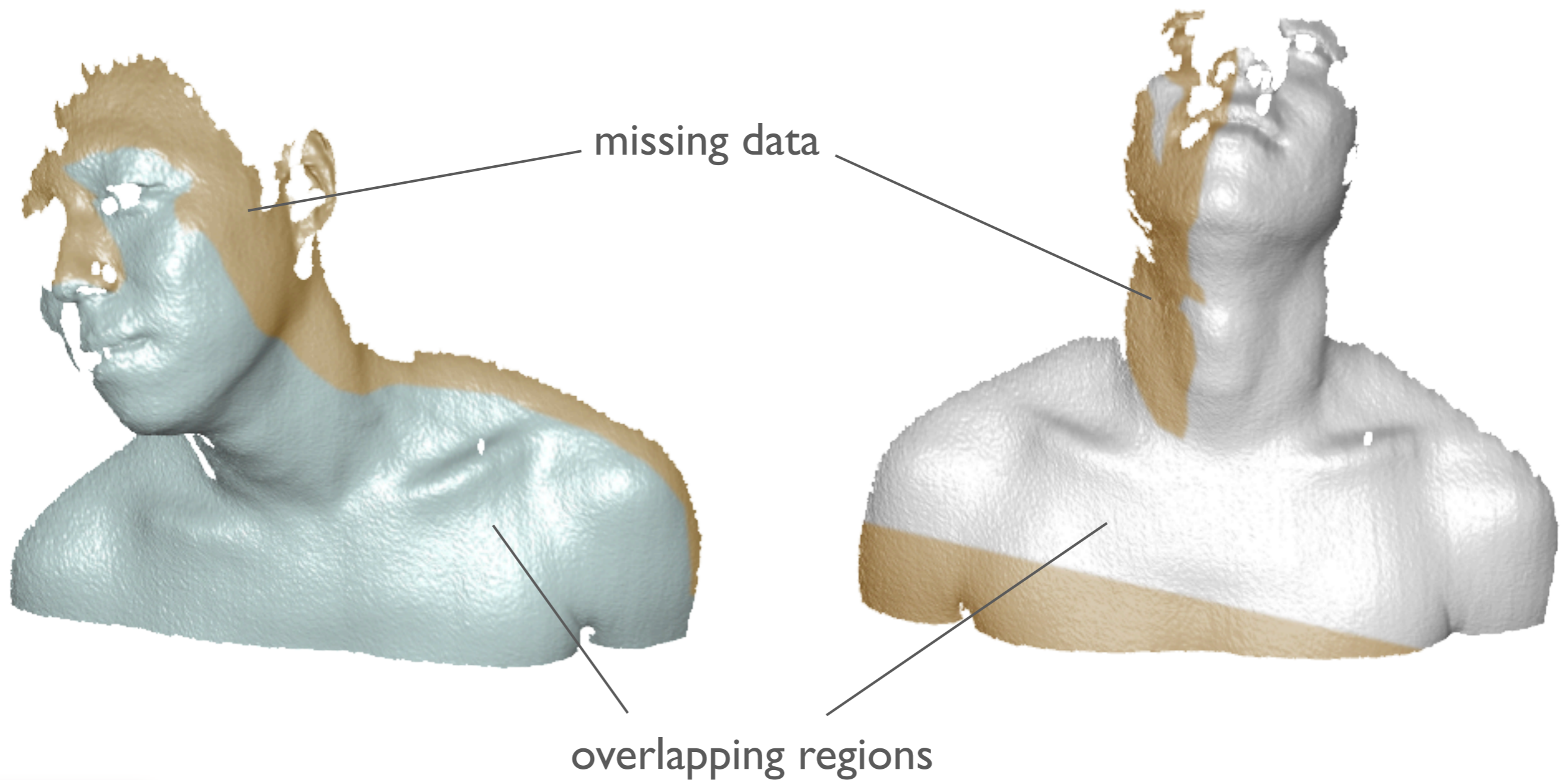


# What are the regions of overlap?

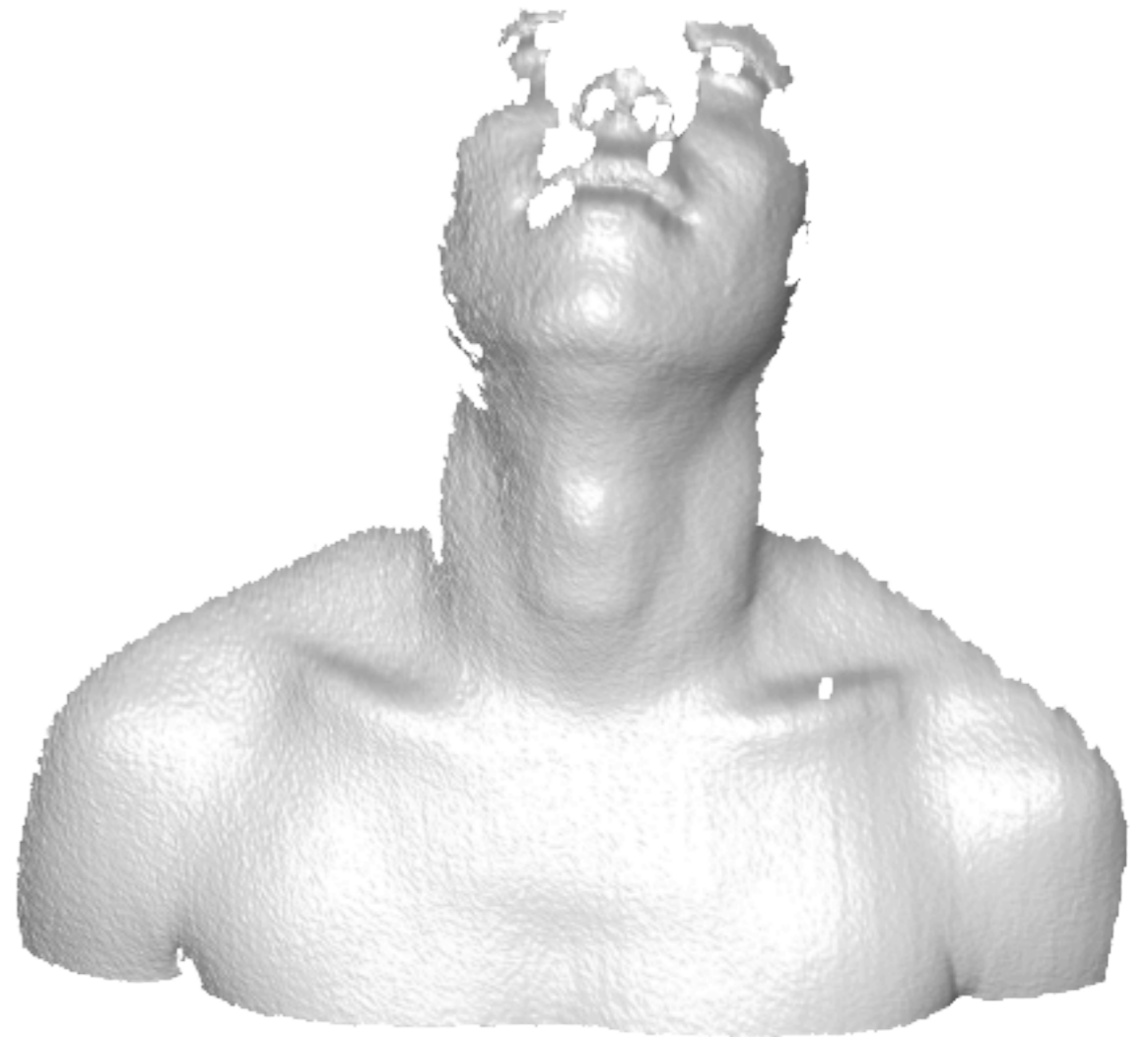
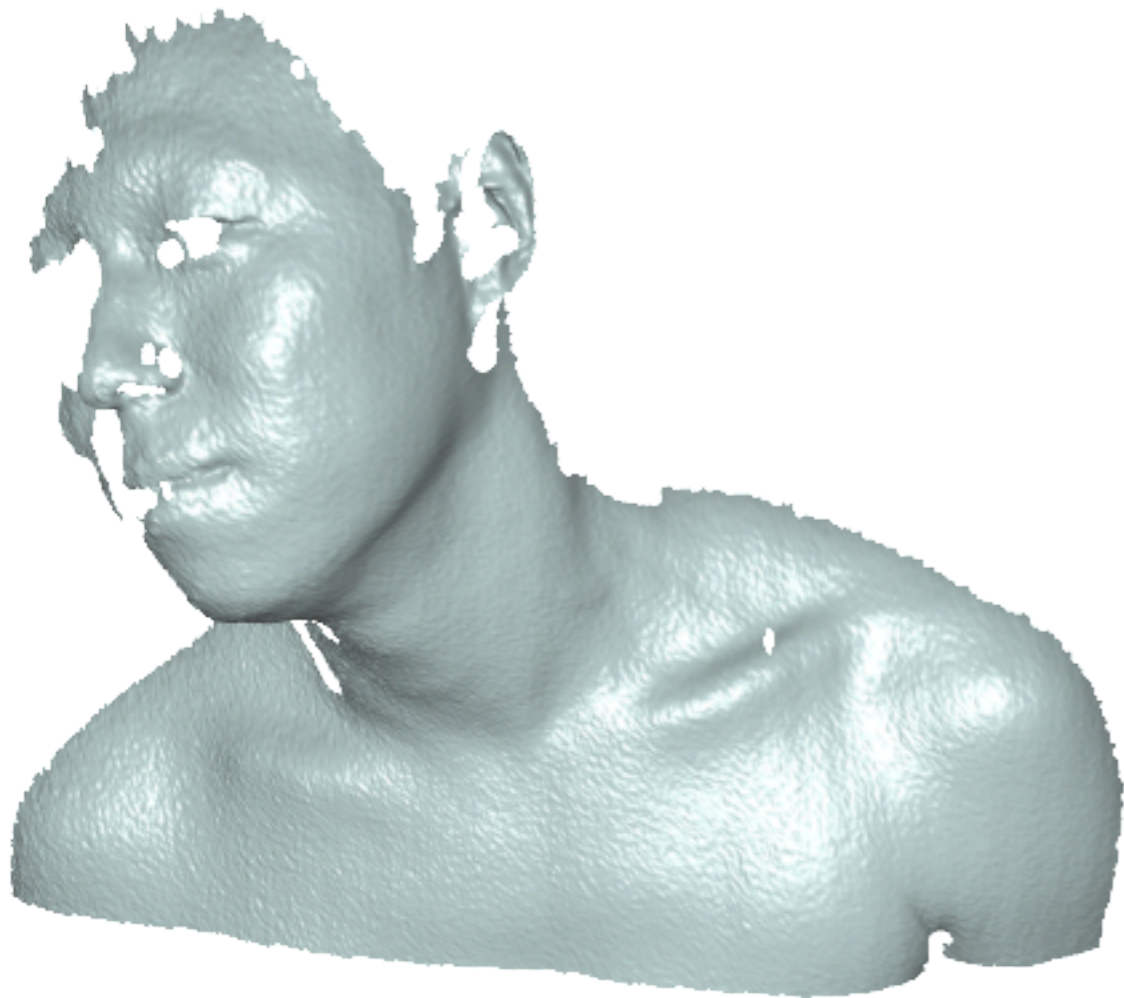


overlapping regions

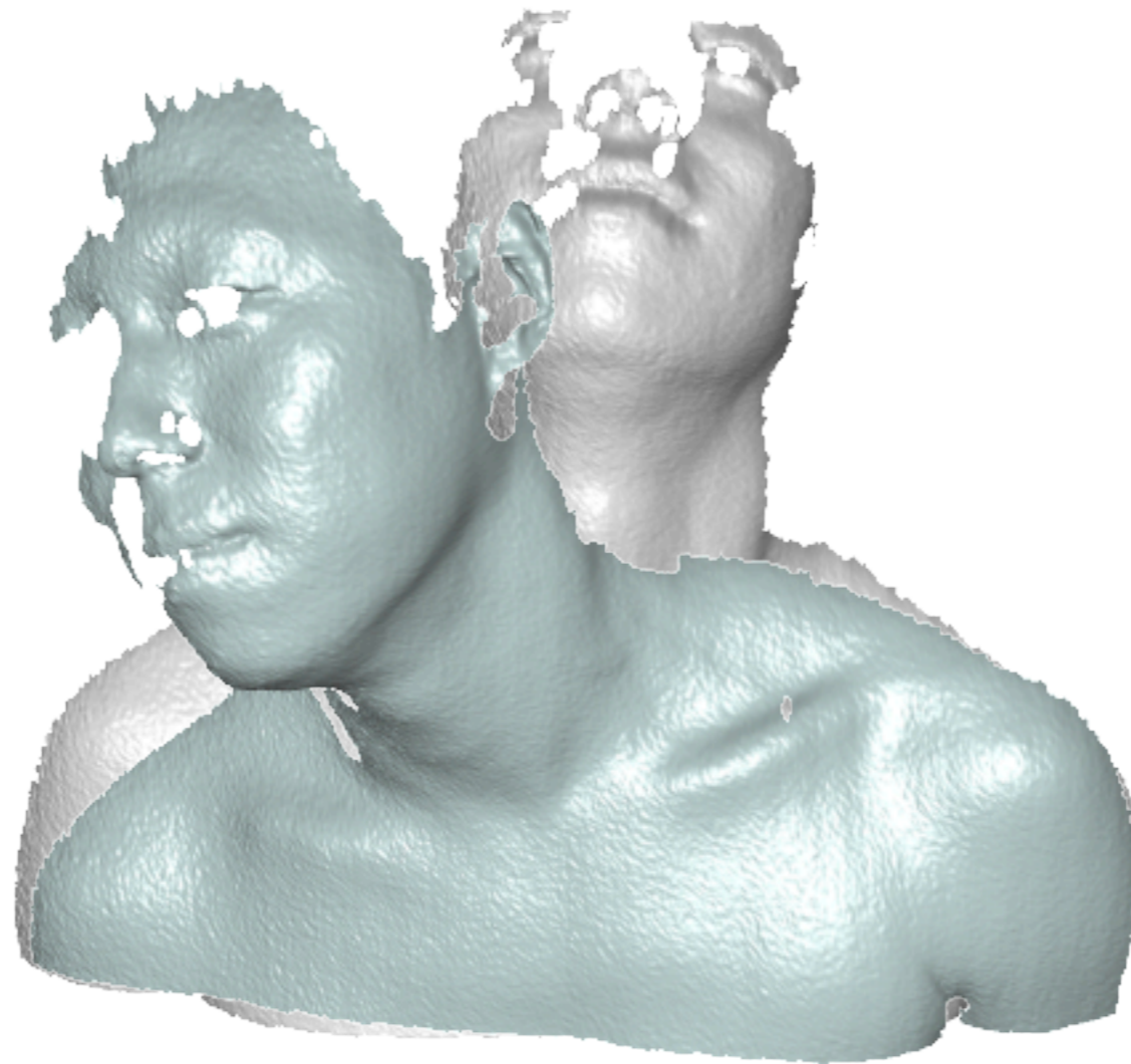
# What are the regions of overlap?



# What are the regions of overlap?



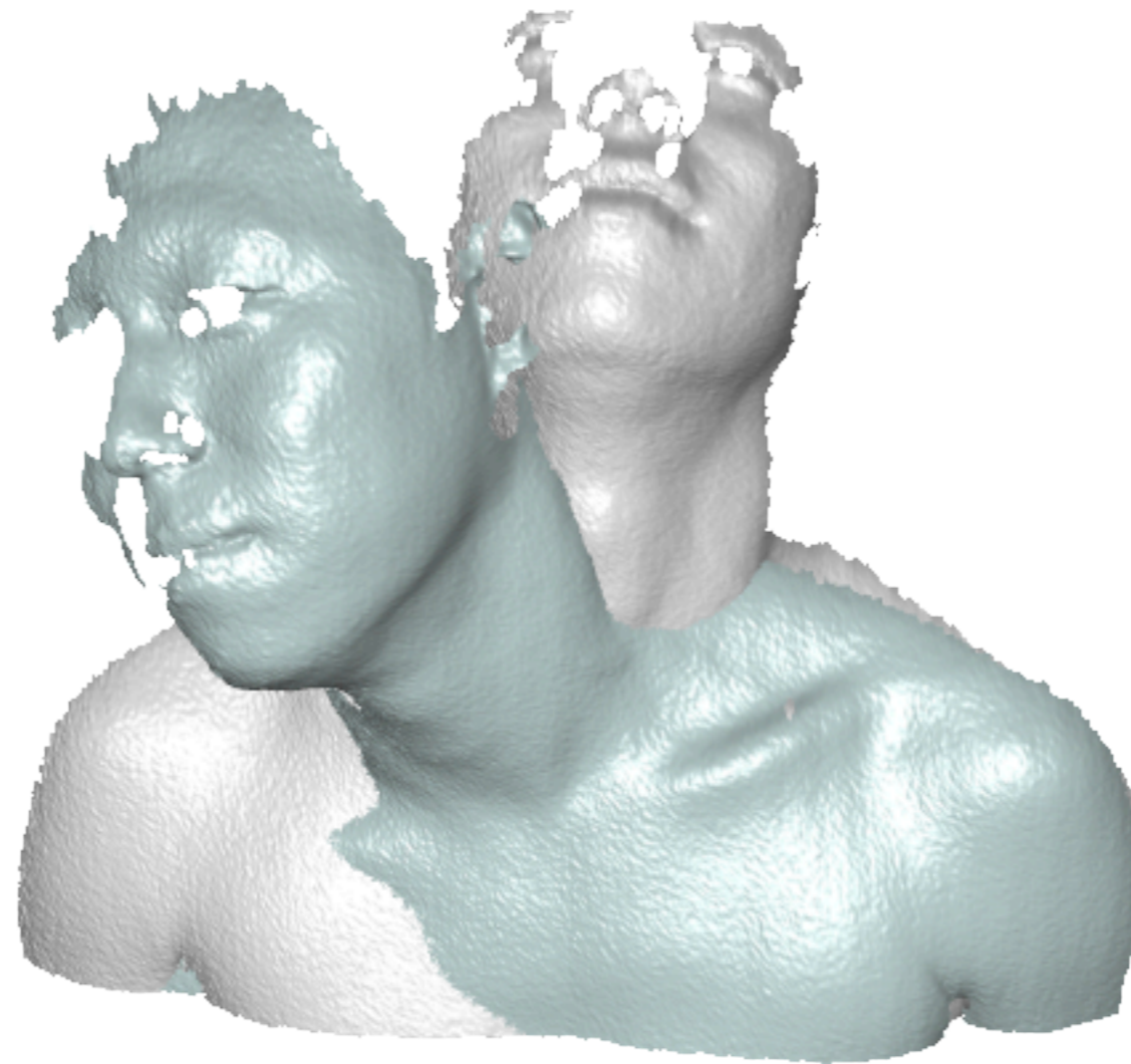
# What are the regions of overlap?



# Non-Rigid Registration



# Non-Rigid Registration



# Ingredients

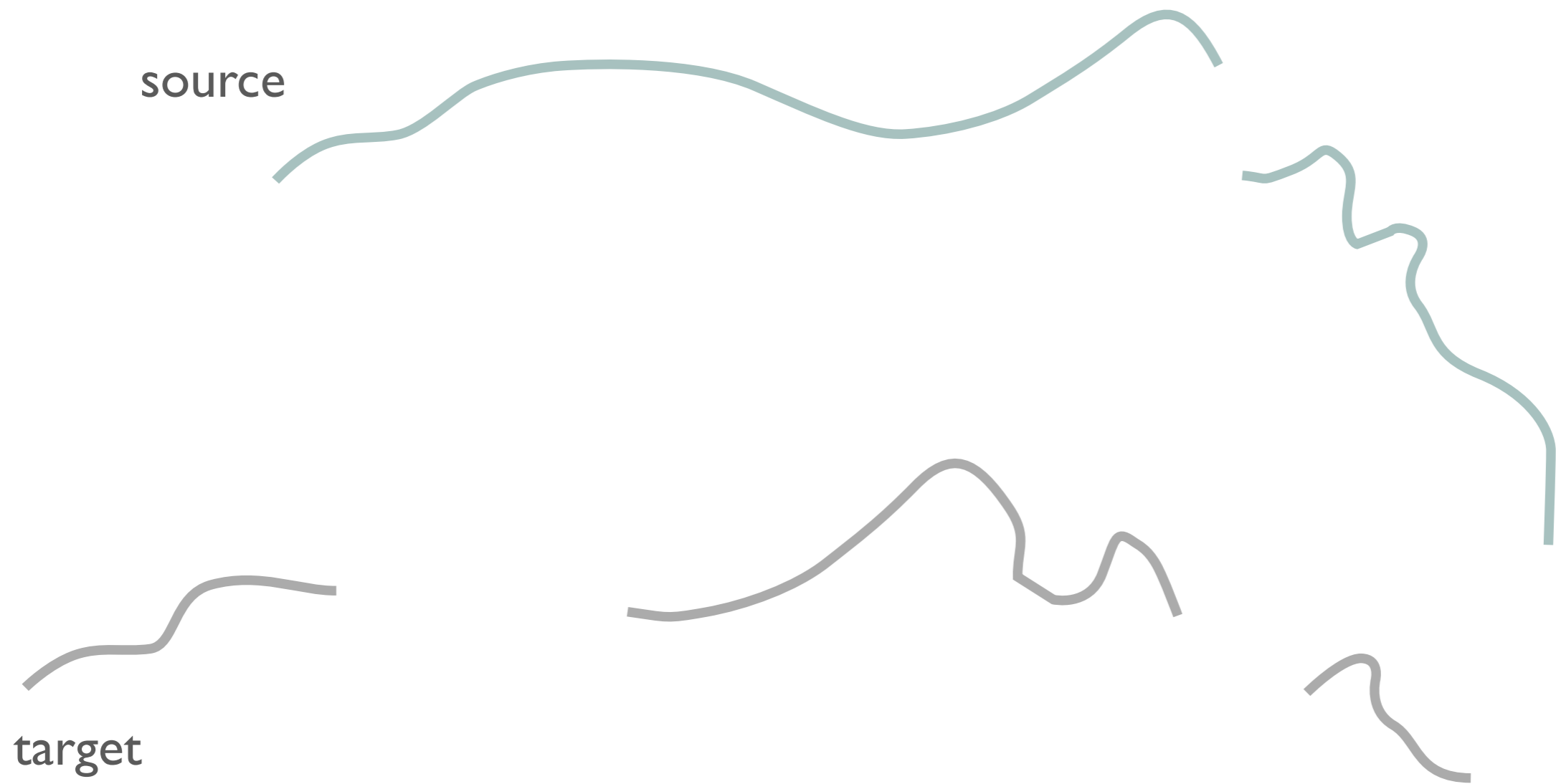


# Ingredients

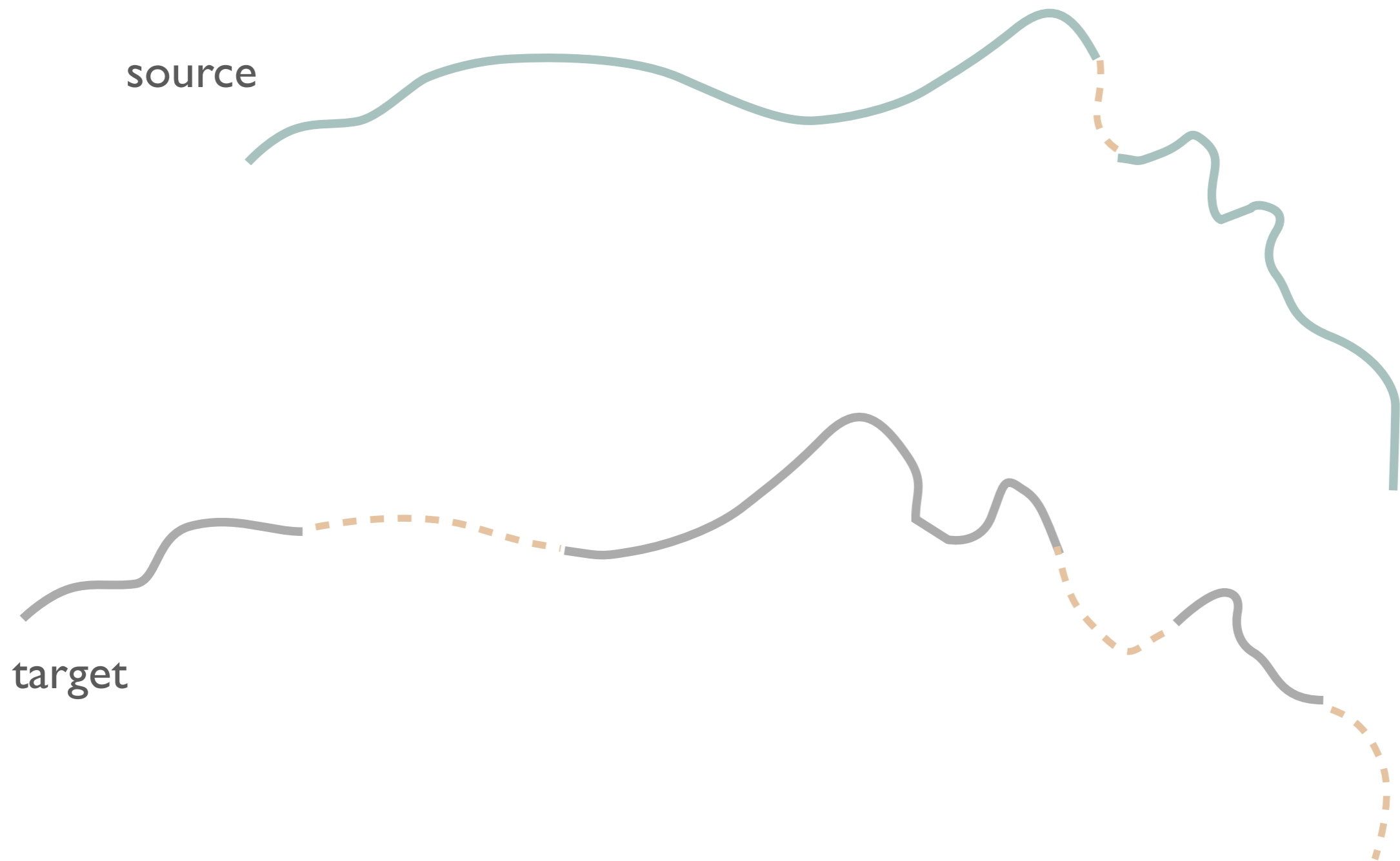
source



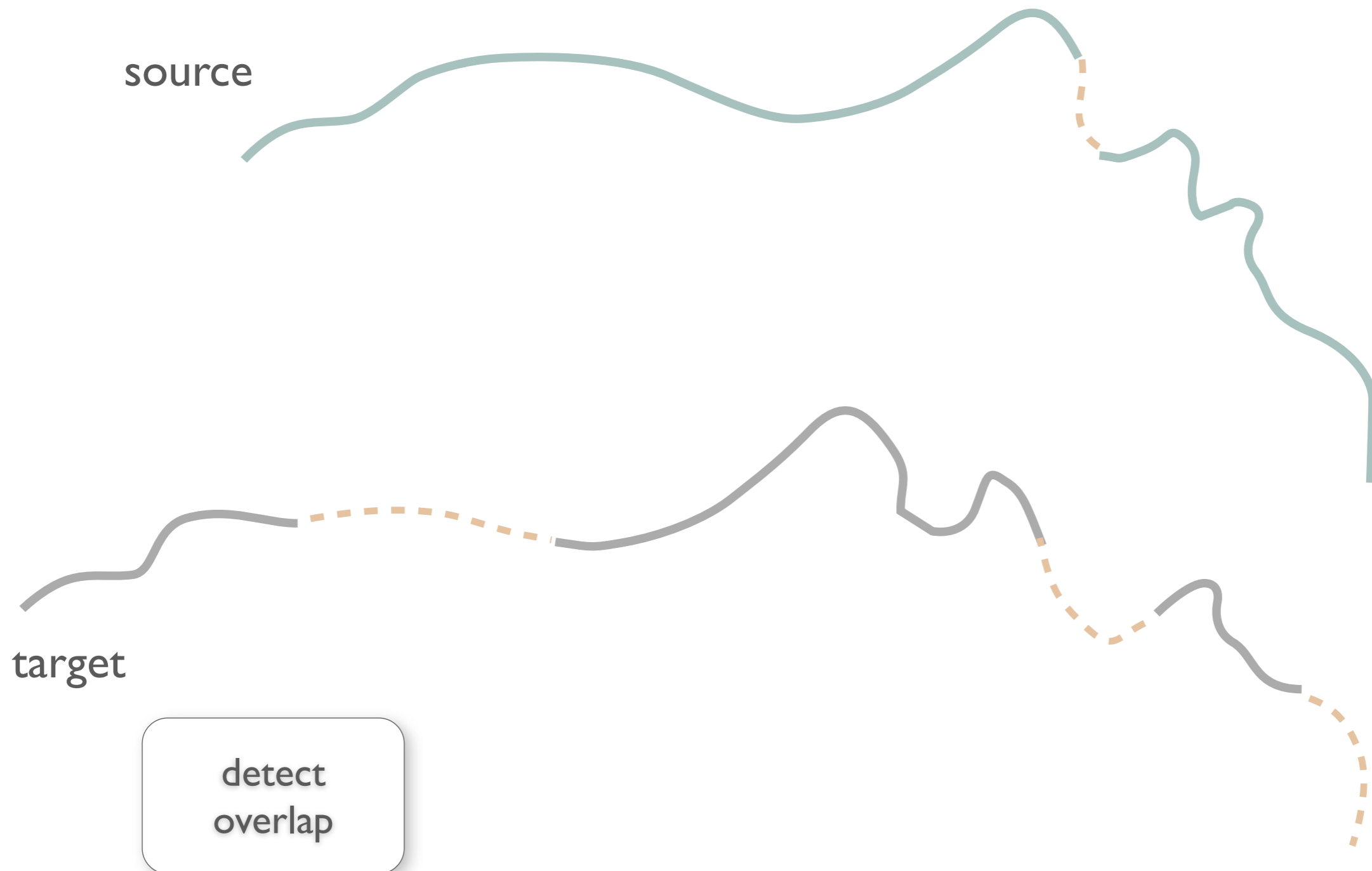
# Ingredients



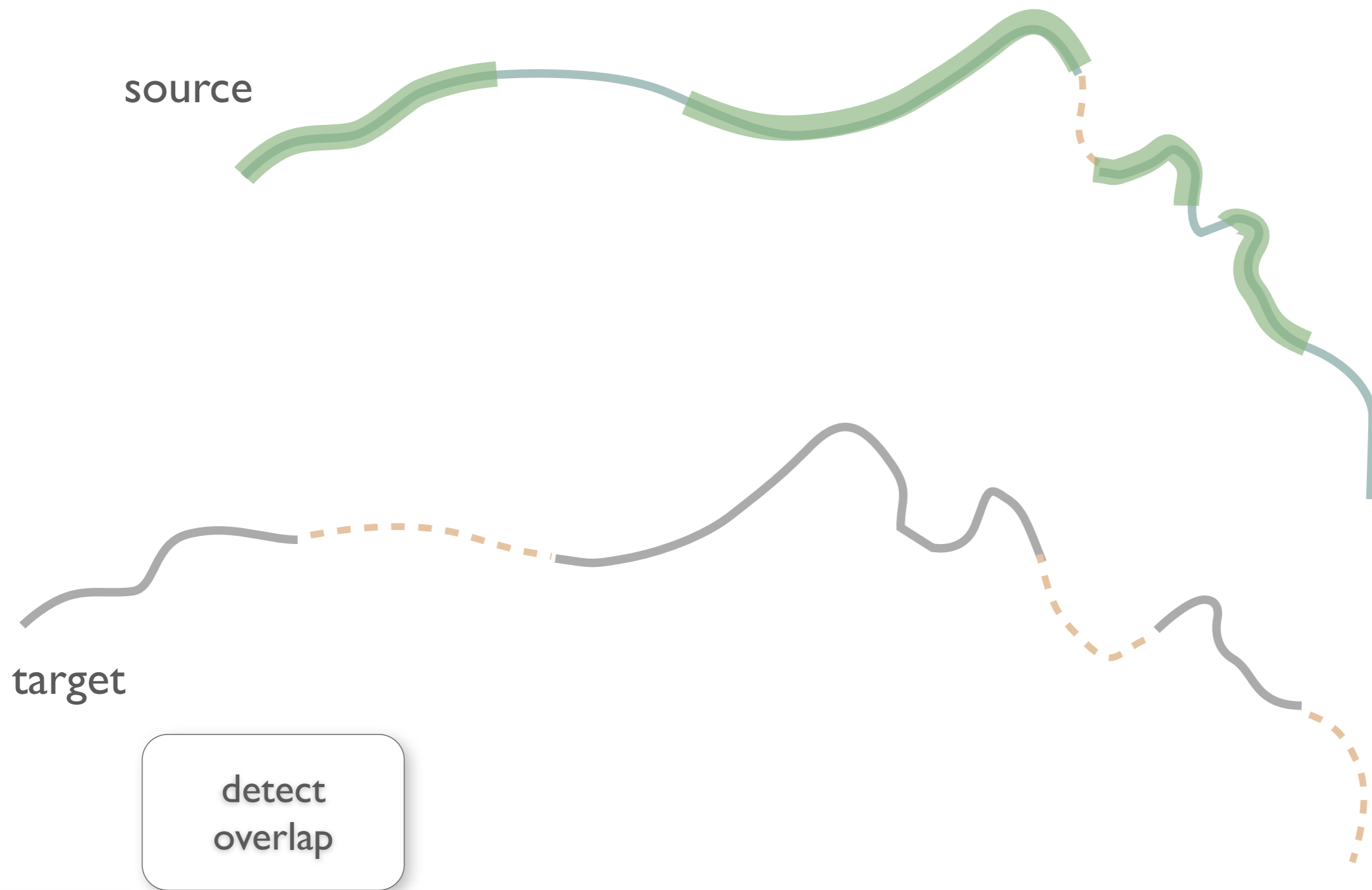
# Ingredients



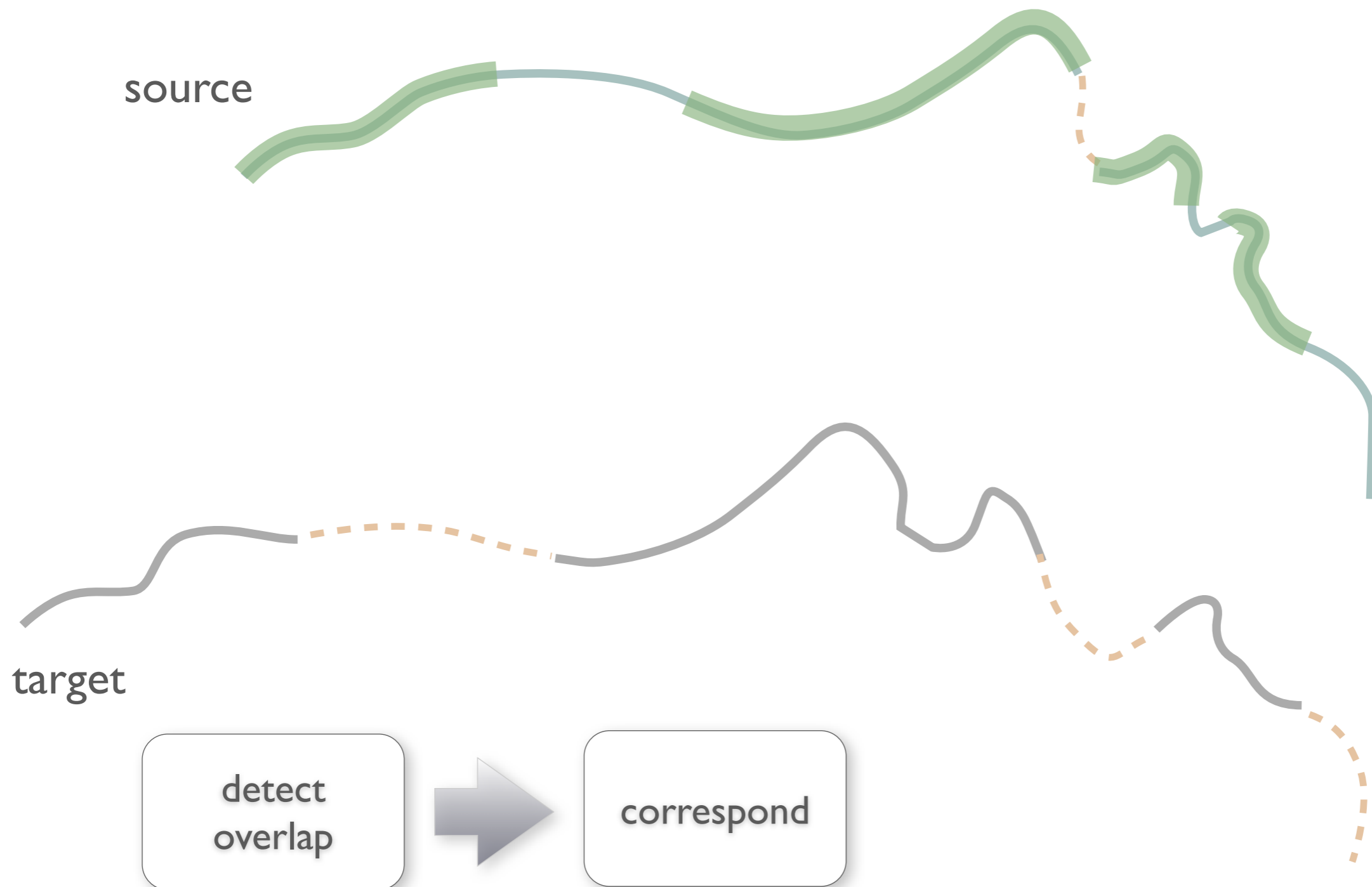
# Ingredients



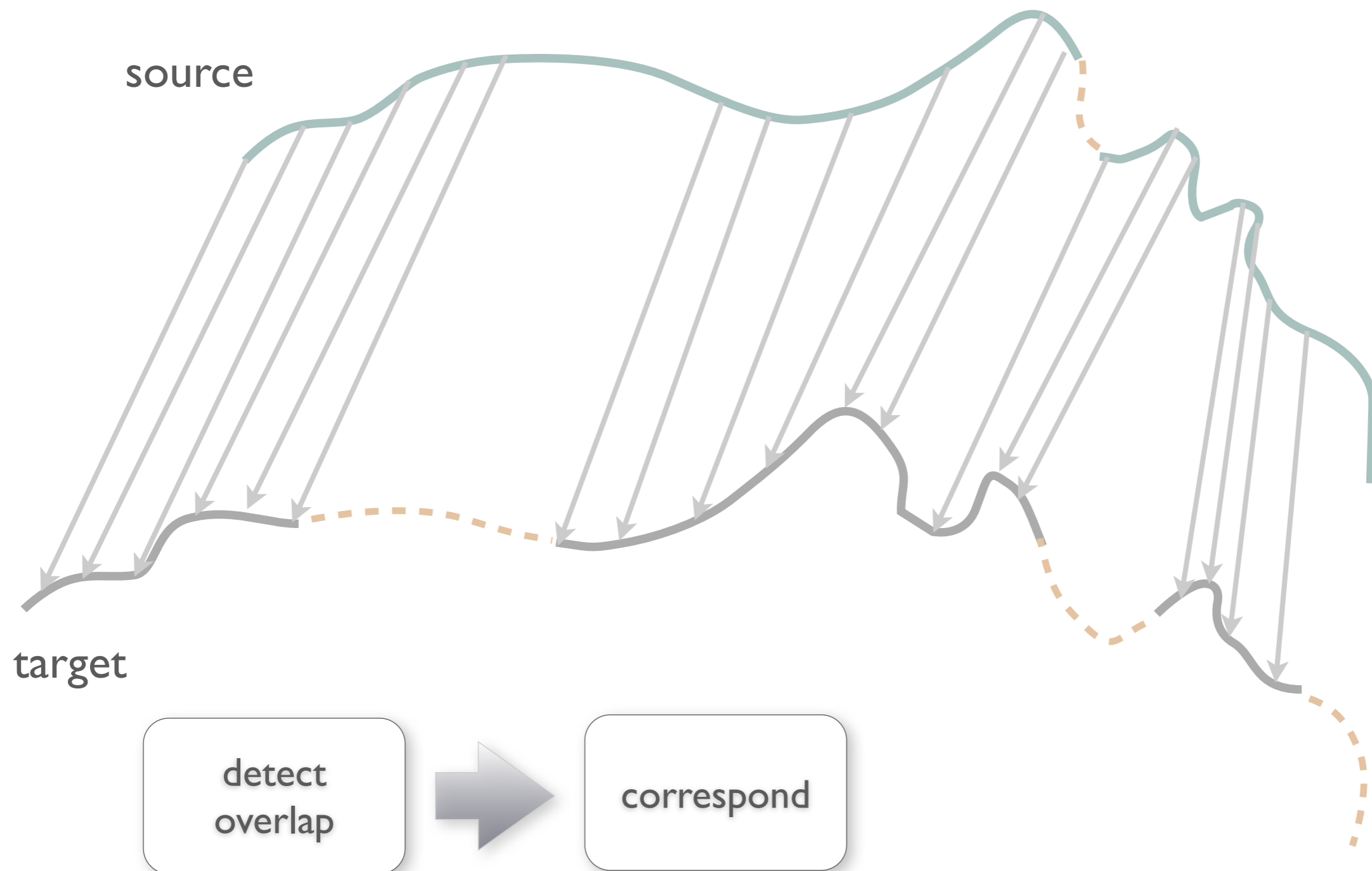
# Ingredients



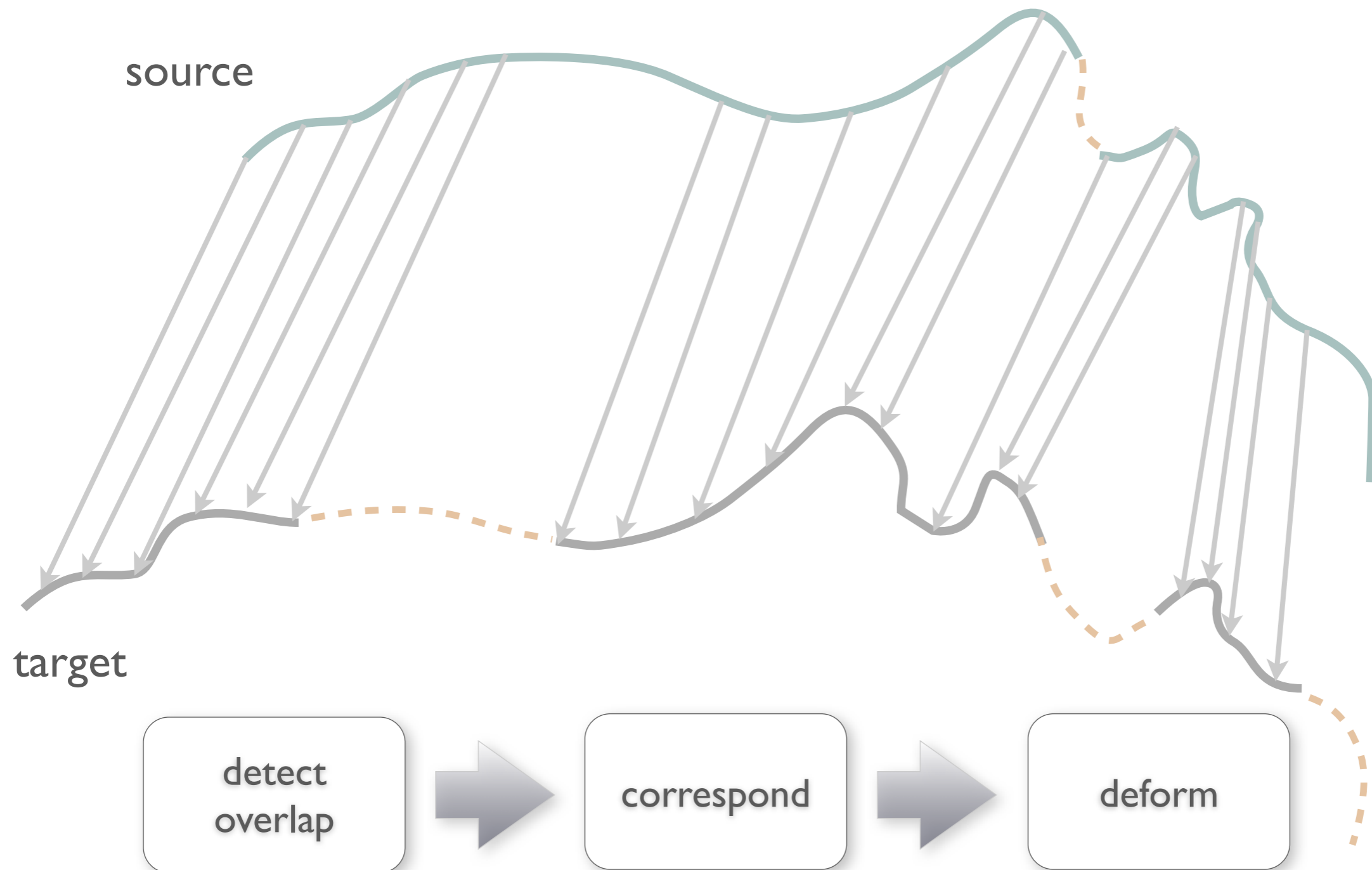
# Ingredients



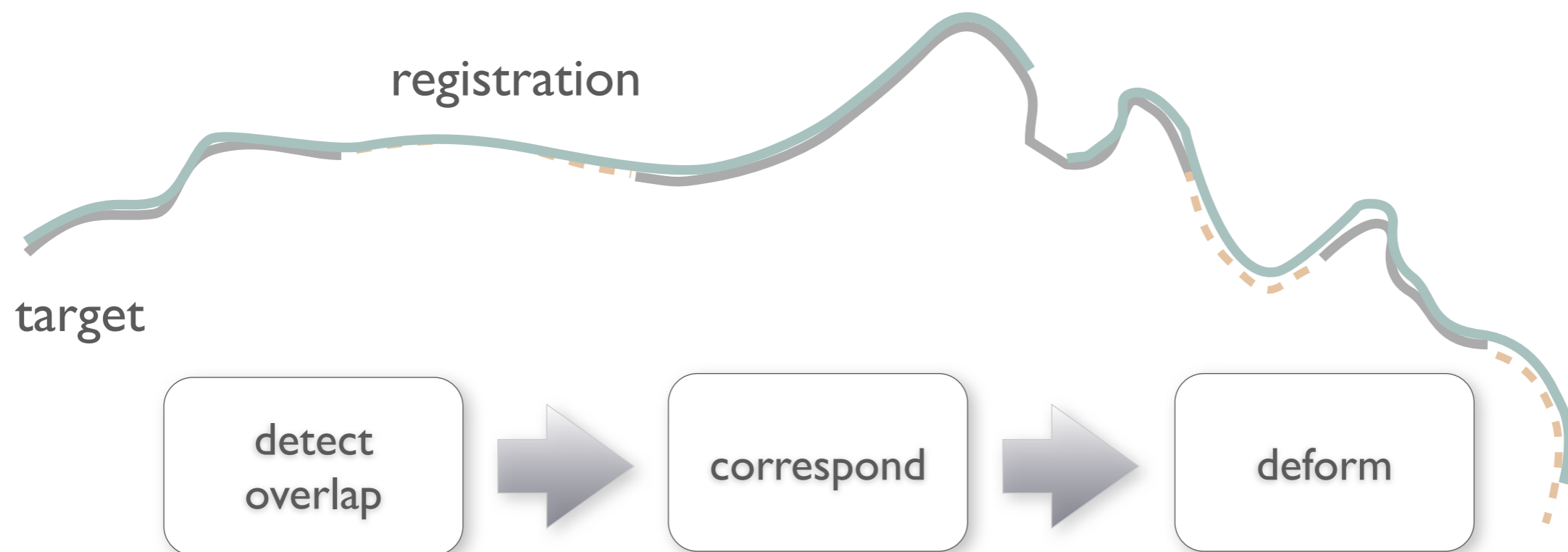
# Ingredients



# Ingredients



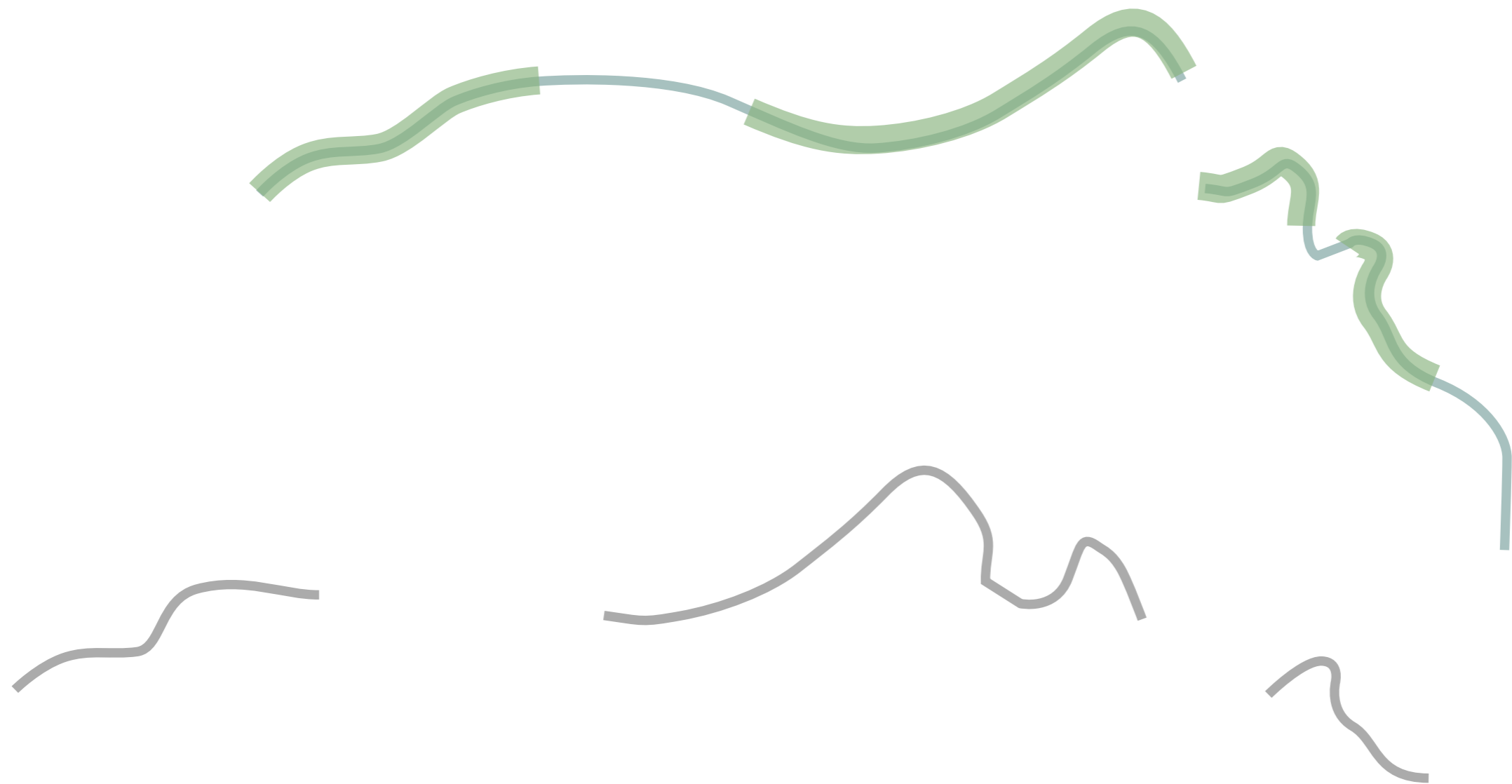
# Ingredients



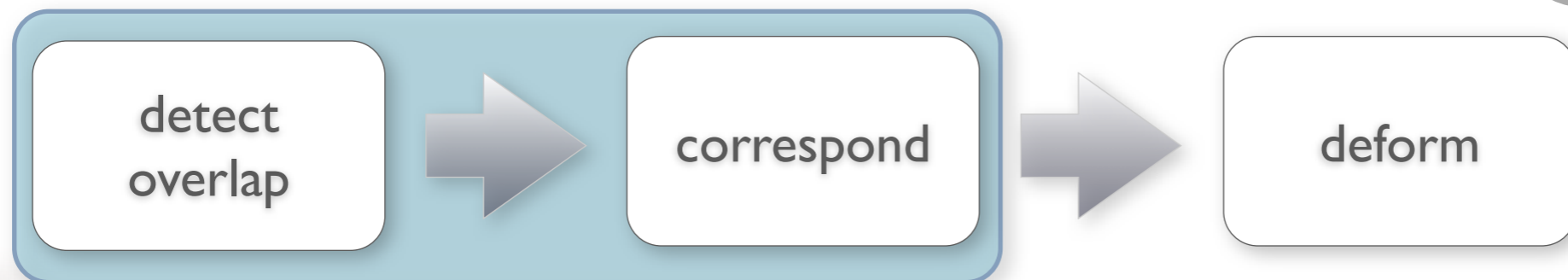
# What's so difficult?



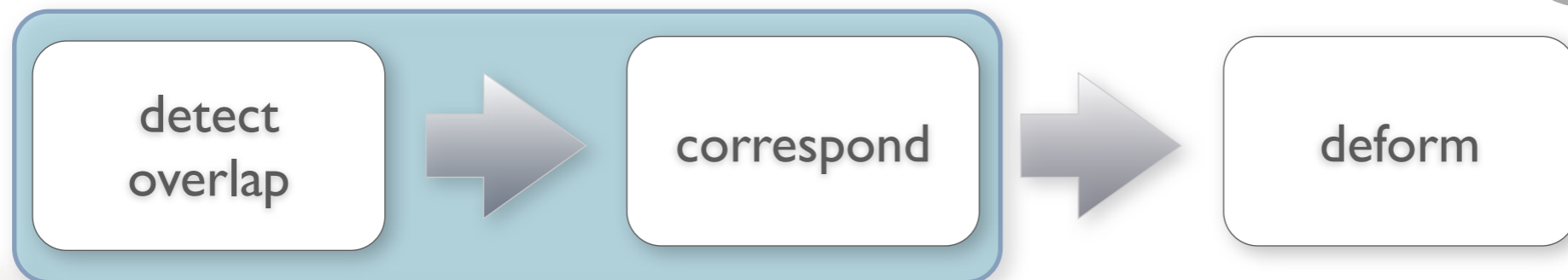
# What's so difficult?



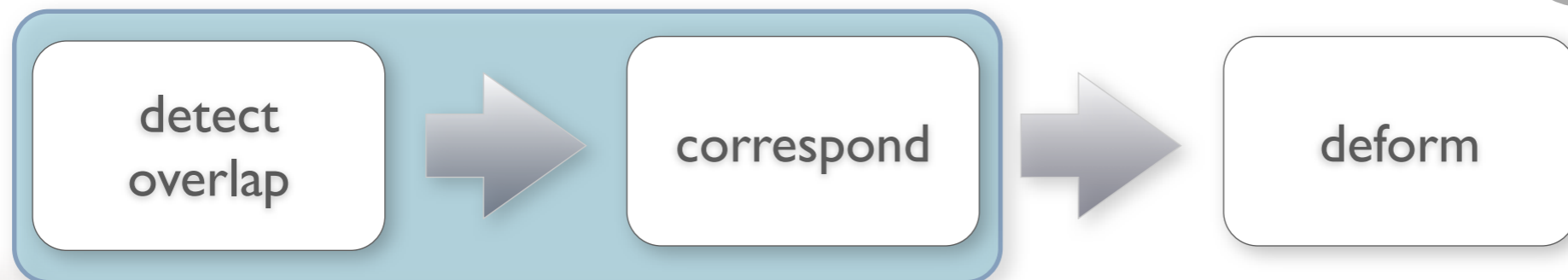
# What's so difficult?



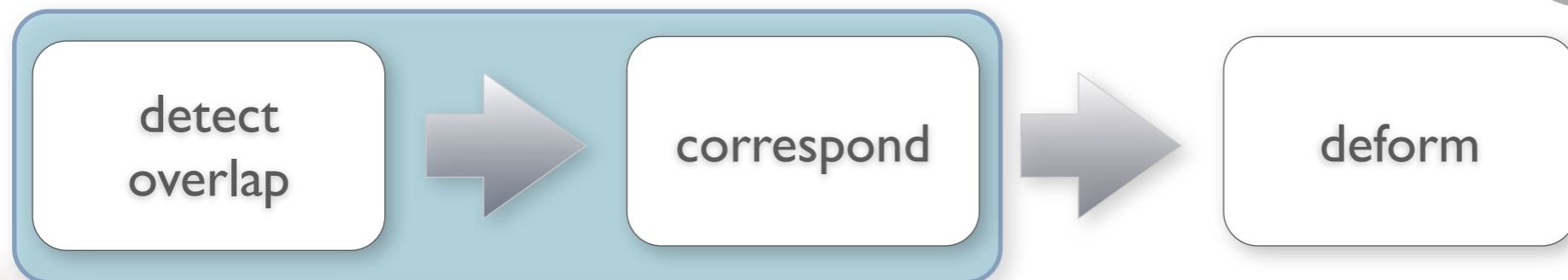
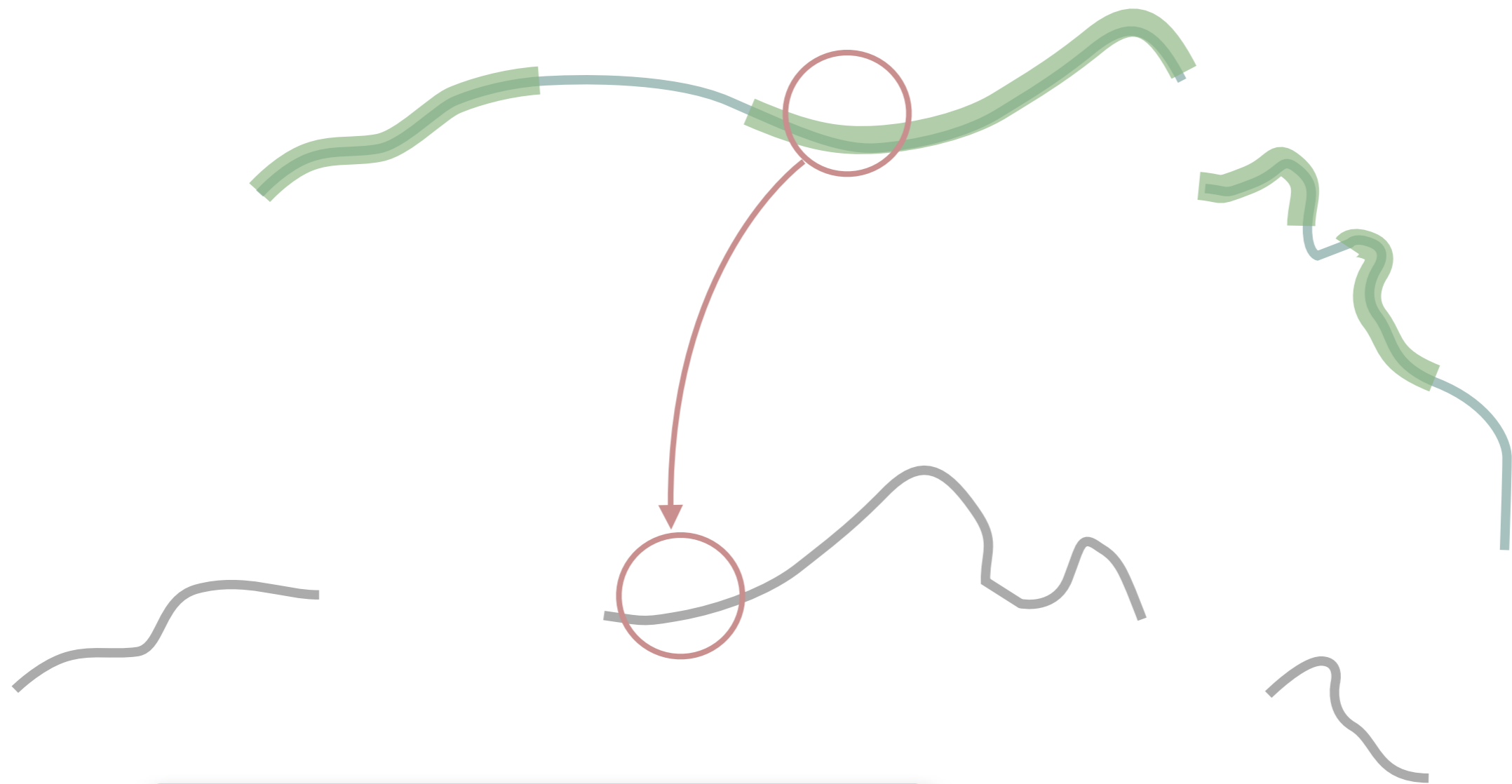
# What's so difficult?



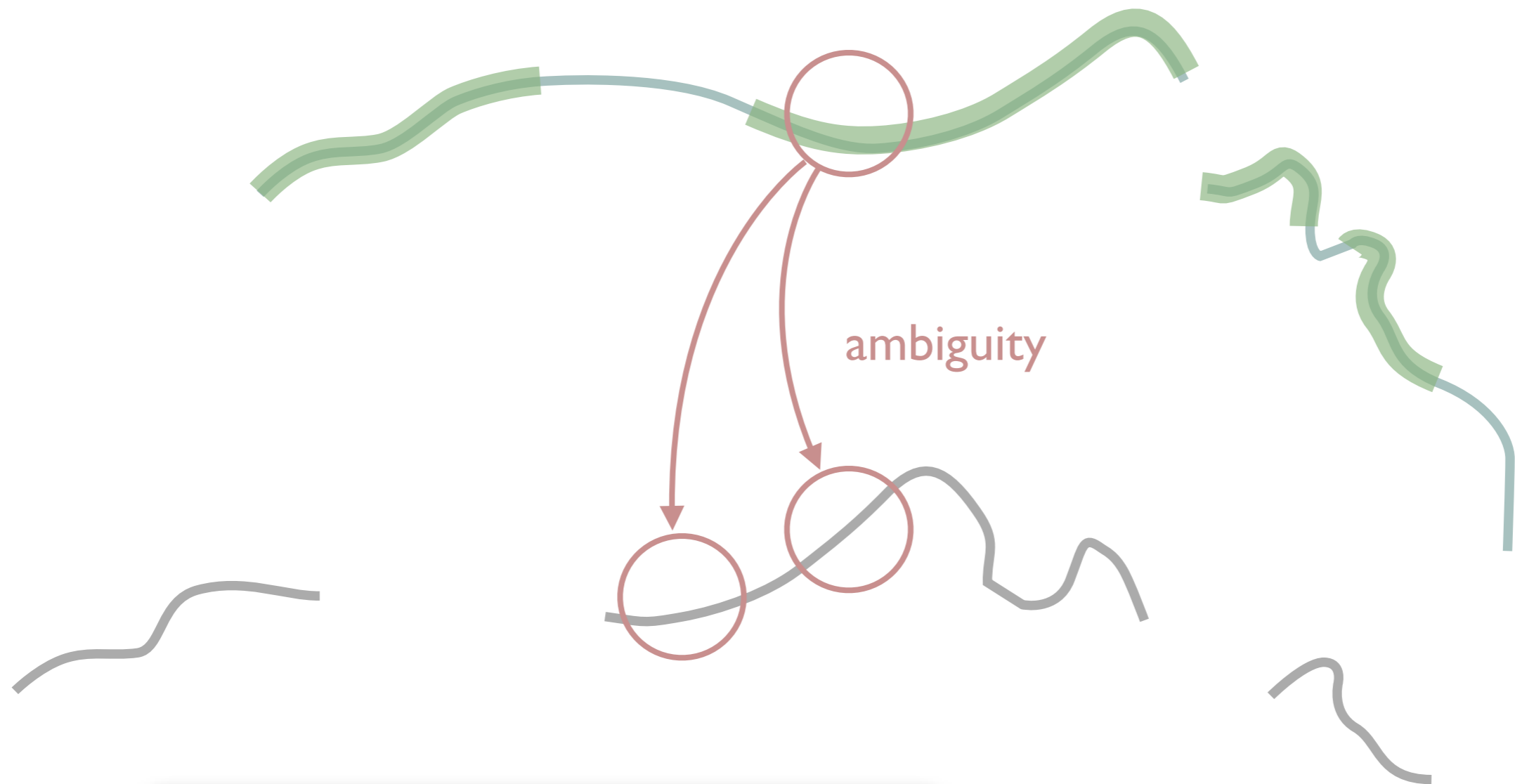
# What's so difficult?



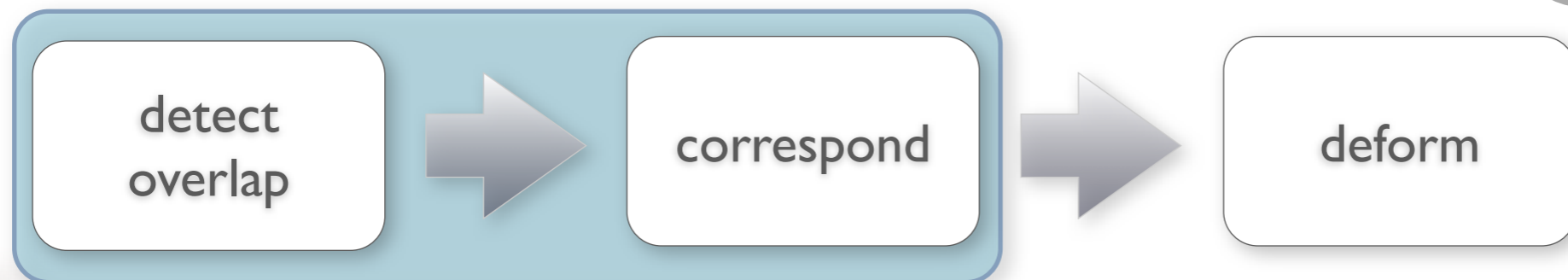
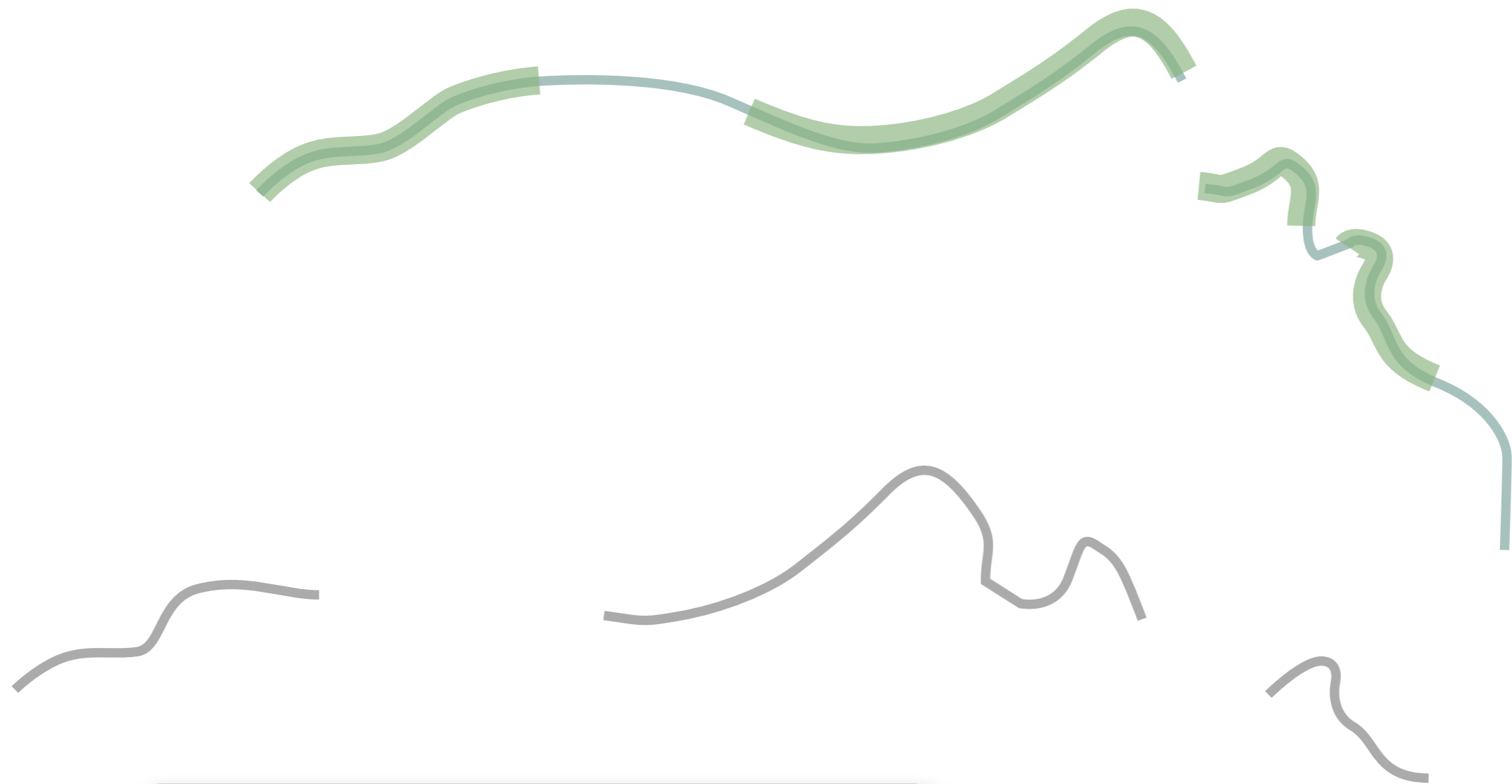
# What's so difficult?



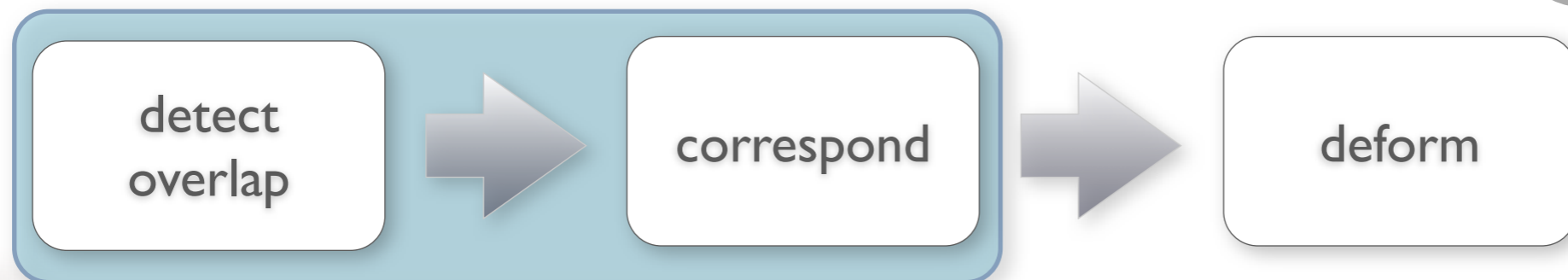
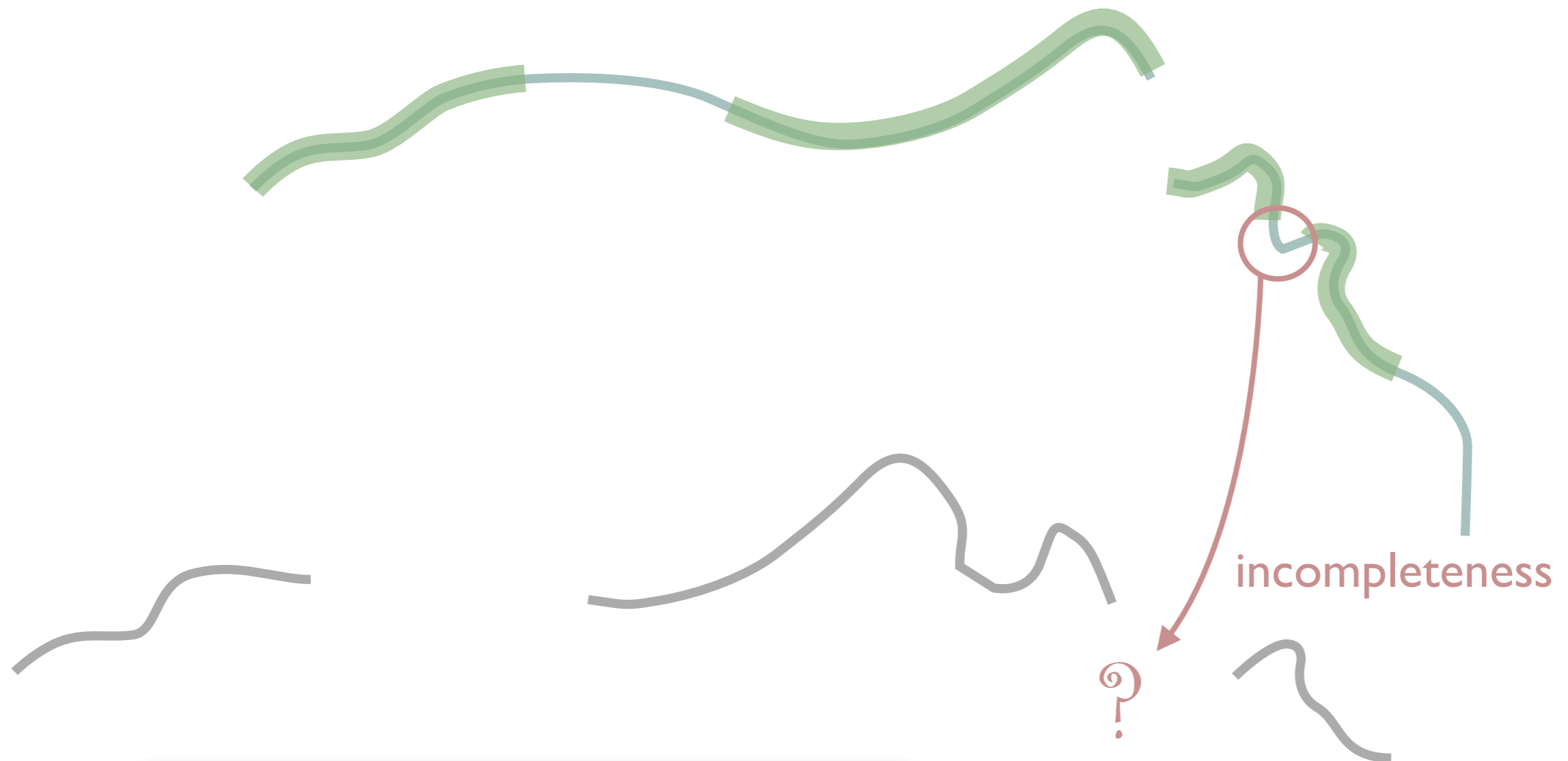
# What's so difficult?



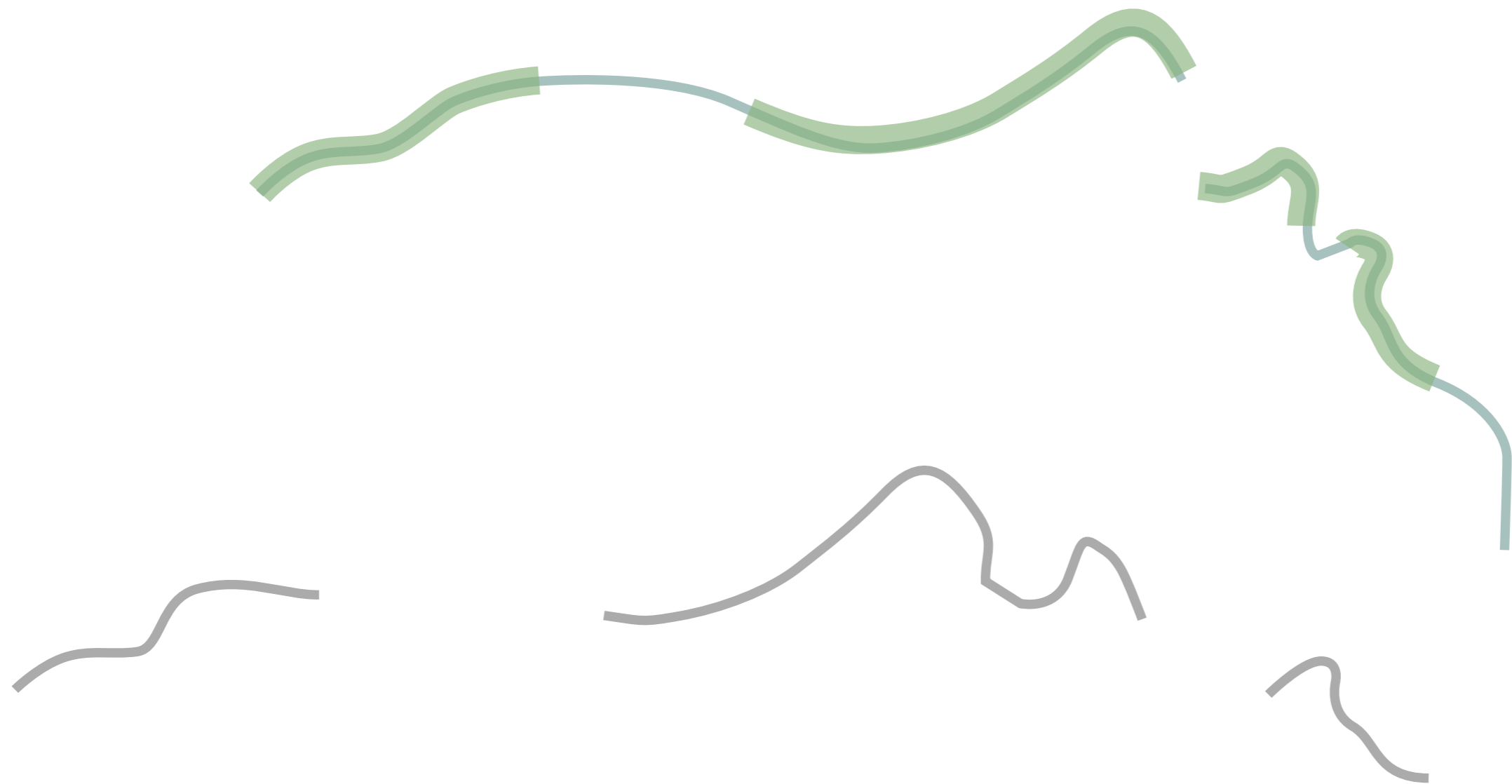
# What's so difficult?



# What's so difficult?



# What's so difficult?



detect  
overlap

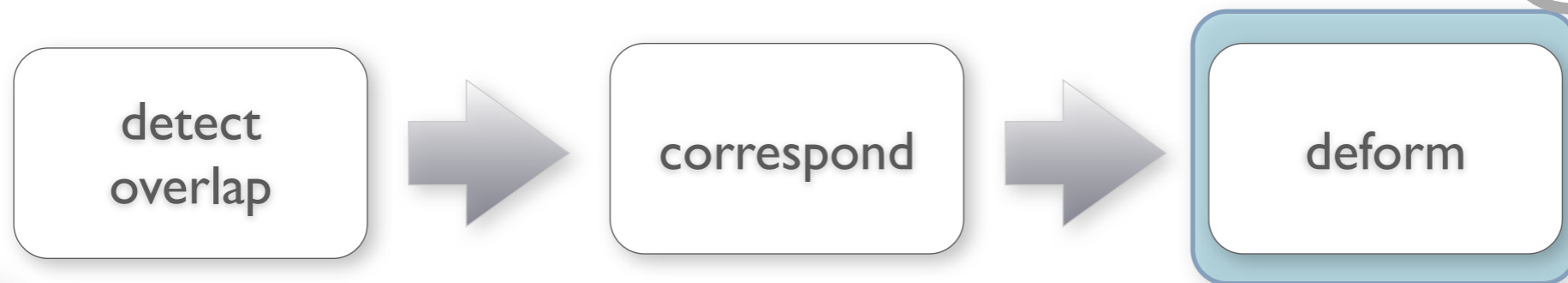


correspond

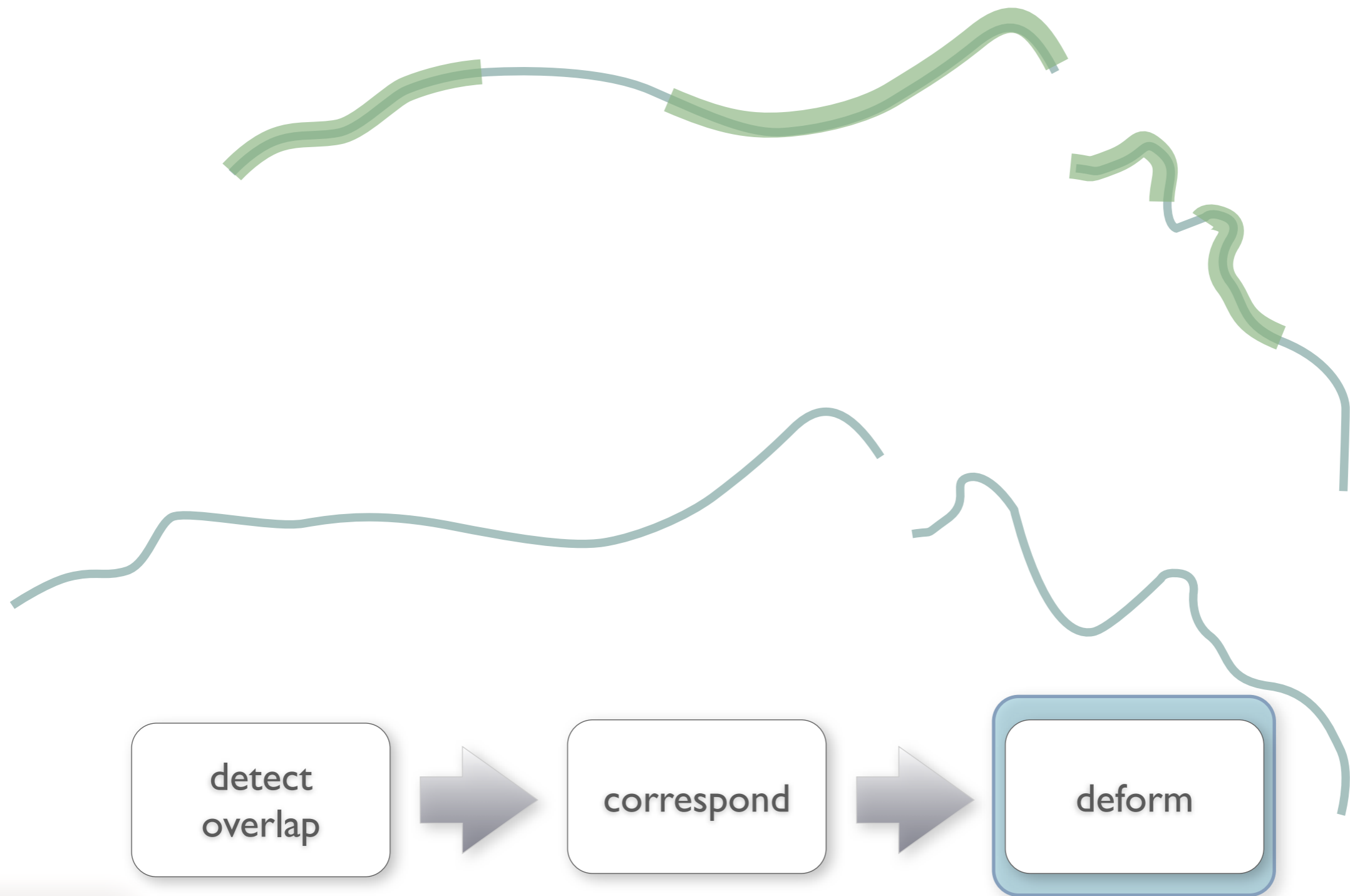


deform

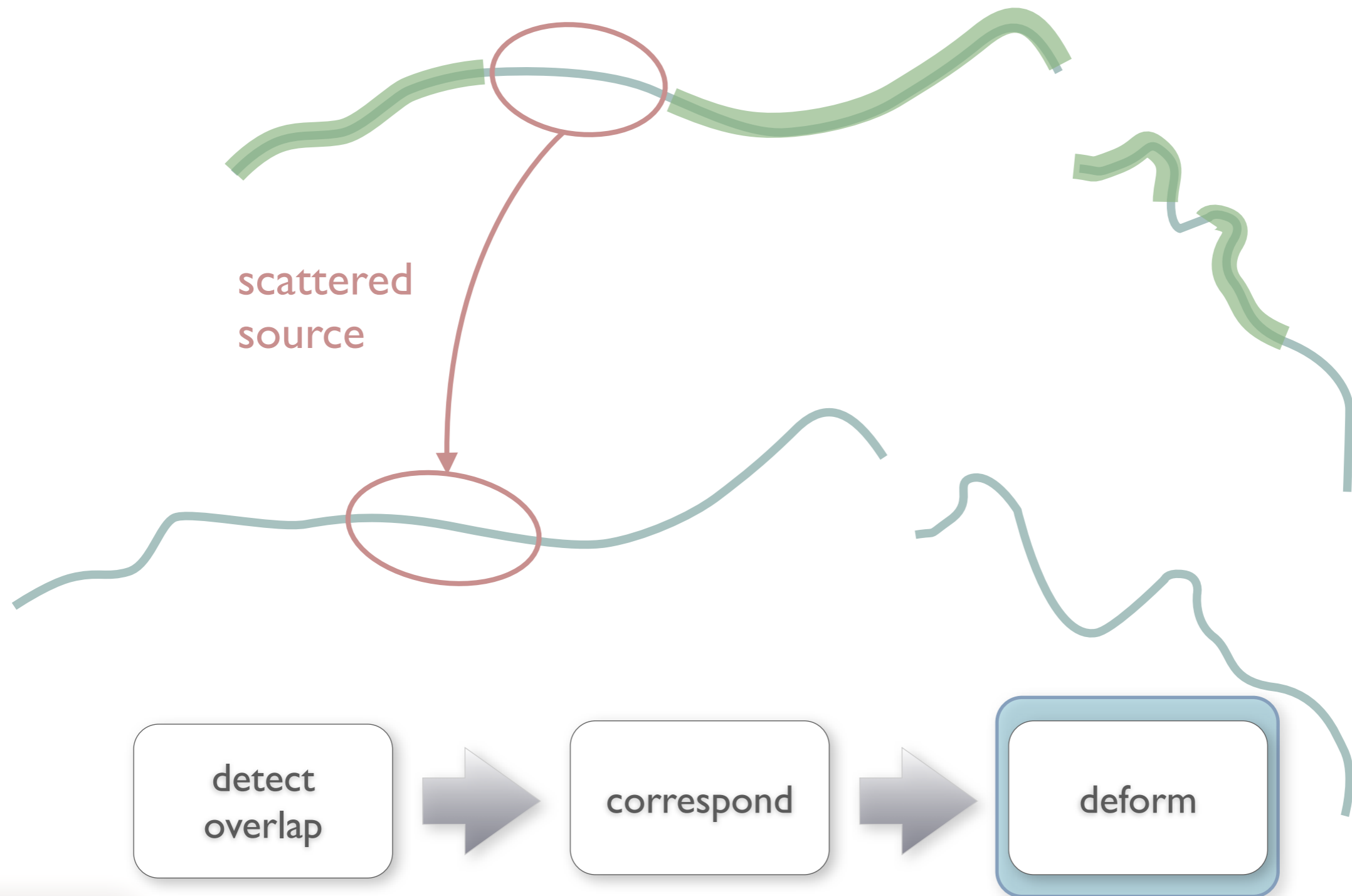
# What's so difficult?



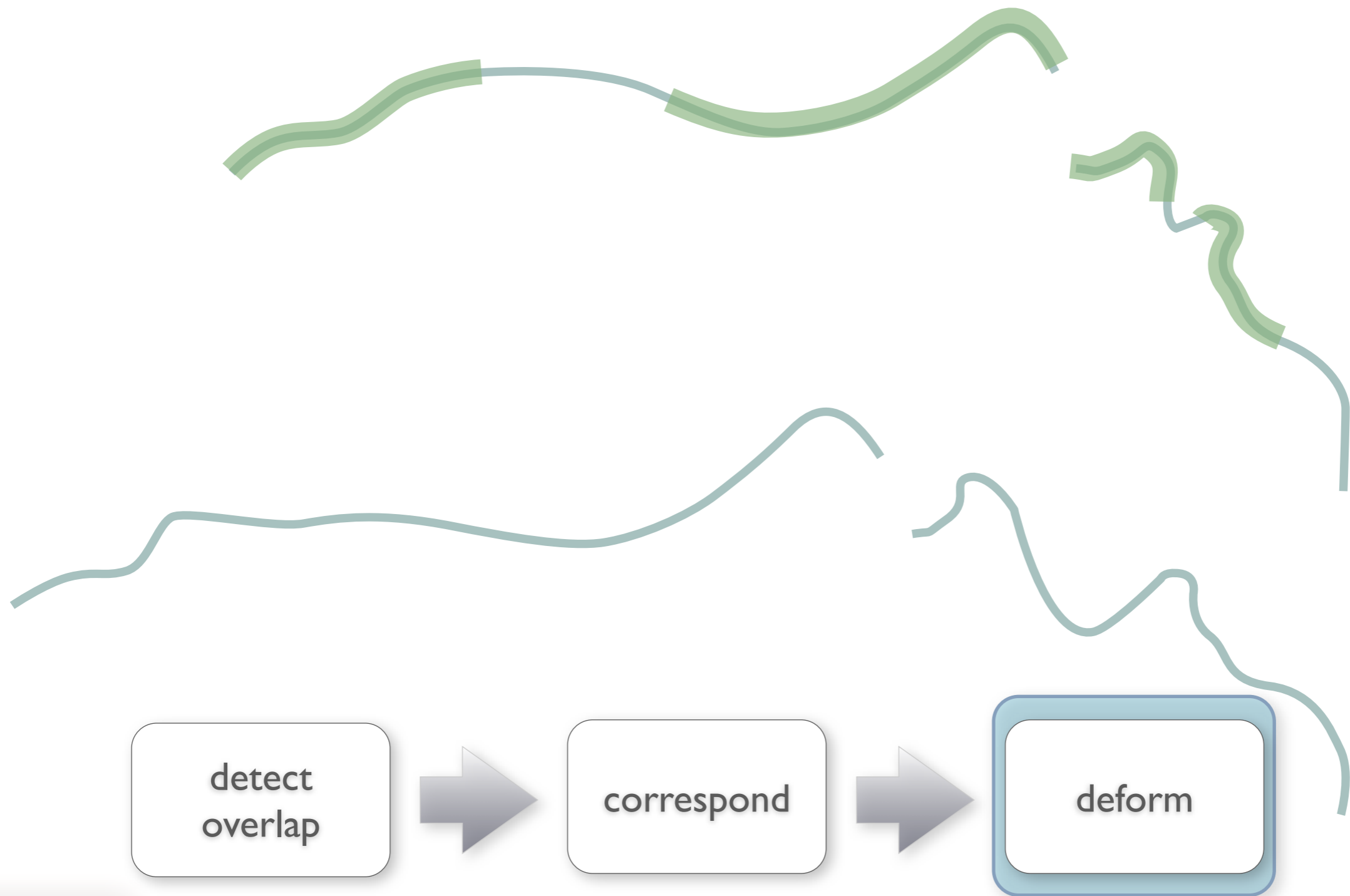
# What's so difficult?



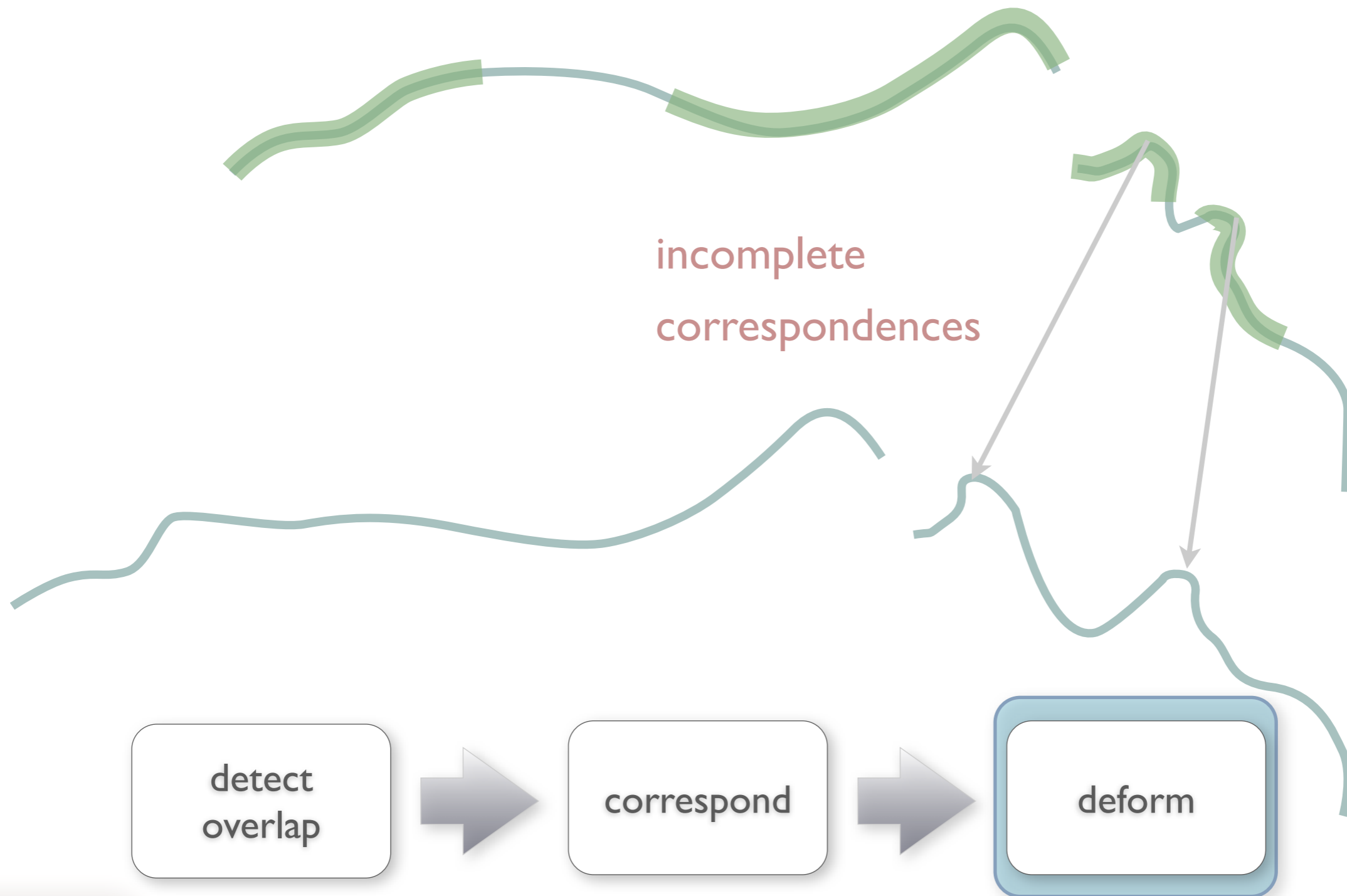
# What's so difficult?



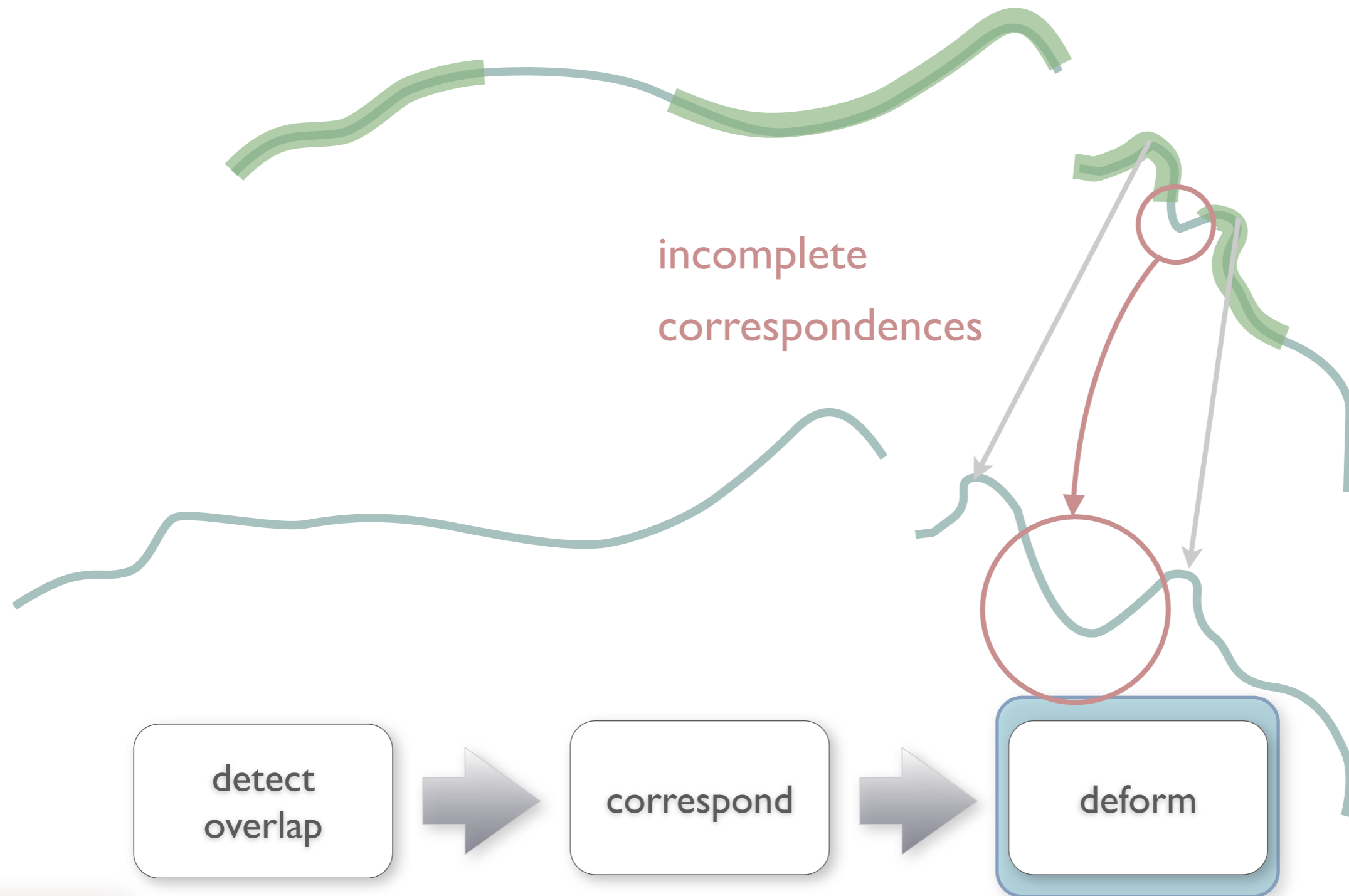
# What's so difficult?



# What's so difficult?



# What's so difficult?



# Challenges

detect  
overlap

correspond

deform

# Challenges

correspond

detect  
overlap

deform

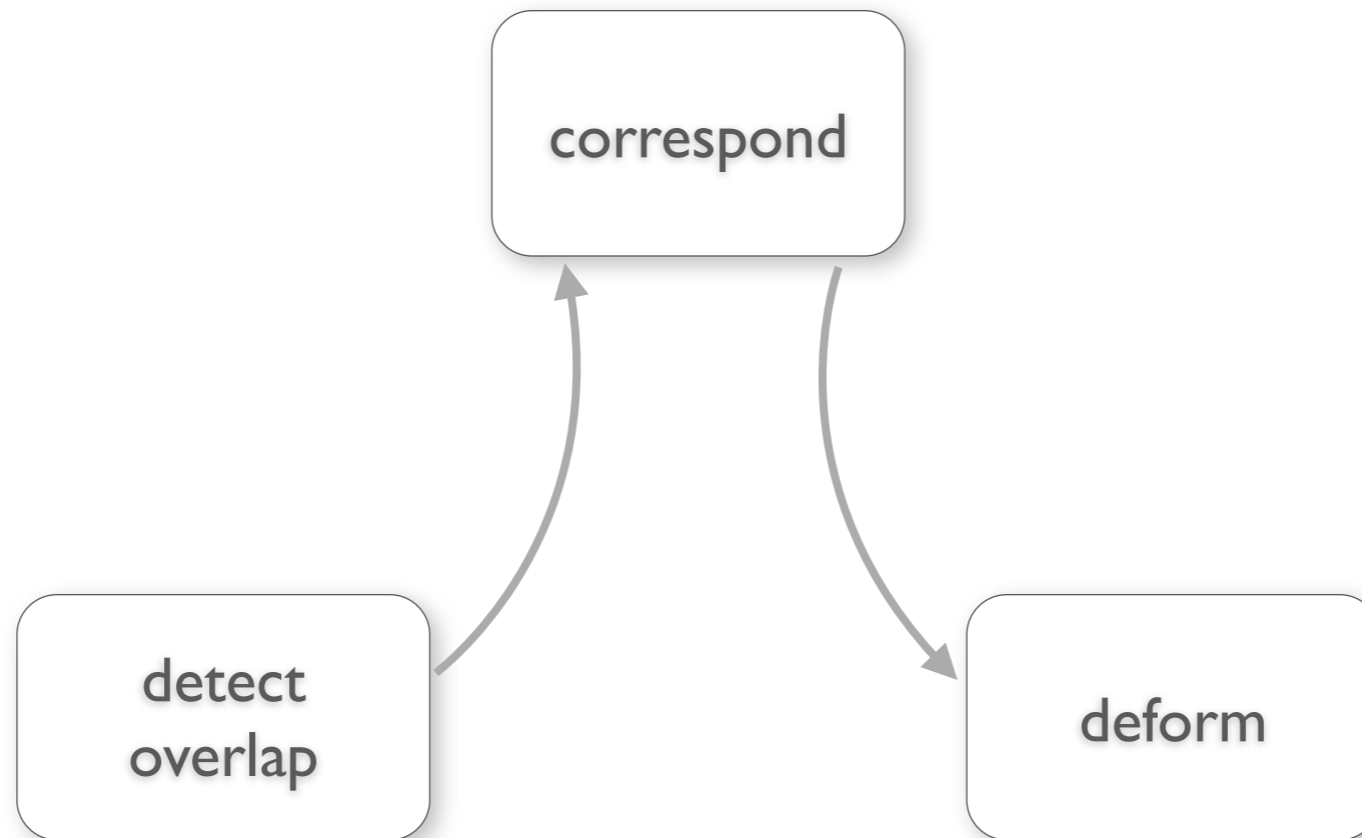
# Observation

correspond

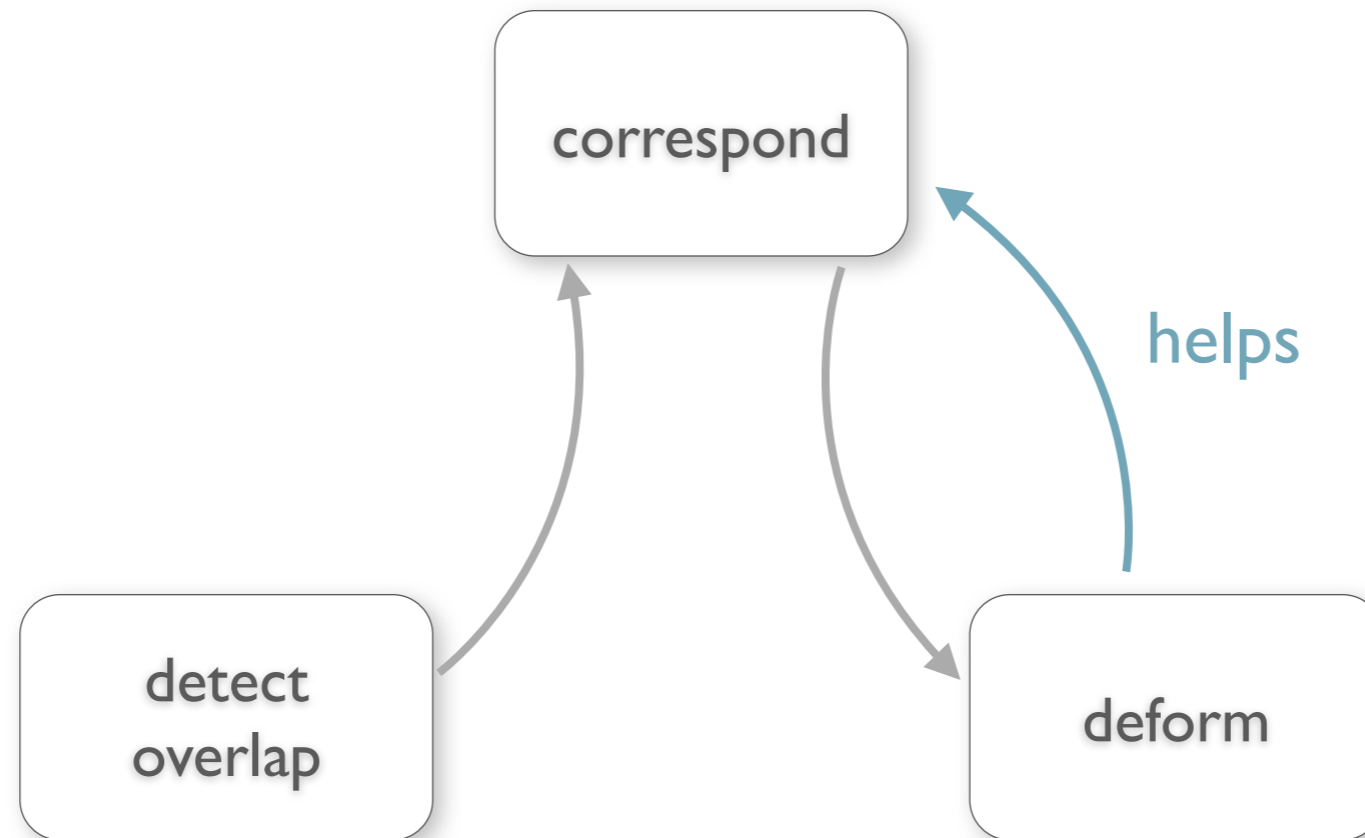
detect  
overlap

deform

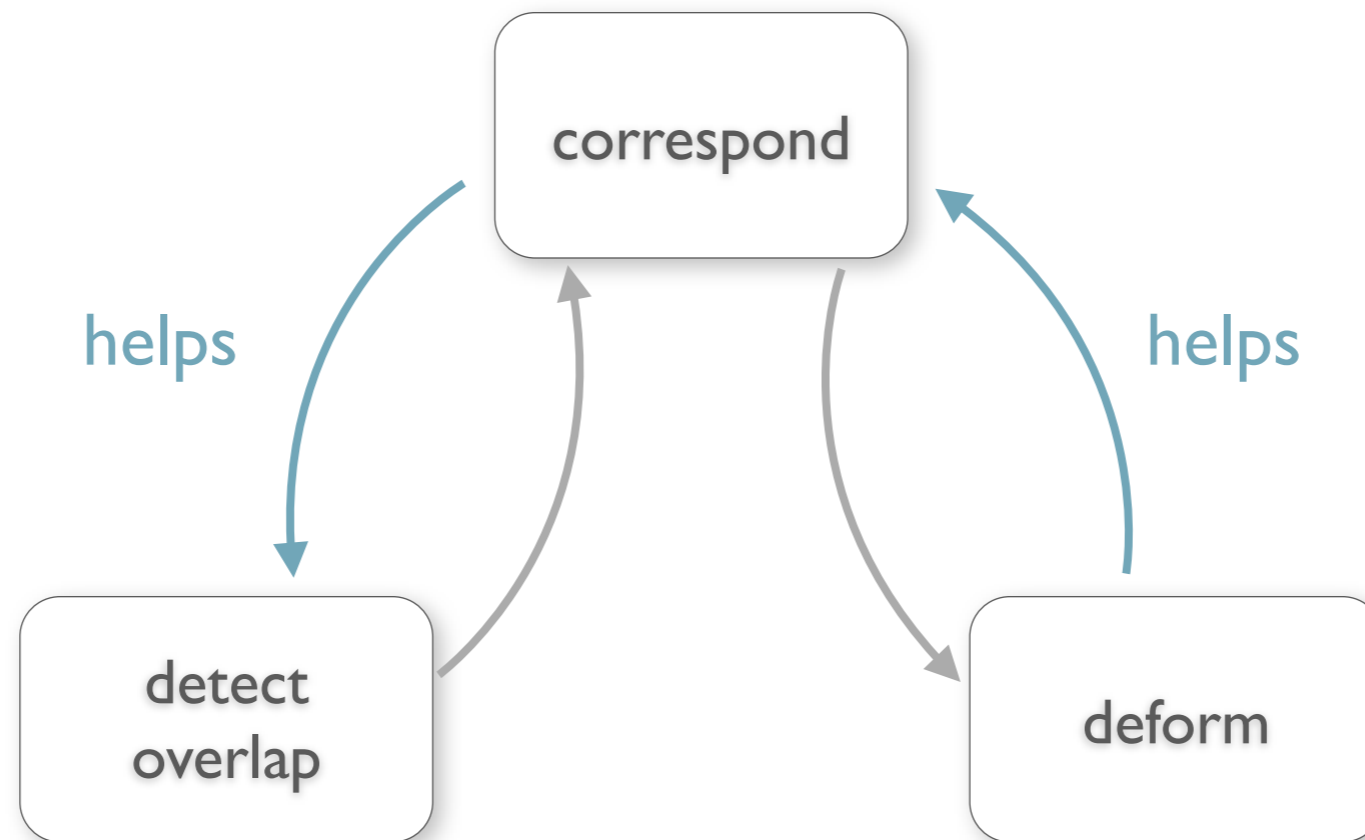
# Observation



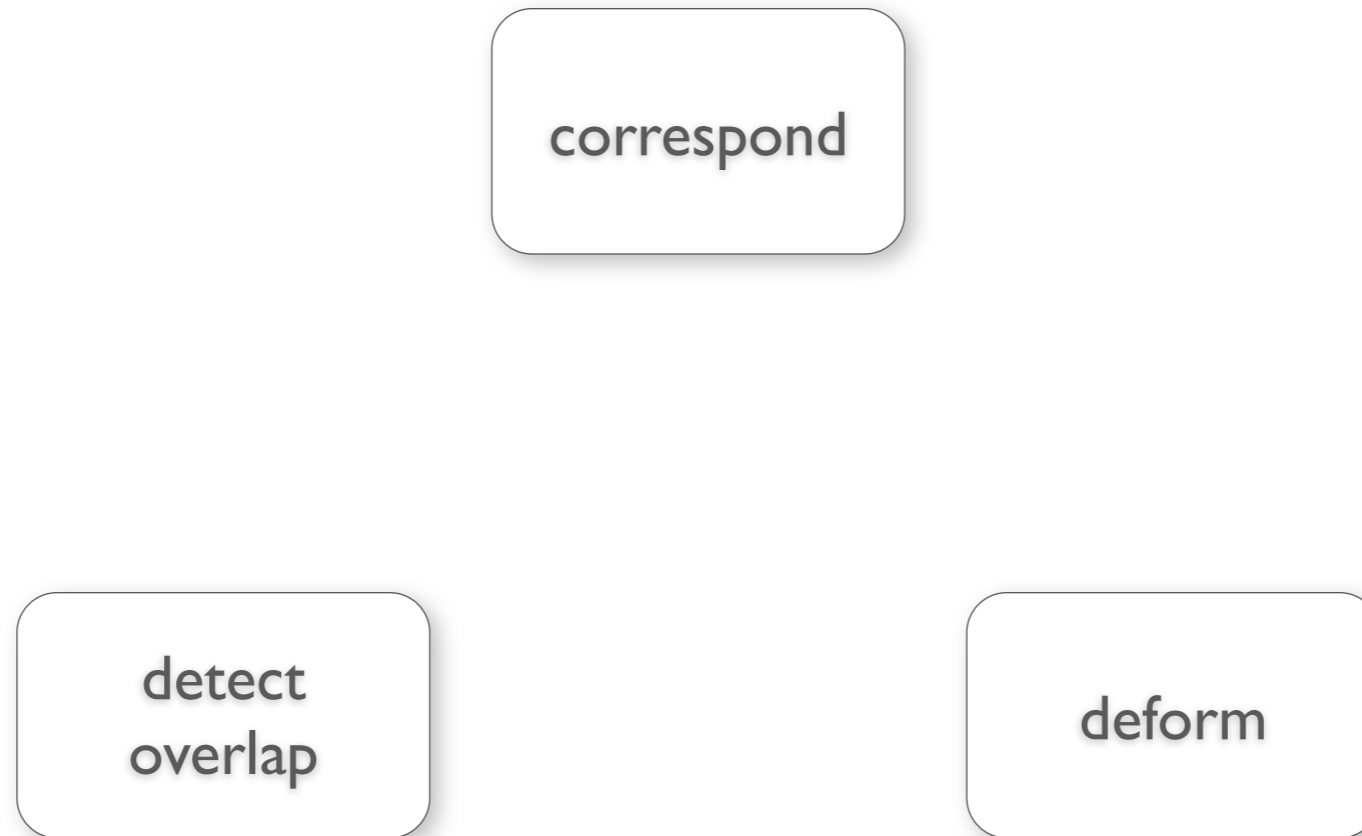
# Observation



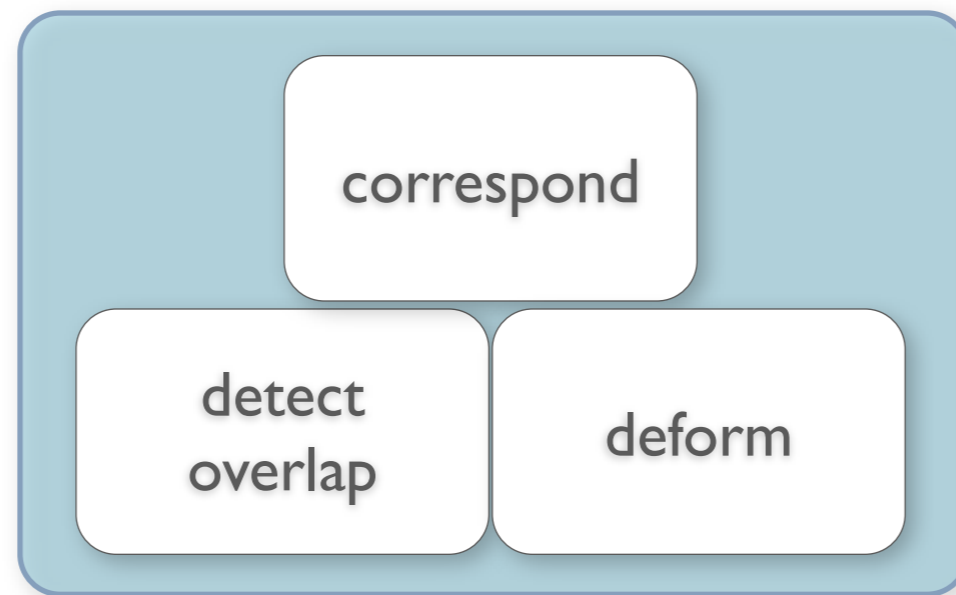
# Observation



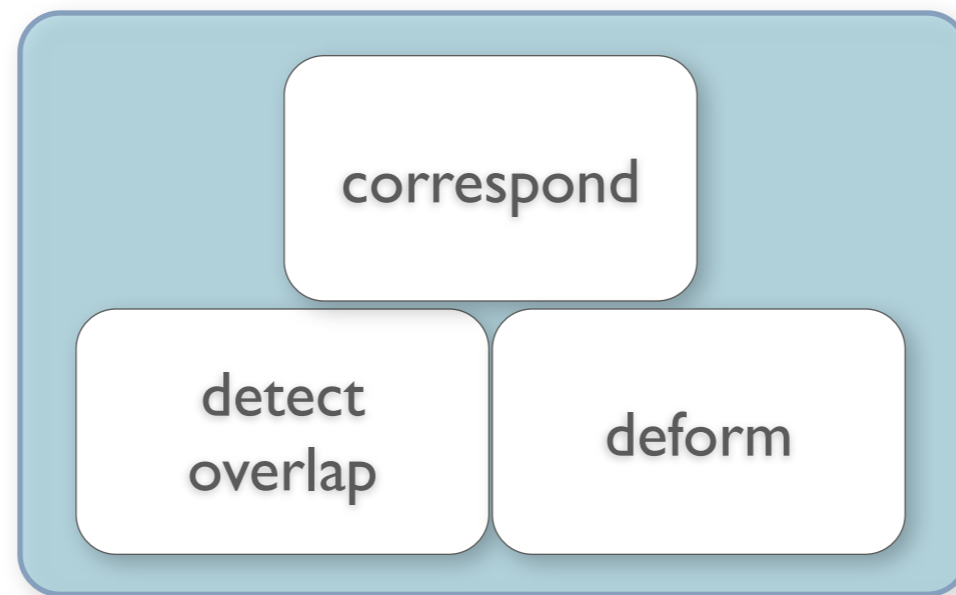
# Observation



# Observation



# Observation



global optimization via local refinement



# Iterative Global Optimization

correspond

detect  
overlap

deform



# Iterative Global Optimization



correspond

detect  
overlap

deform

# Iterative Global Optimization



correspond

detect  
overlap

deform

# Iterative Global Optimization



correspond

detect  
overlap

deform

# Iterative Global Optimization

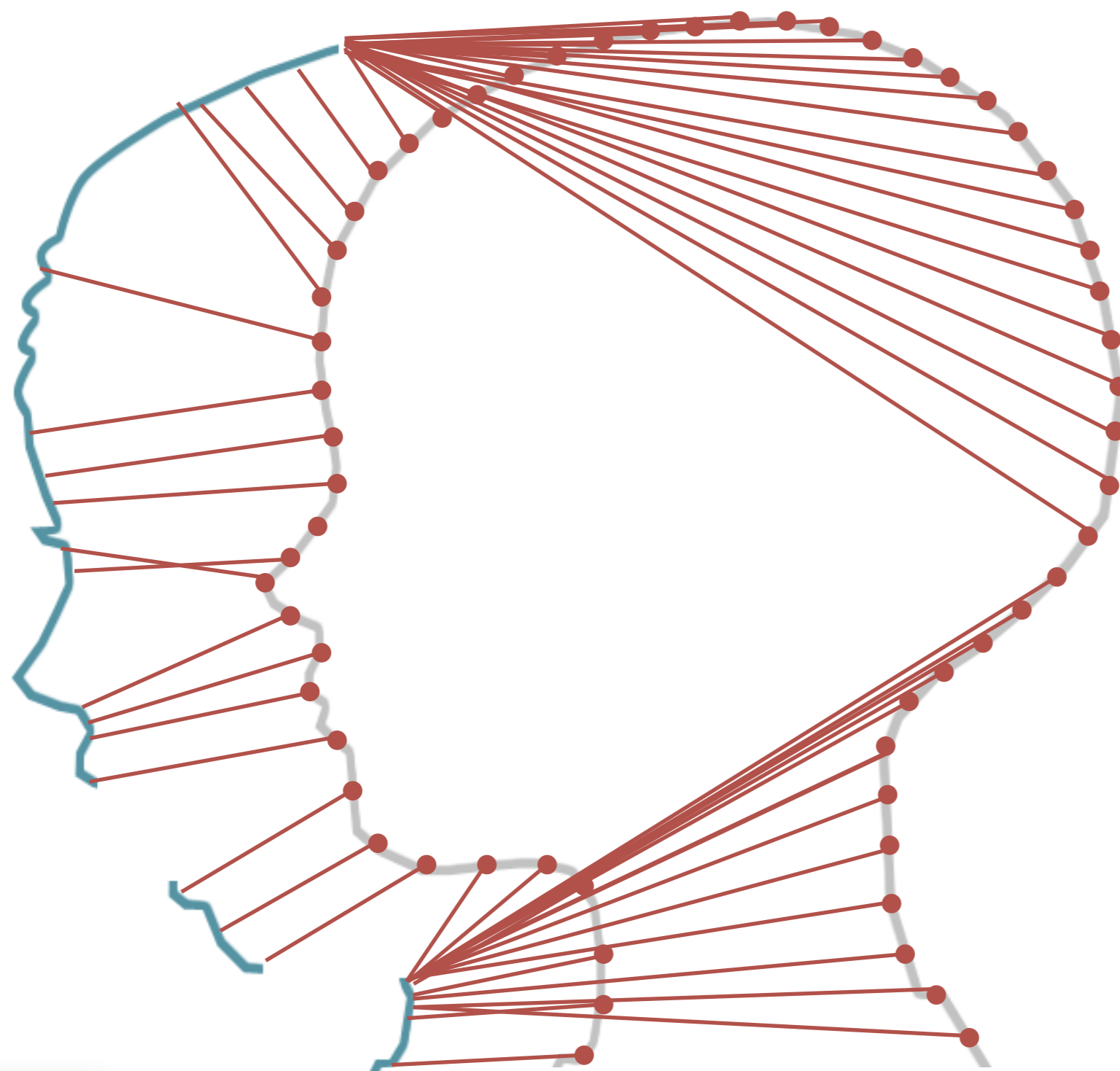


closest point

detect  
overlap

deform

# Iterative Global Optimization

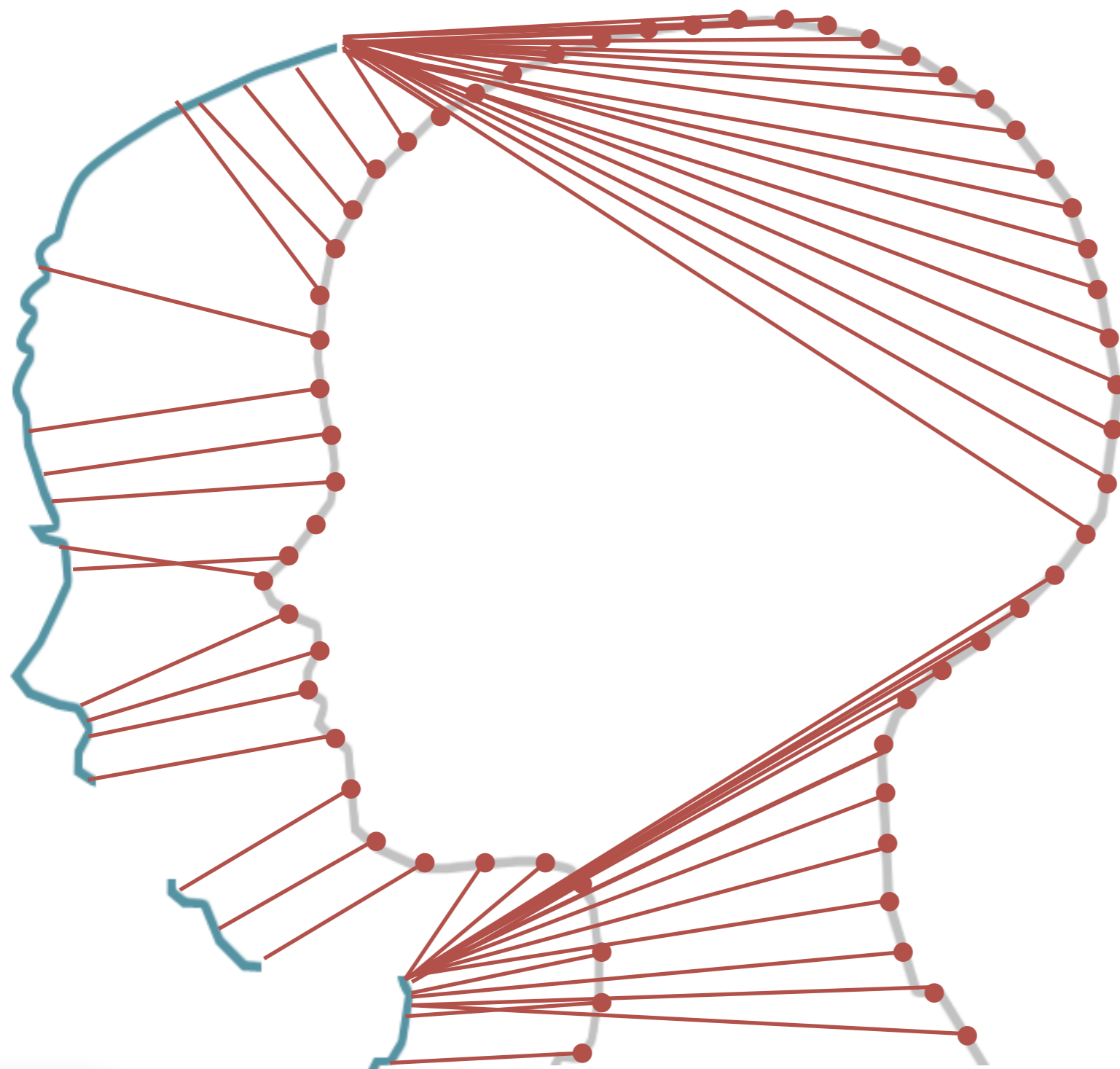


closest point

detect  
overlap

deform

# Iterative Global Optimization



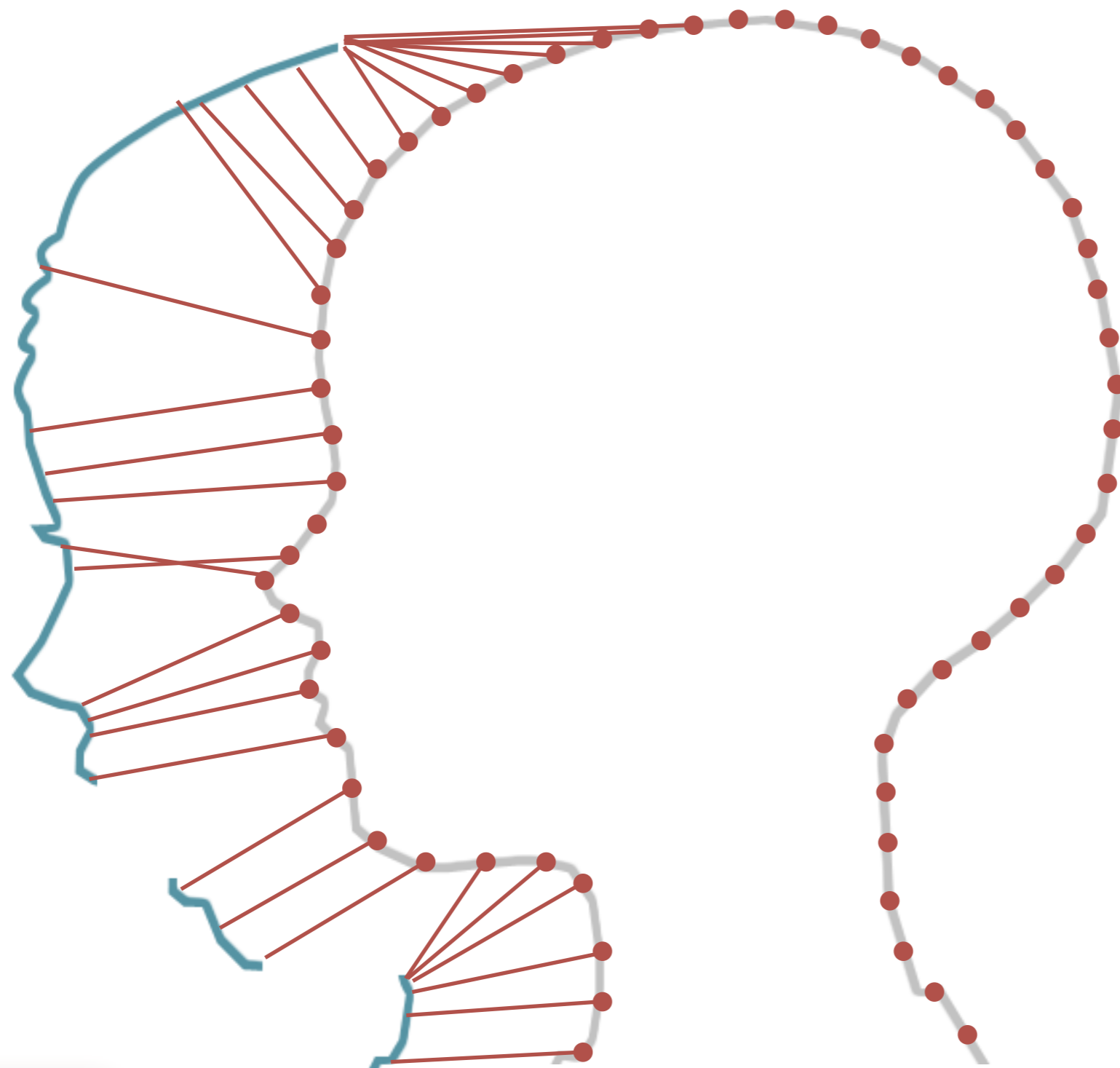
closest point



pruning

deform

# Iterative Global Optimization



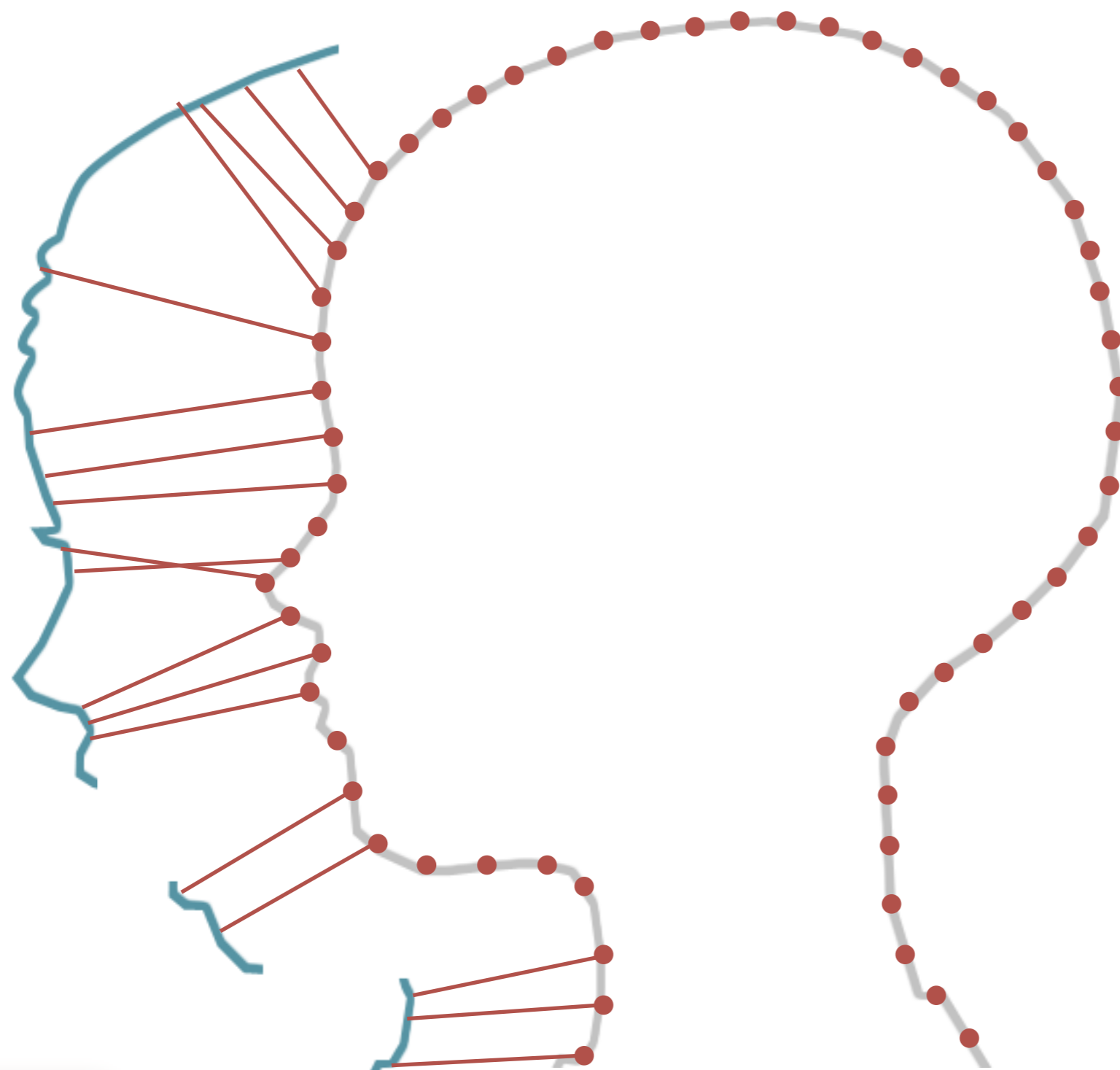
closest point



pruning

deform

# Iterative Global Optimization



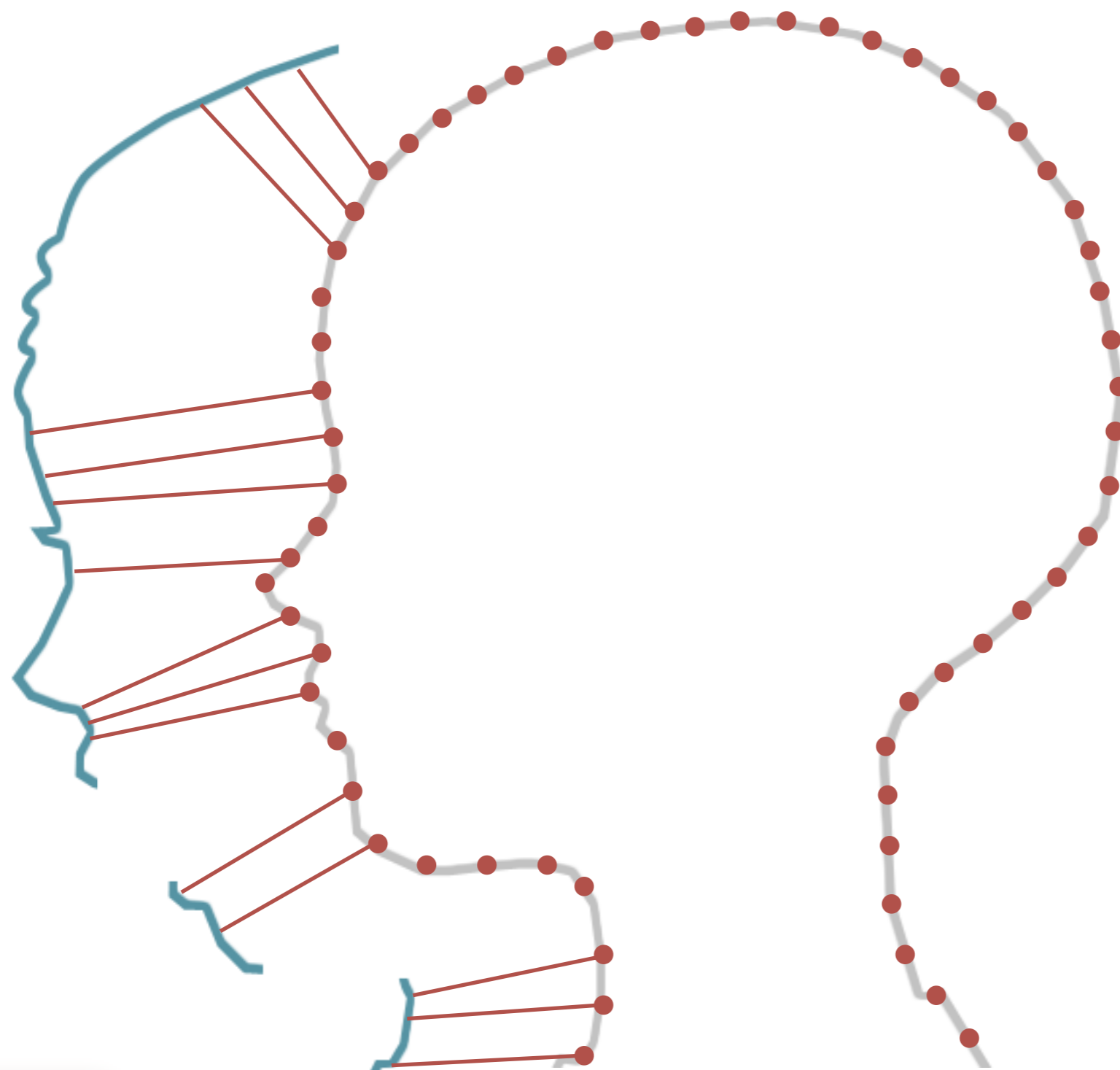
closest point



pruning

deform

# Iterative Global Optimization



closest point



pruning

deform

# Iterative Global Optimization



closest point



pruning

deform

# Iterative Global Optimization



closest point

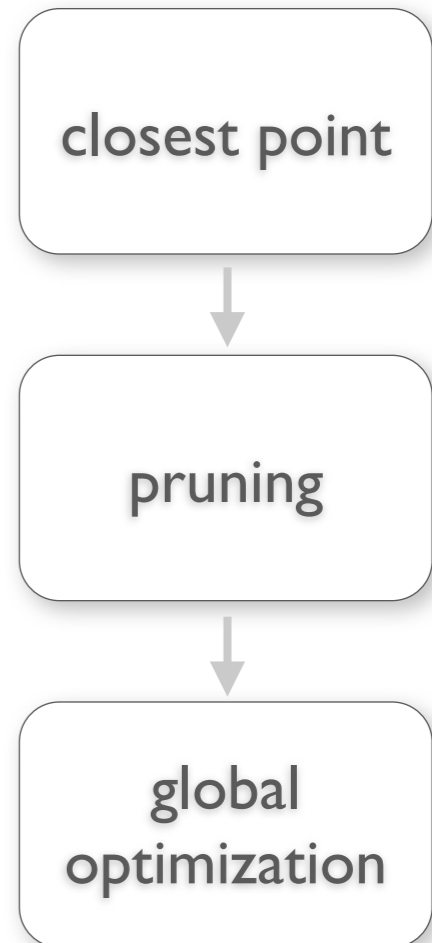


pruning

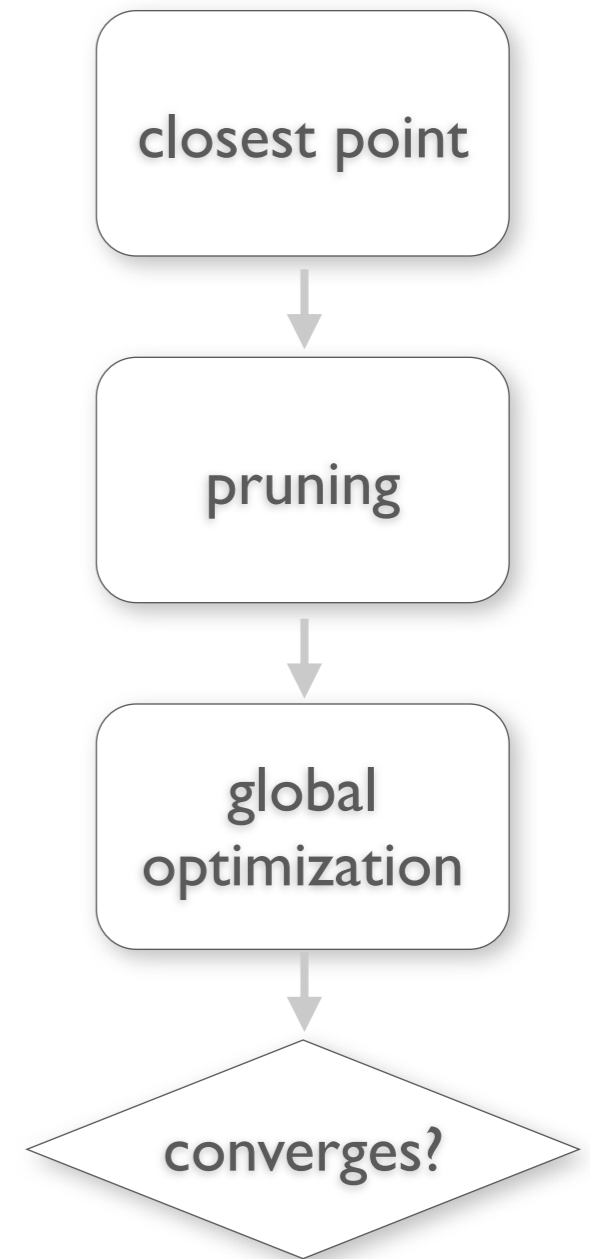


global  
optimization

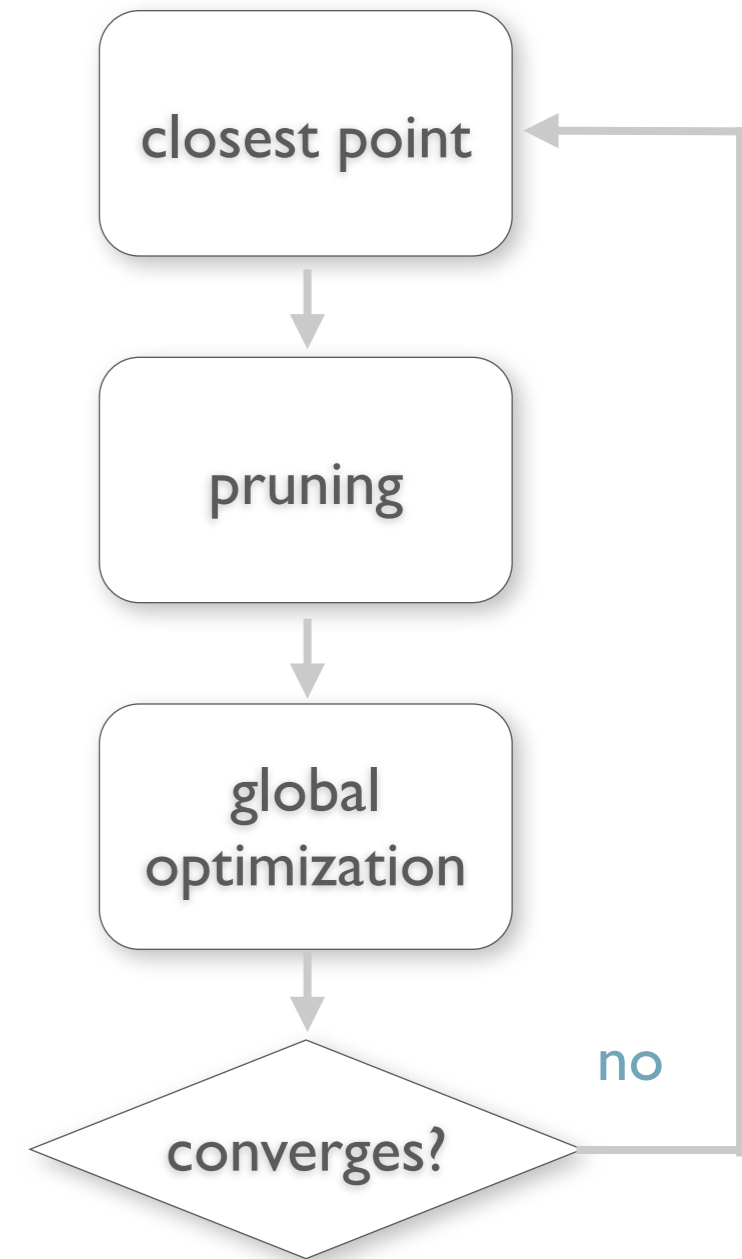
# Iterative Global Optimization



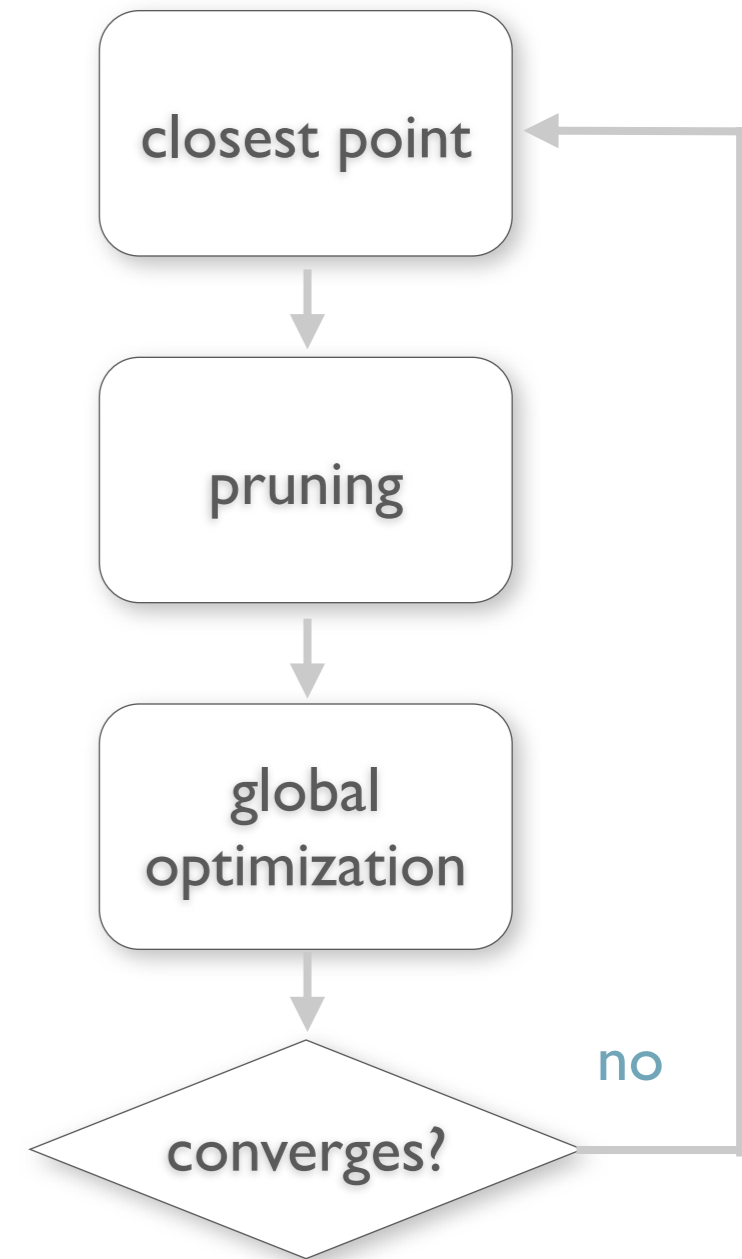
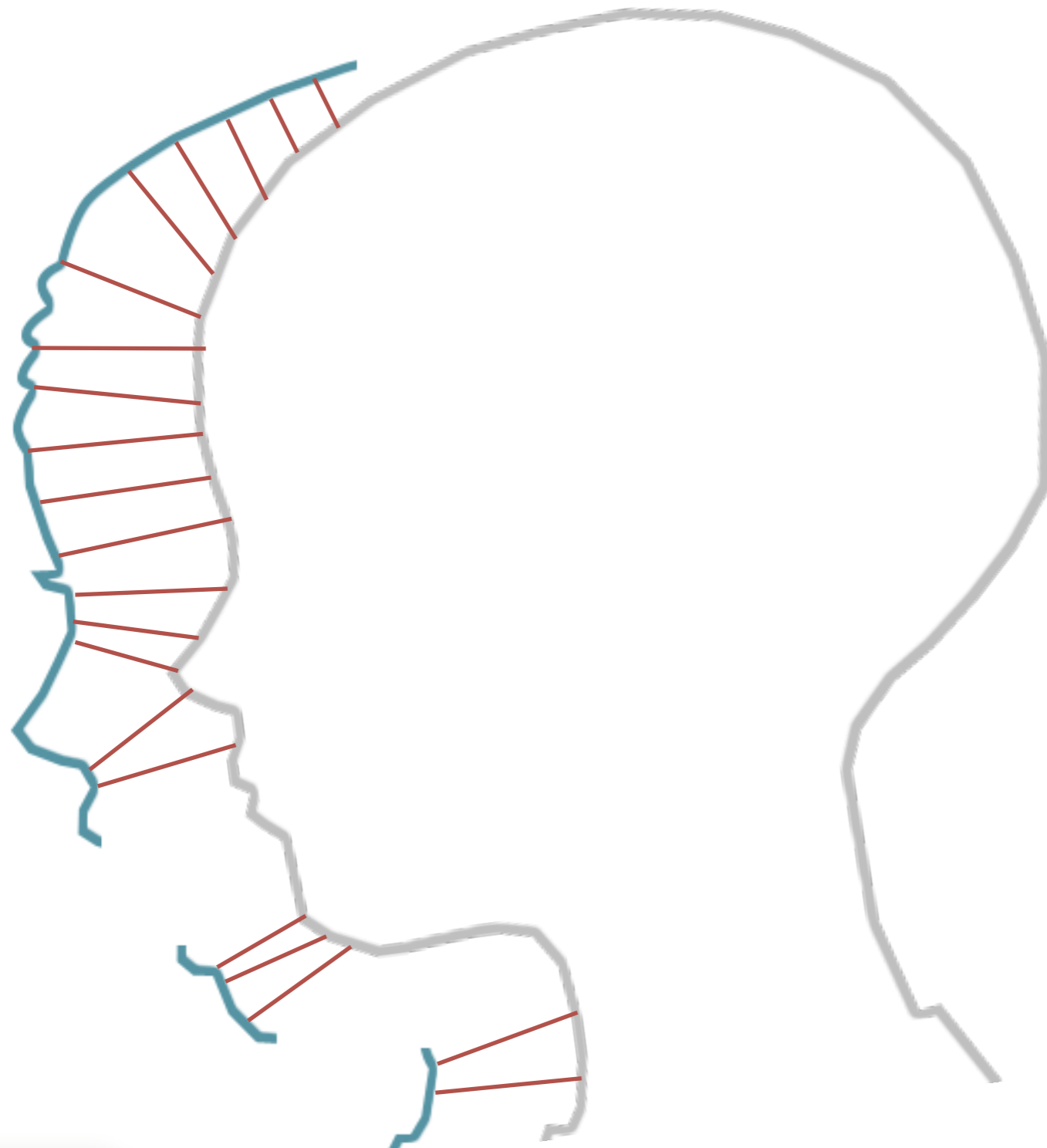
# Iterative Global Optimization



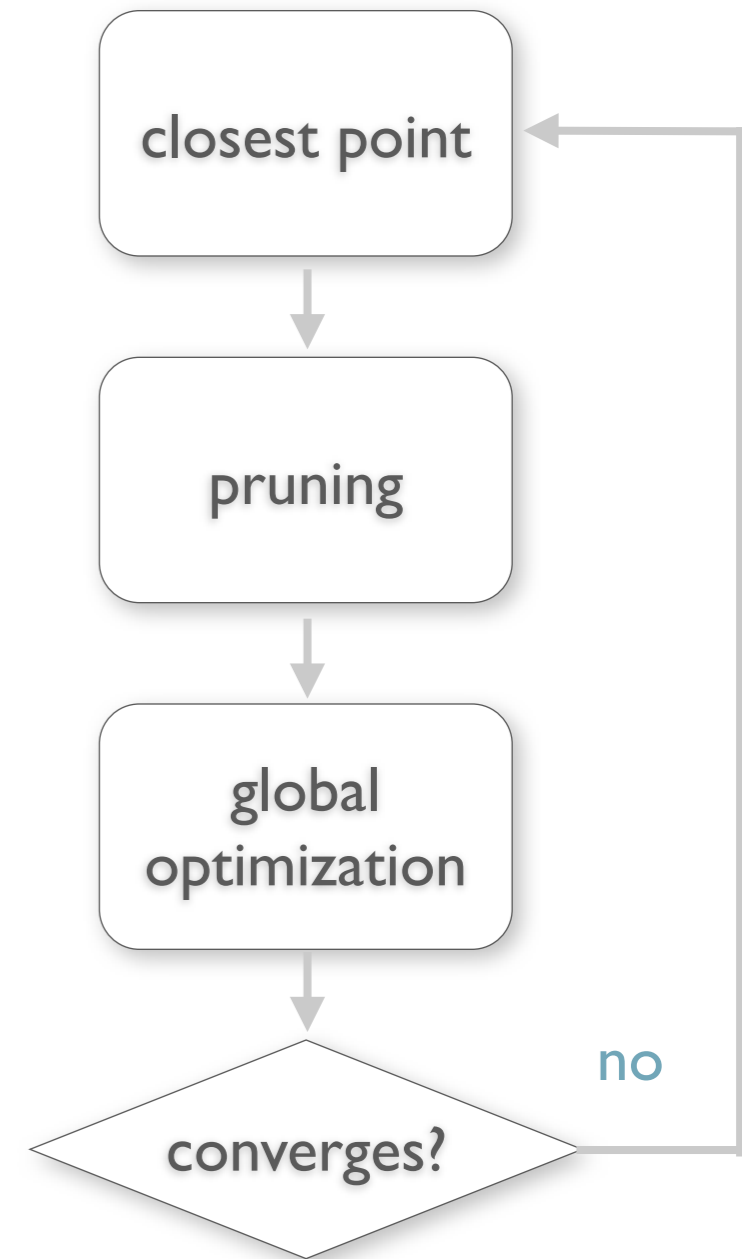
# Iterative Global Optimization



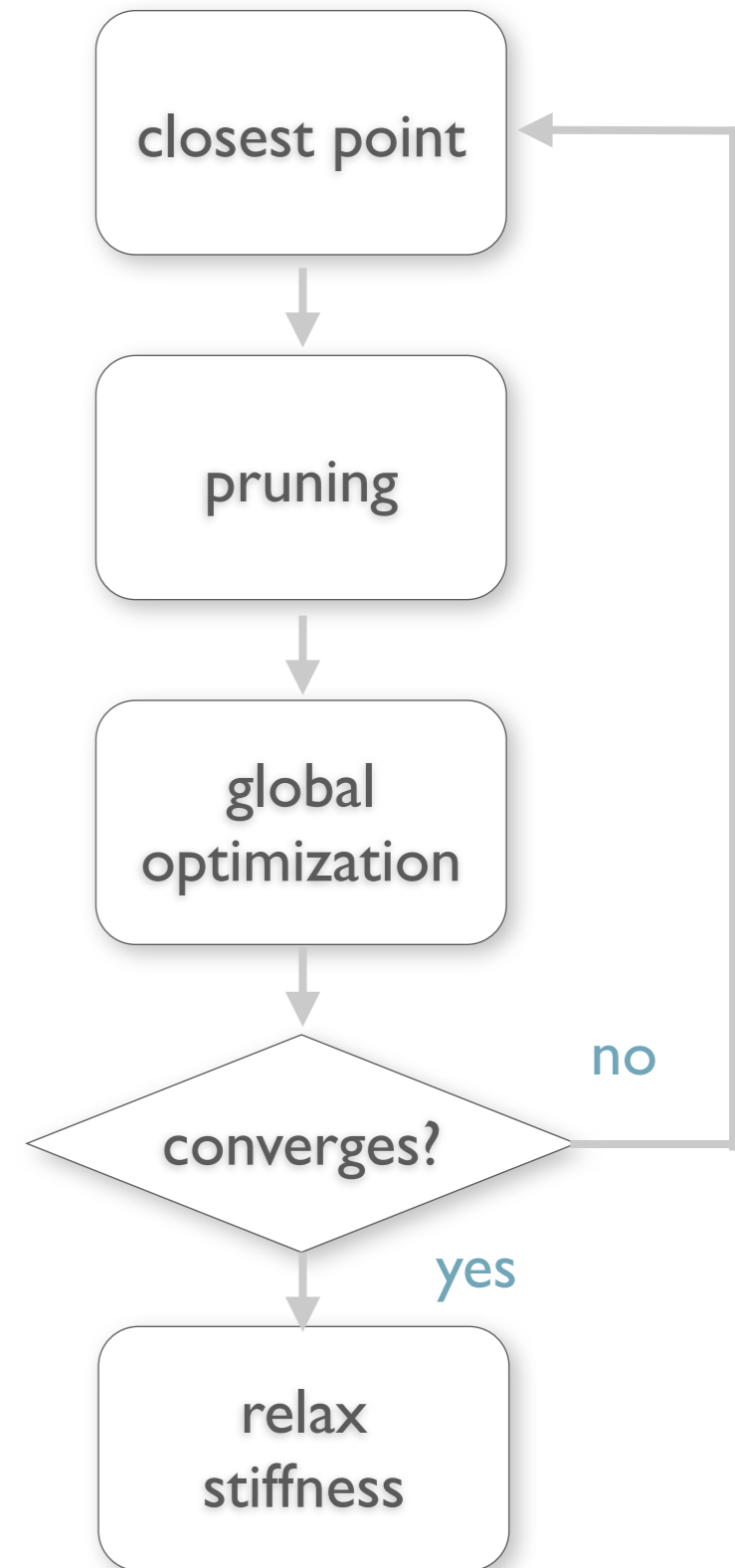
# Iterative Global Optimization



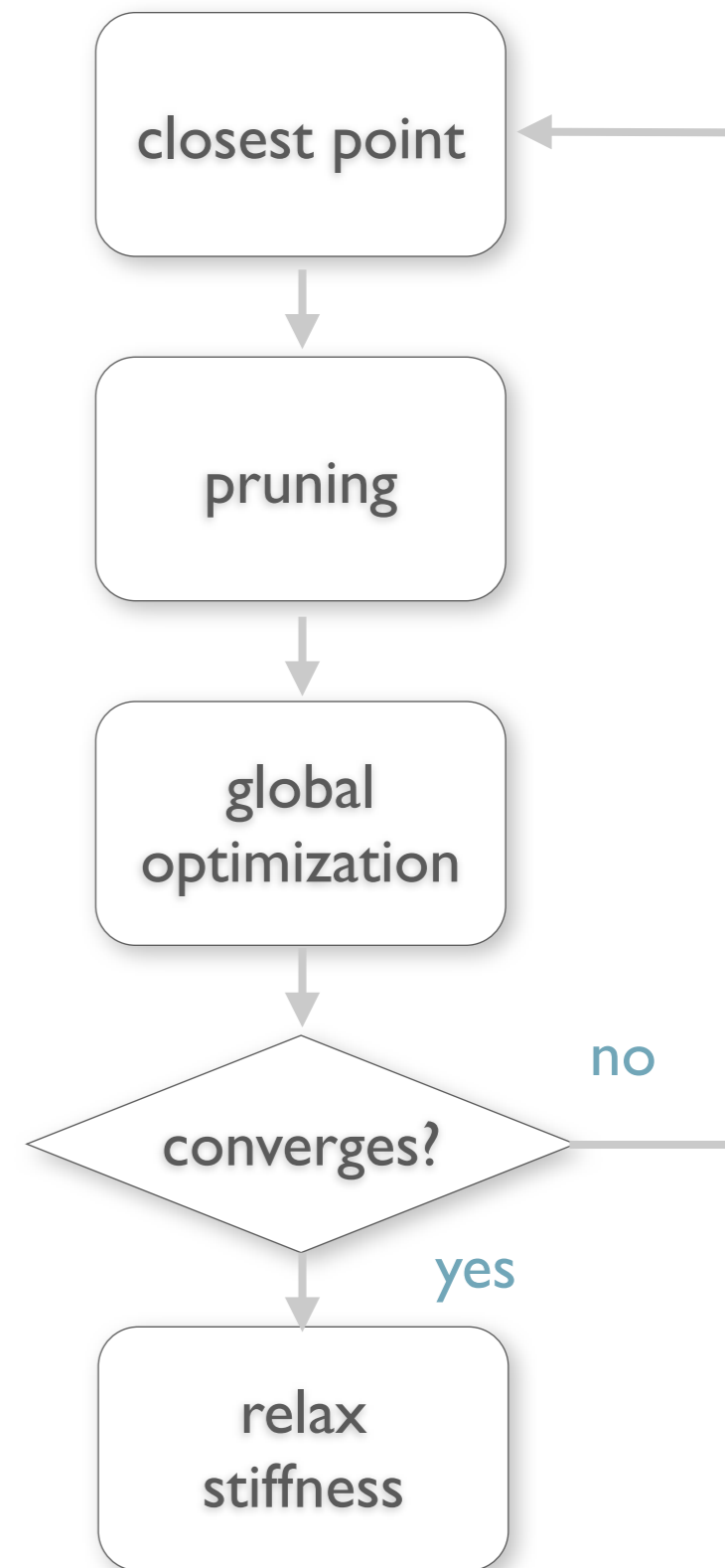
# Iterative Global Optimization



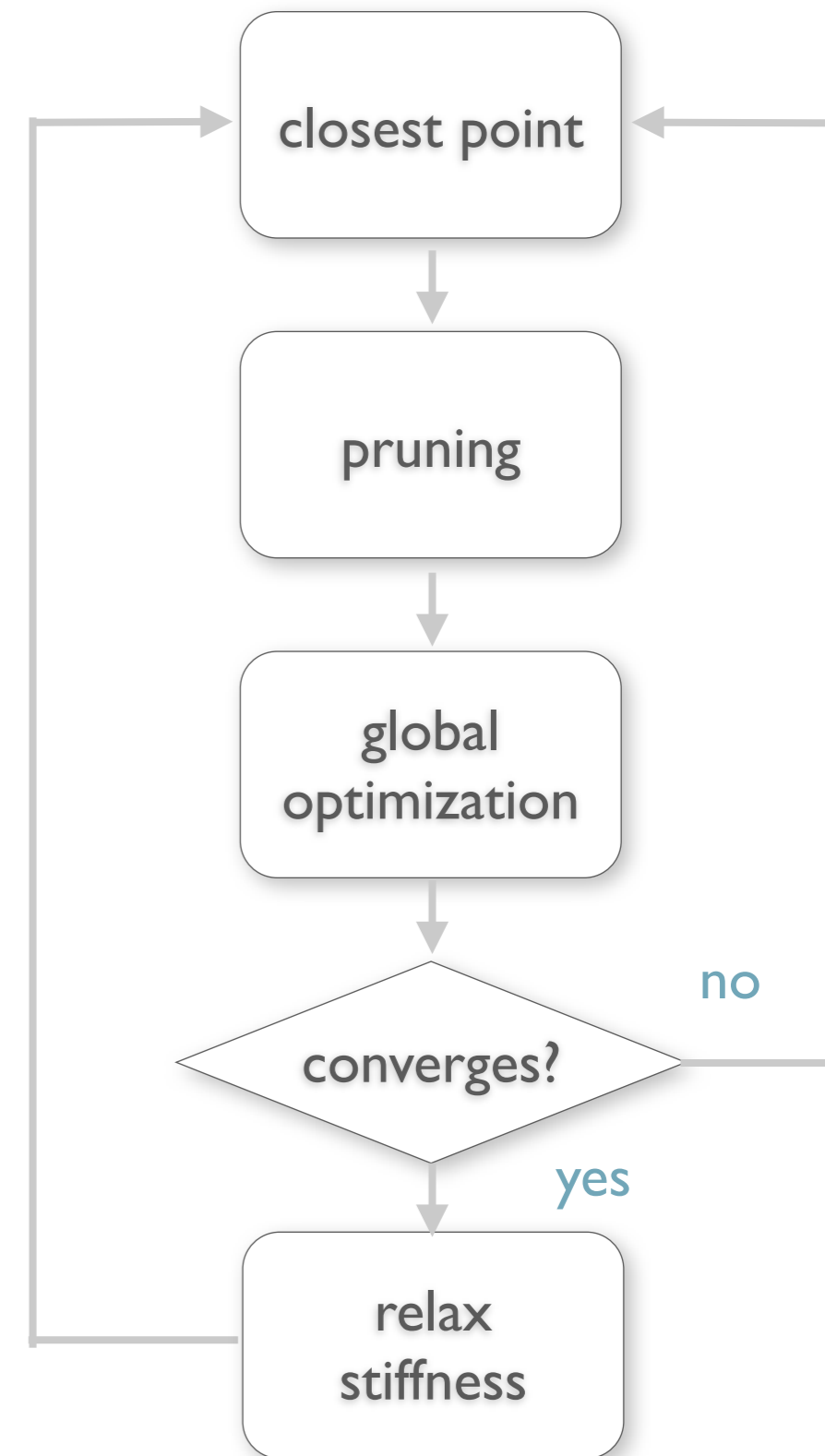
# Iterative Global Optimization



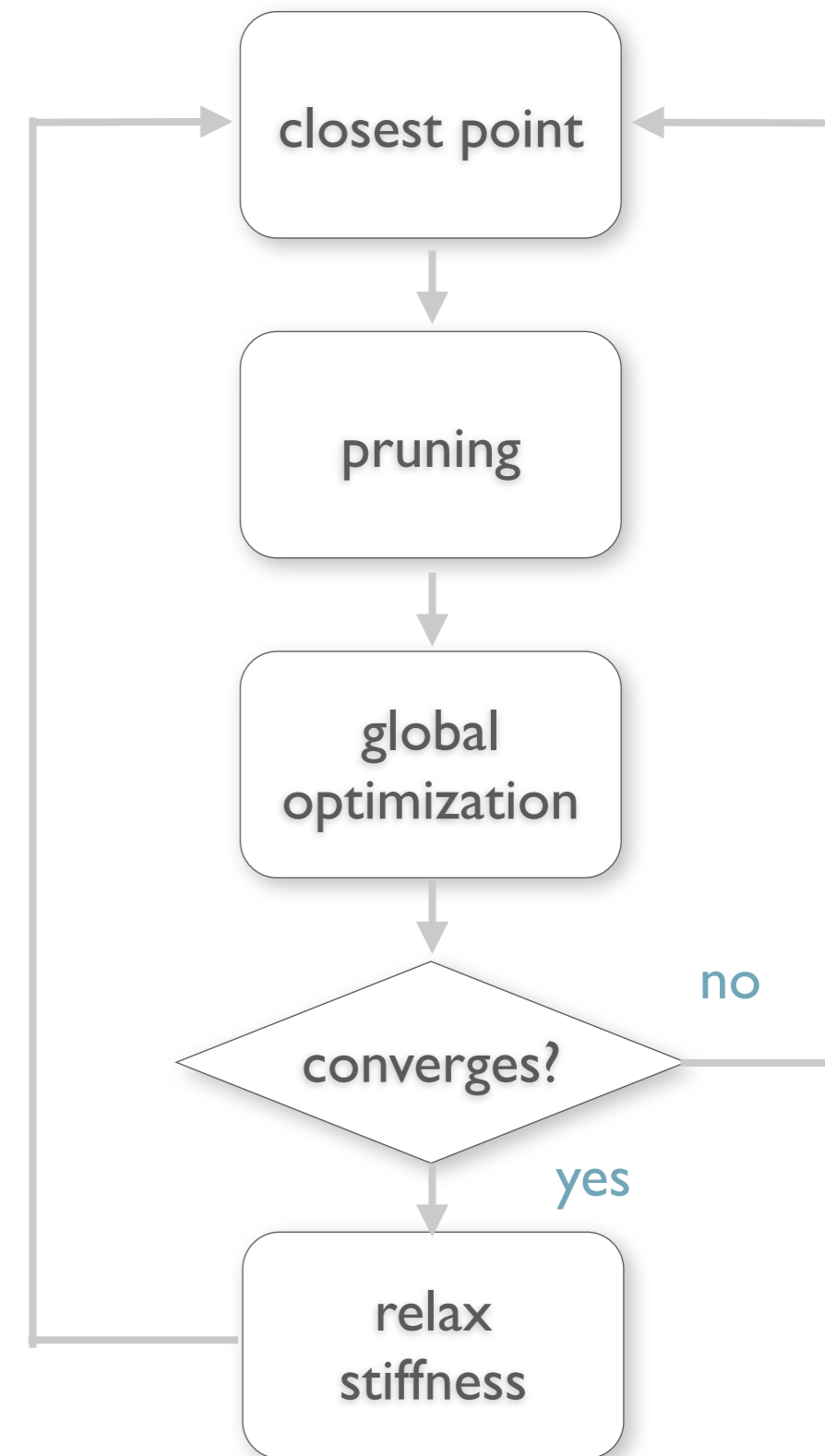
# Iterative Global Optimization



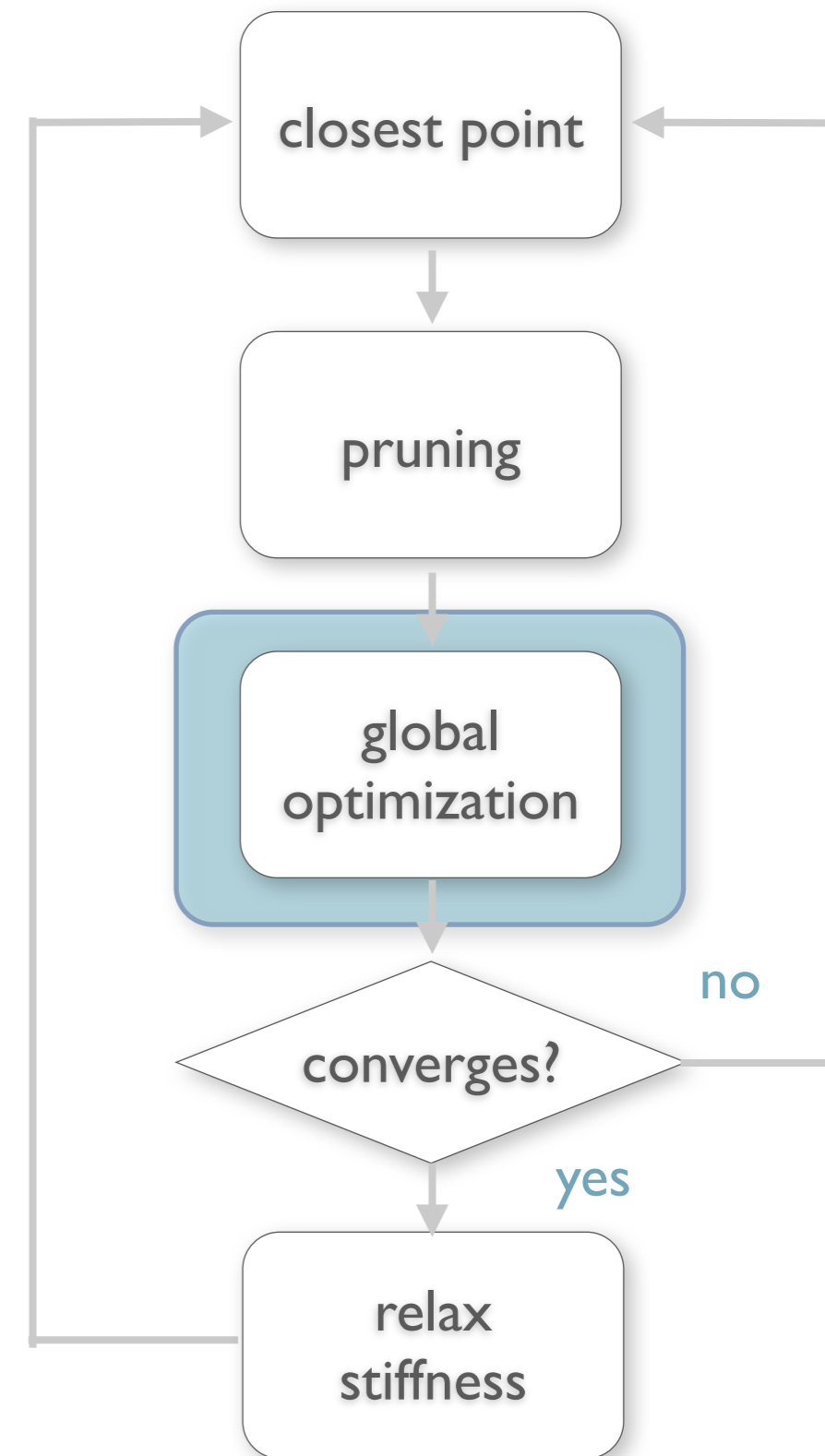
# Iterative Global Optimization



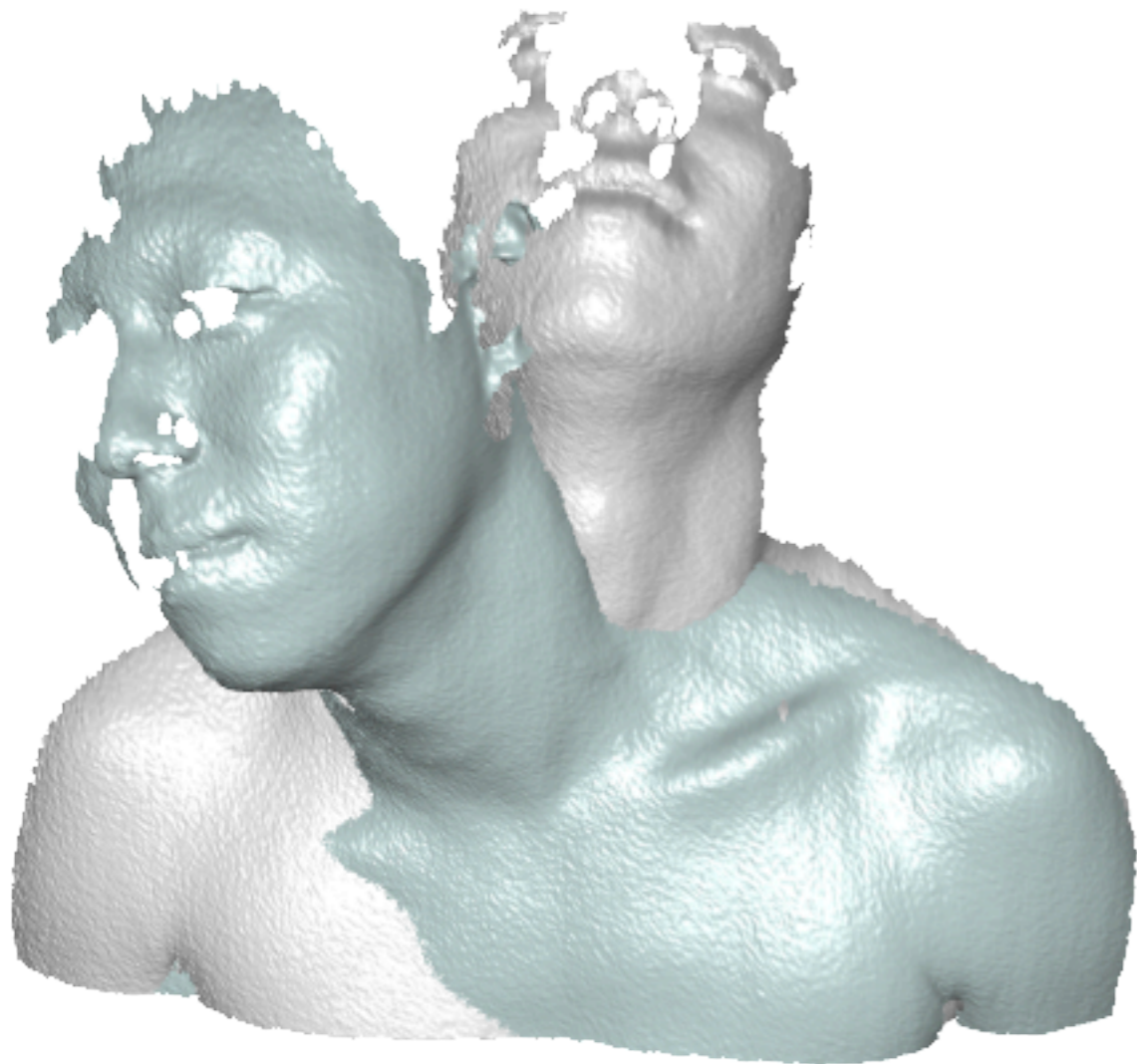
# Iterative Global Optimization



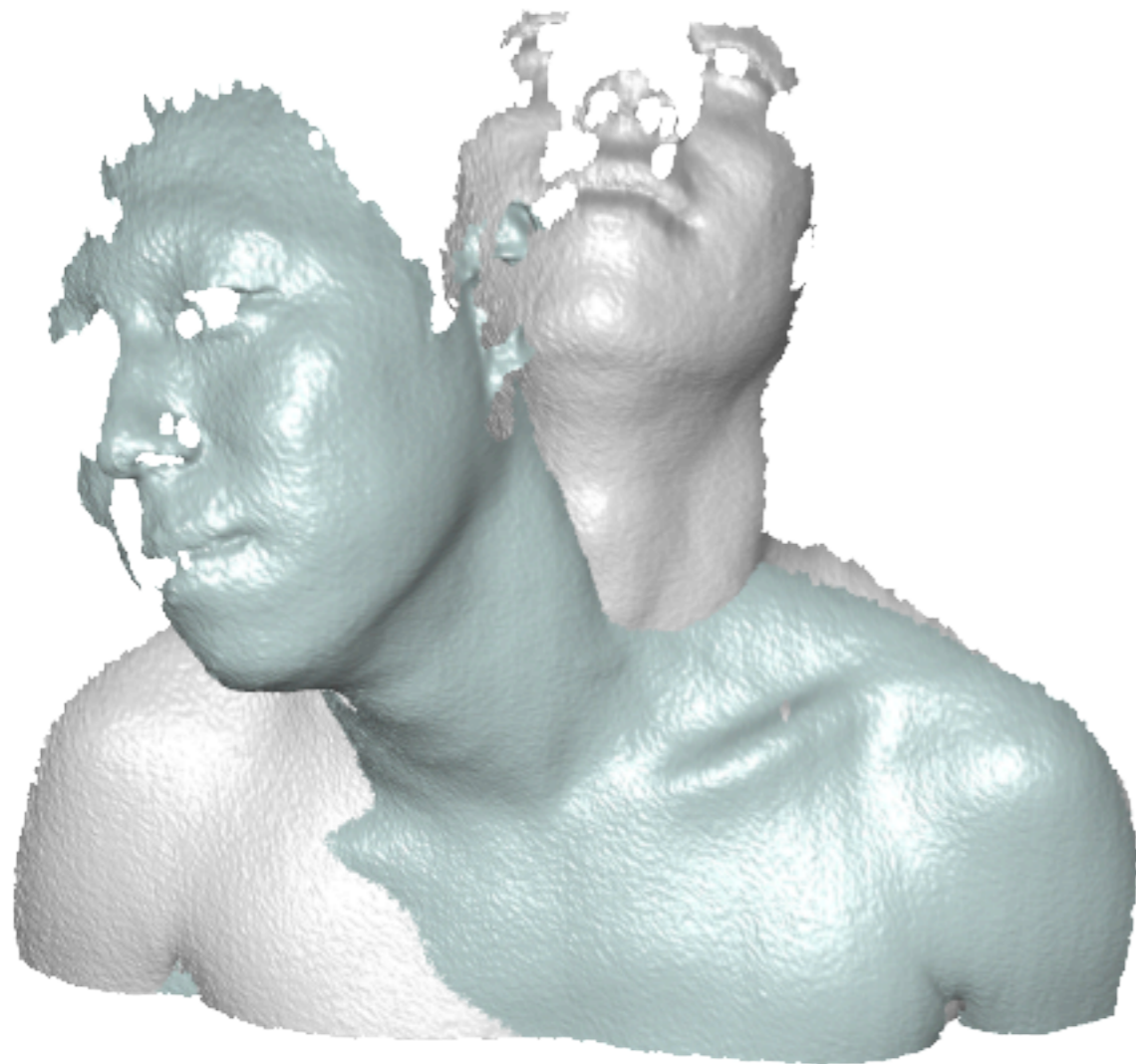
# Iterative Global Optimization



# Deformation Model

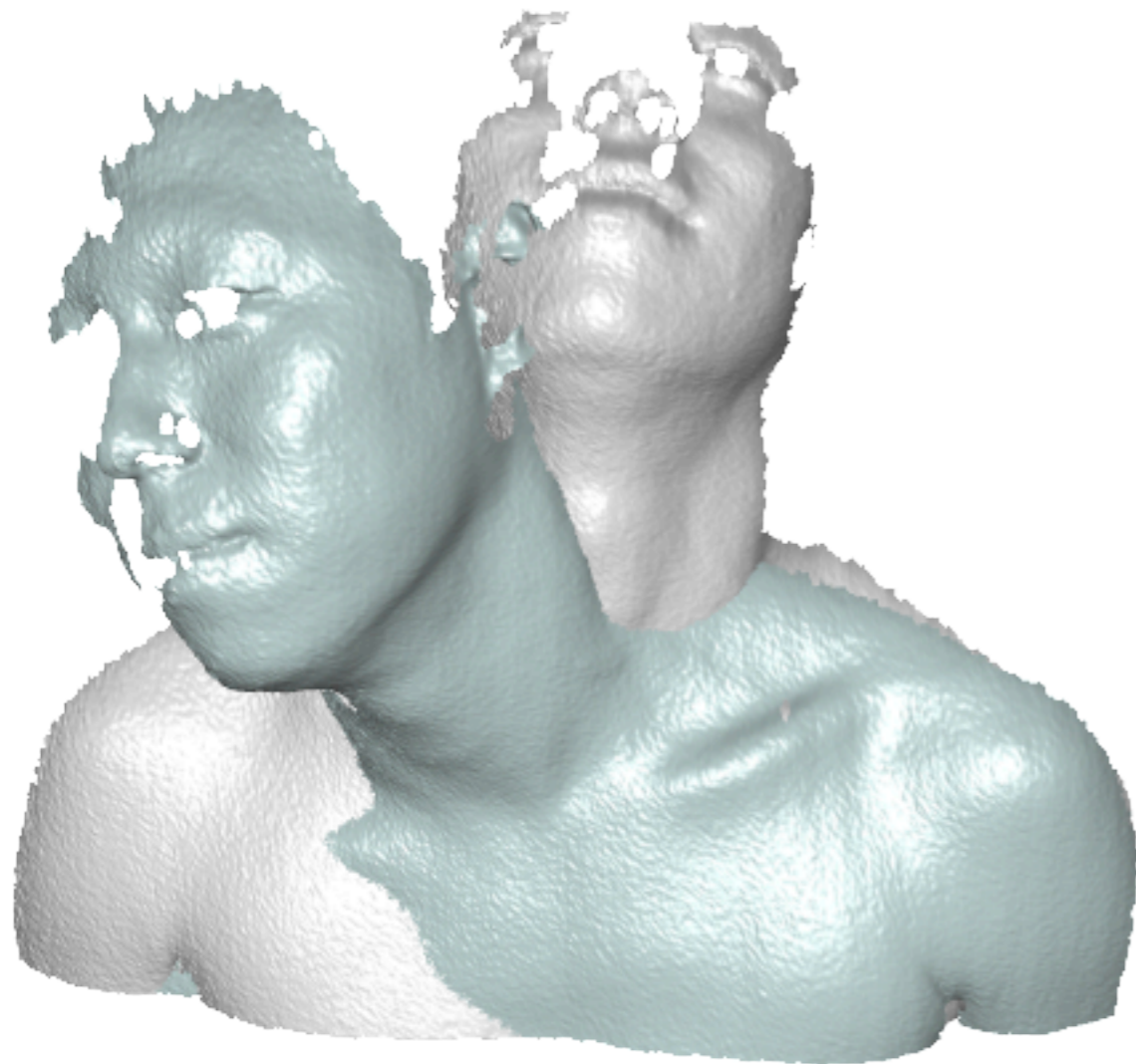


# Deformation Model



Embedded Deformation

# Deformation Model

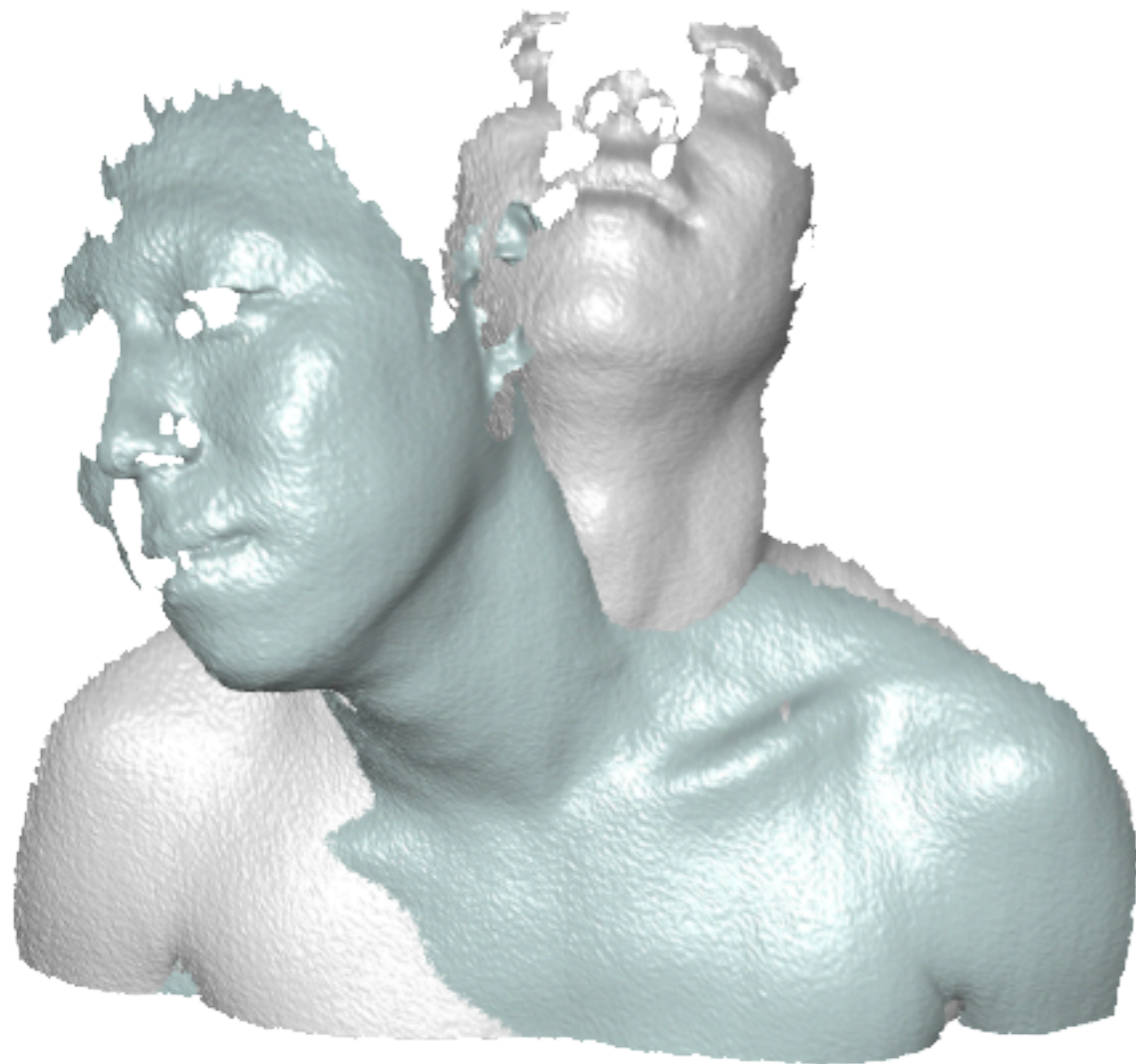


Embedded Deformation

incomplete scan



# Deformation Model

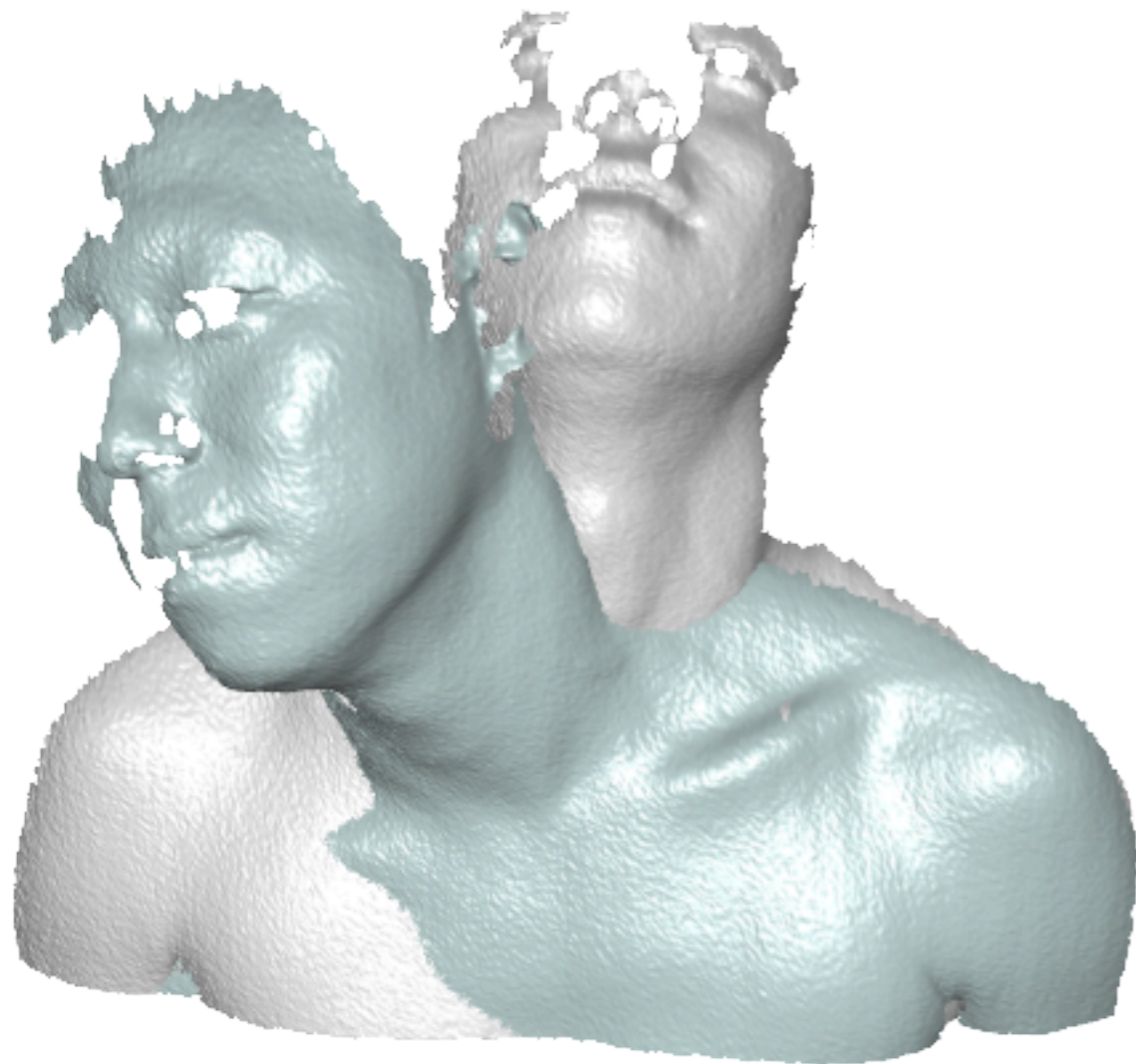


Embedded Deformation

incomplete scan

space deformation

# Deformation Model



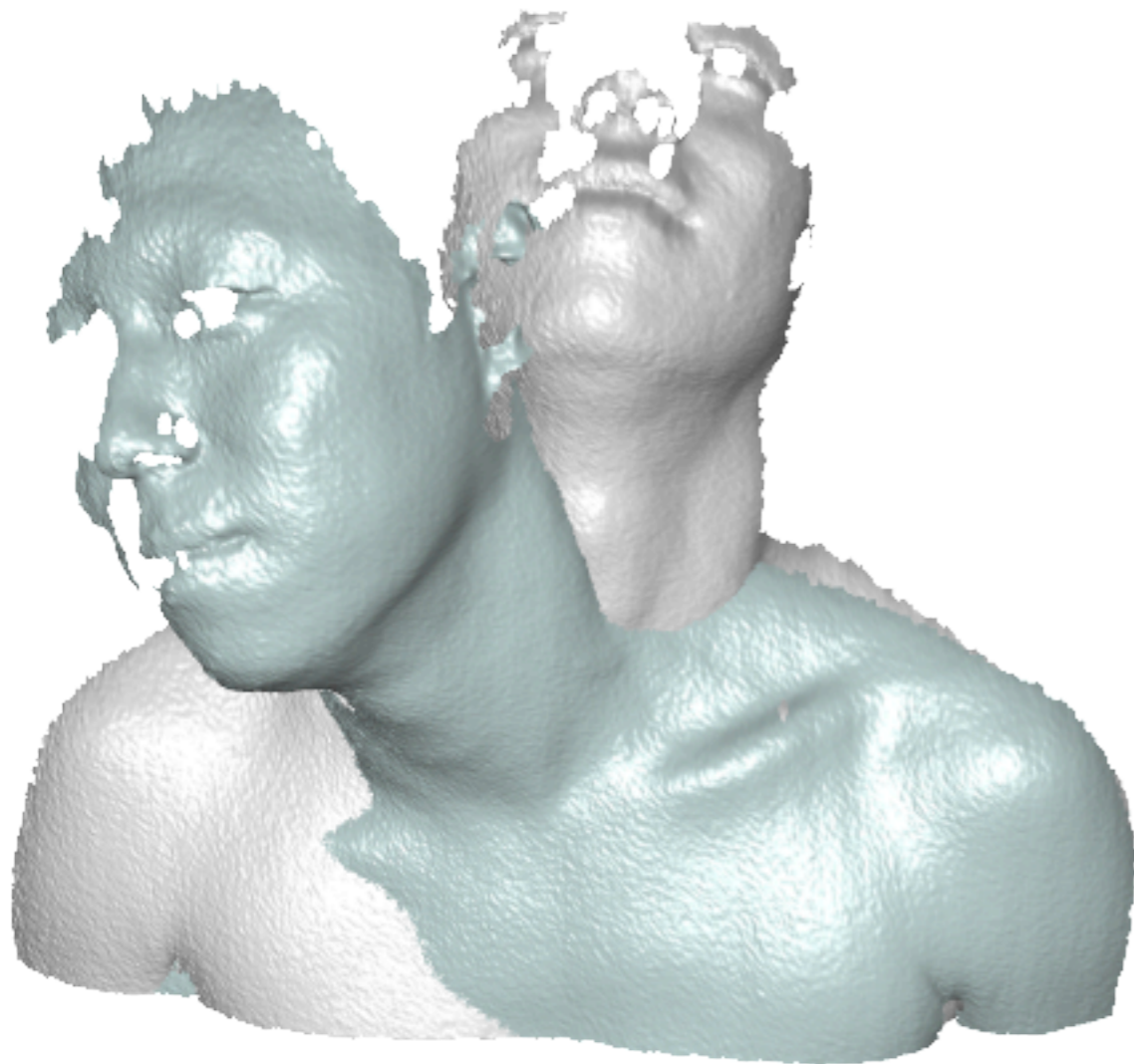
Embedded Deformation

incomplete scan

space deformation

high-resolution

# Deformation Model



Embedded Deformation

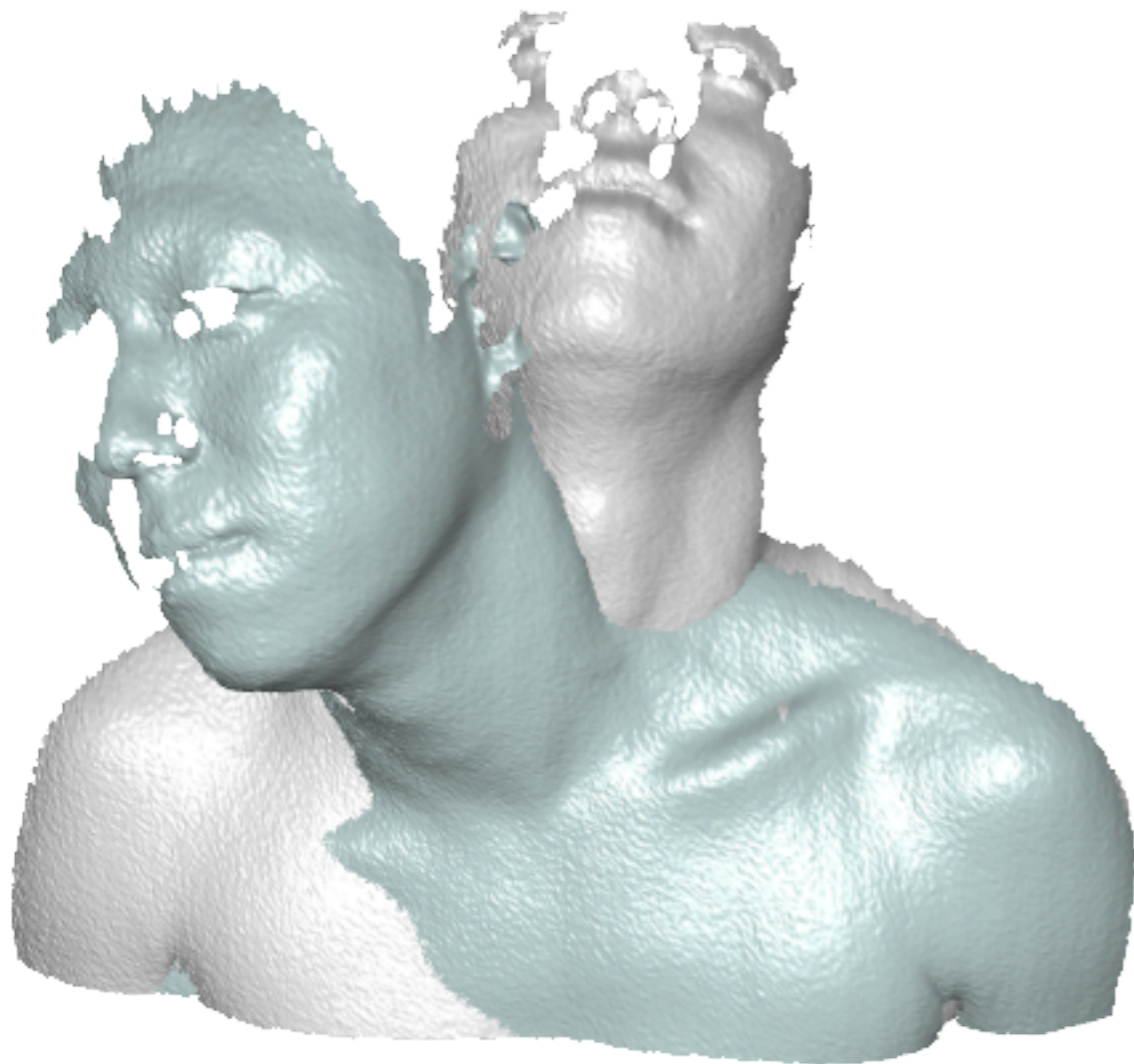
incomplete scan

space deformation

high-resolution

decoupled complexity

# Deformation Model



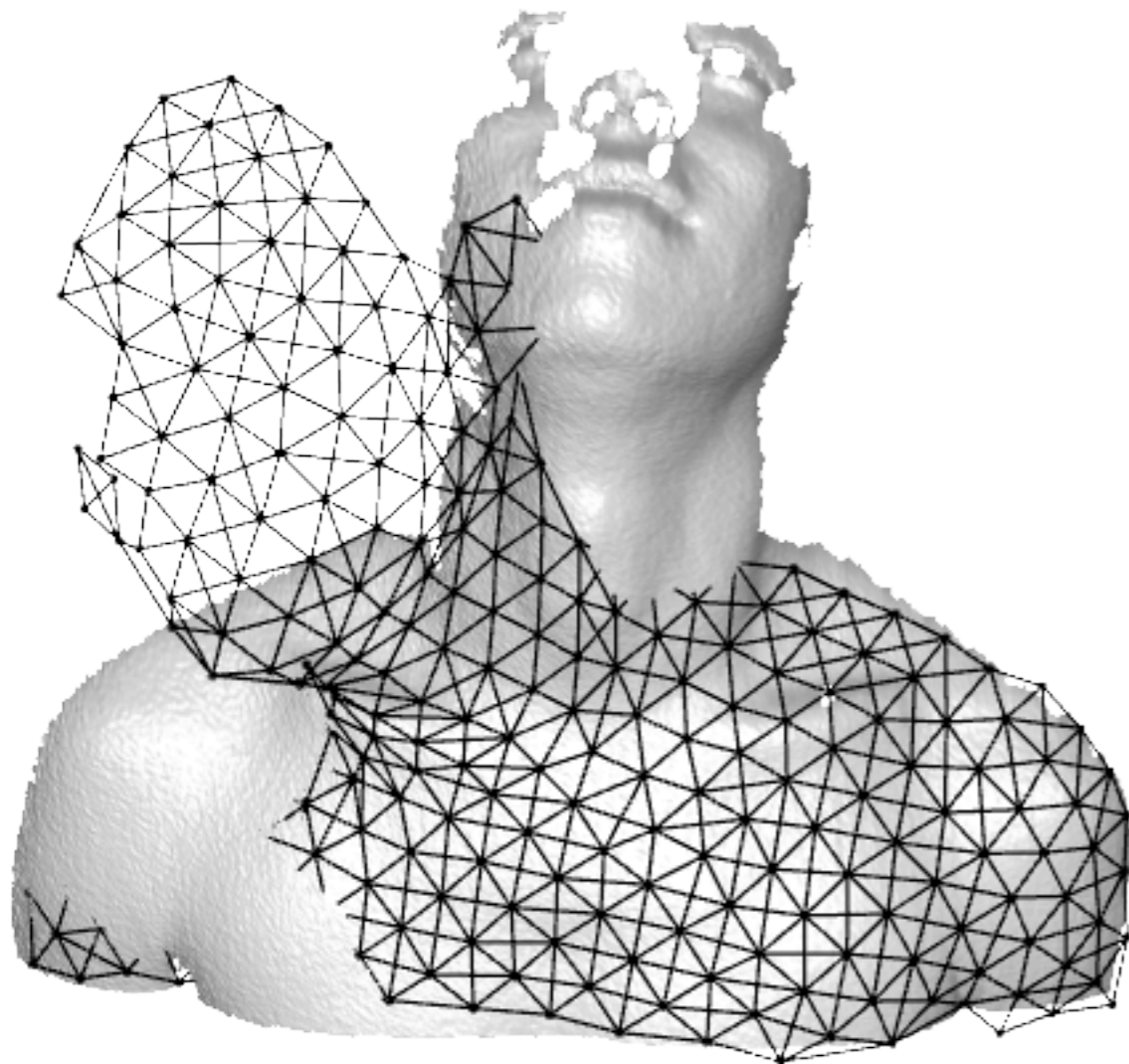
Embedded Deformation

incomplete scan

high-resolution

deformation  
graph

# Deformation Model



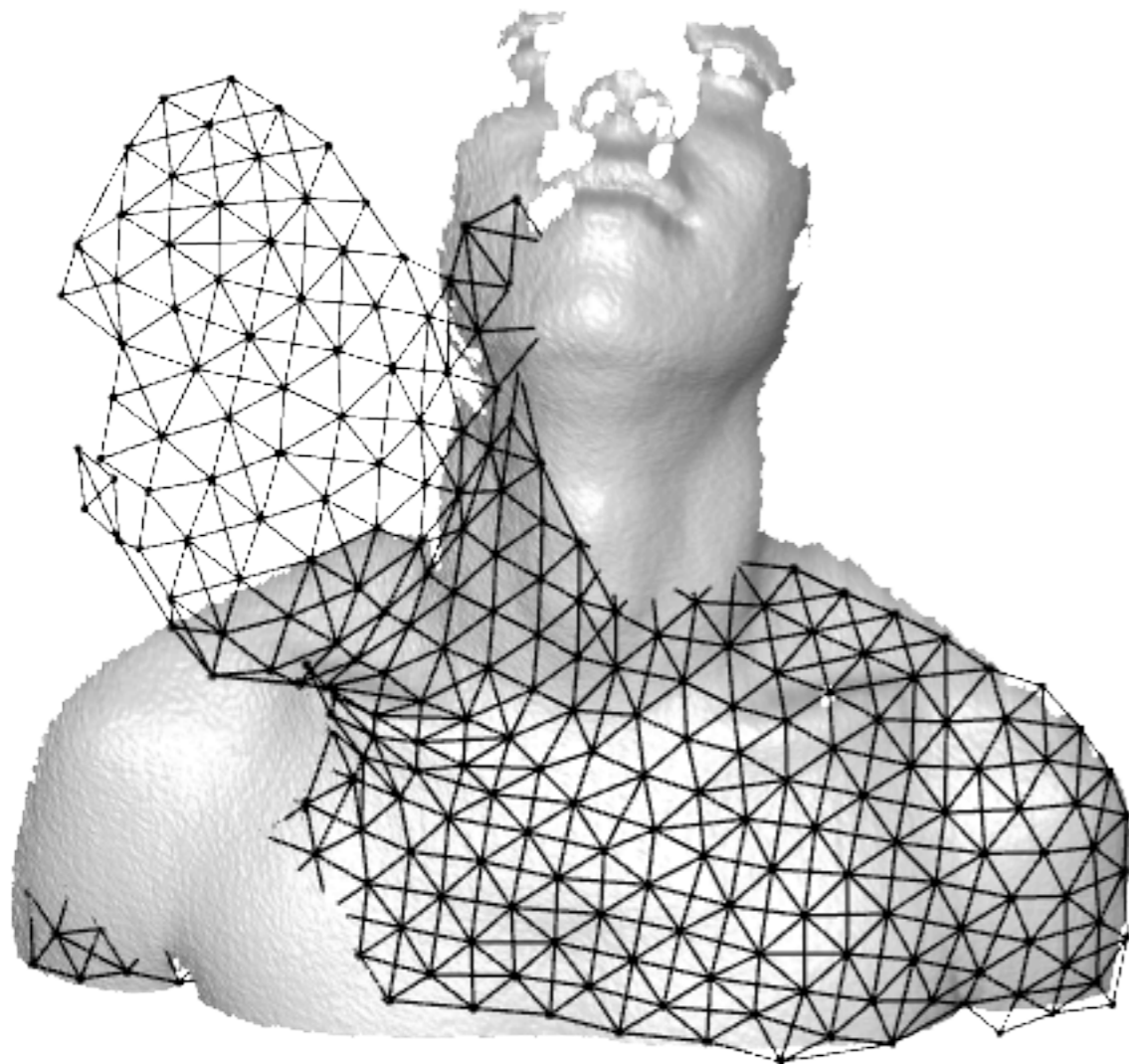
Embedded Deformation

incomplete scan

high-resolution

deformation  
graph

# Deformation Model



Embedded Deformation

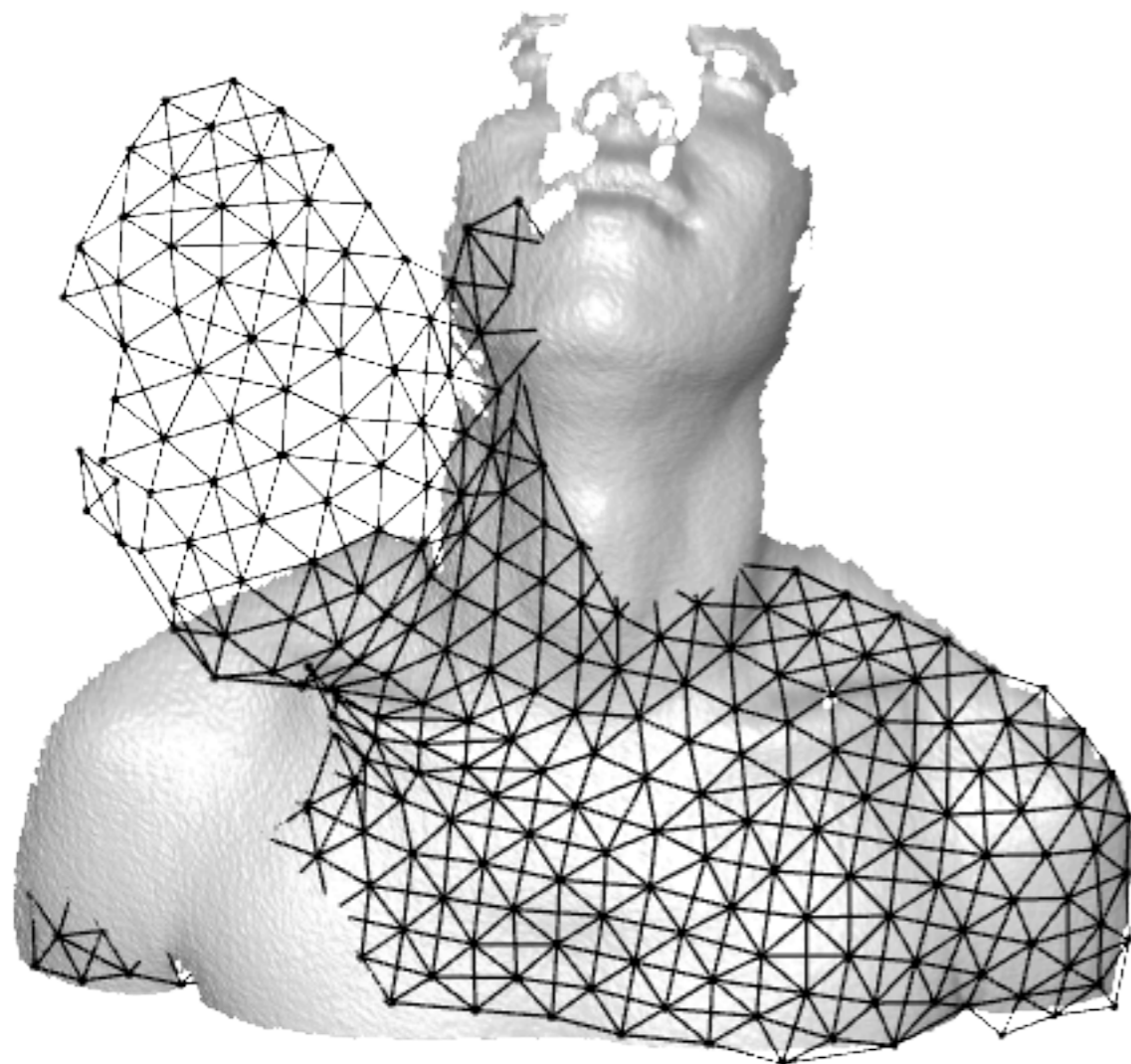
incomplete scan

high-resolution

unknown warp

deformation  
graph

# Deformation Model



Embedded Deformation

incomplete scan

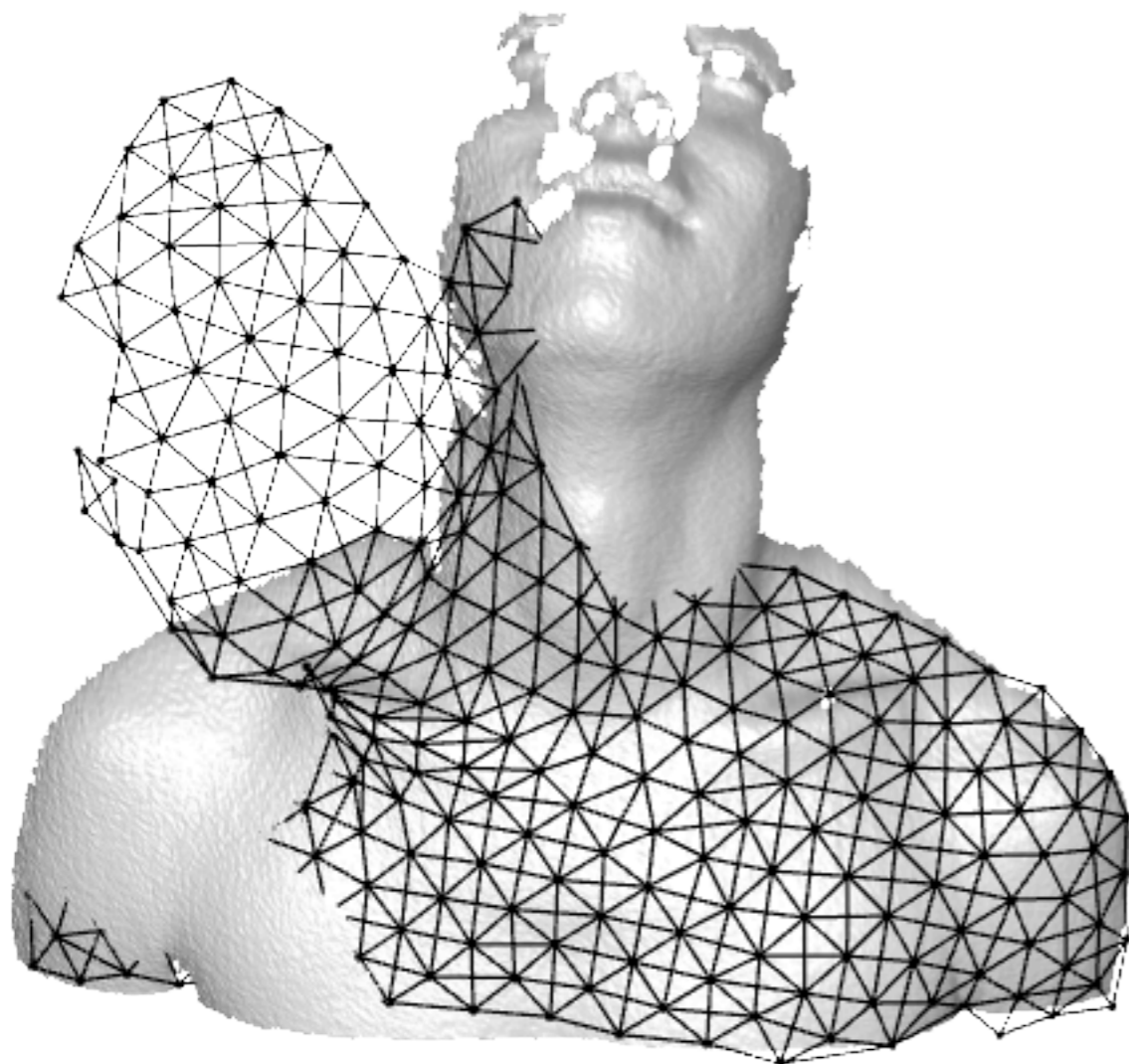
high-resolution

unknown warp

incomplete match

deformation  
graph

# Deformation Model



Embedded Deformation

incomplete scan

high-resolution

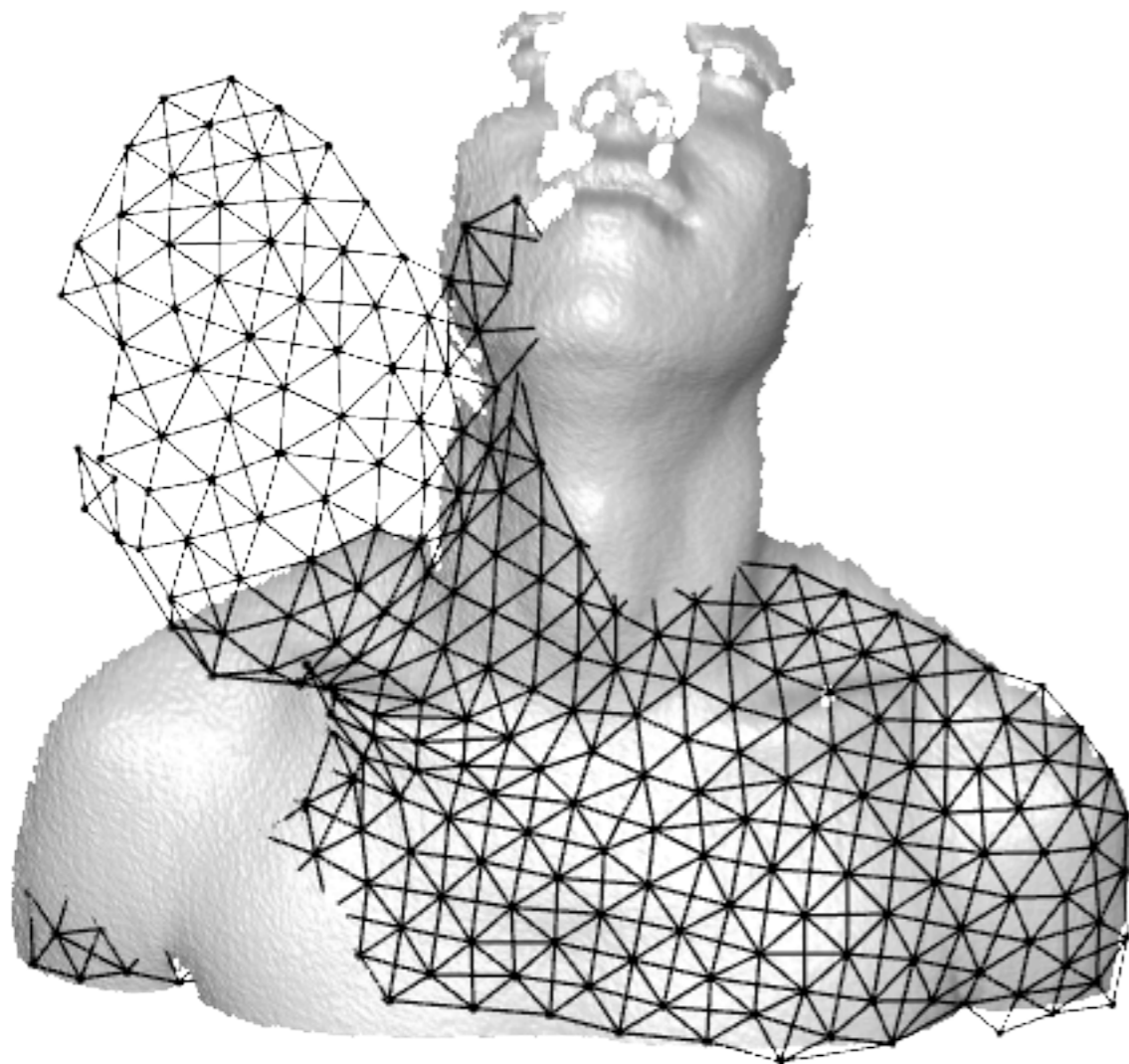
unknown warp

incomplete match

deformation  
graph

detail preservation  
global consistency

# Deformation Model



Embedded Deformation

incomplete scan

high-resolution

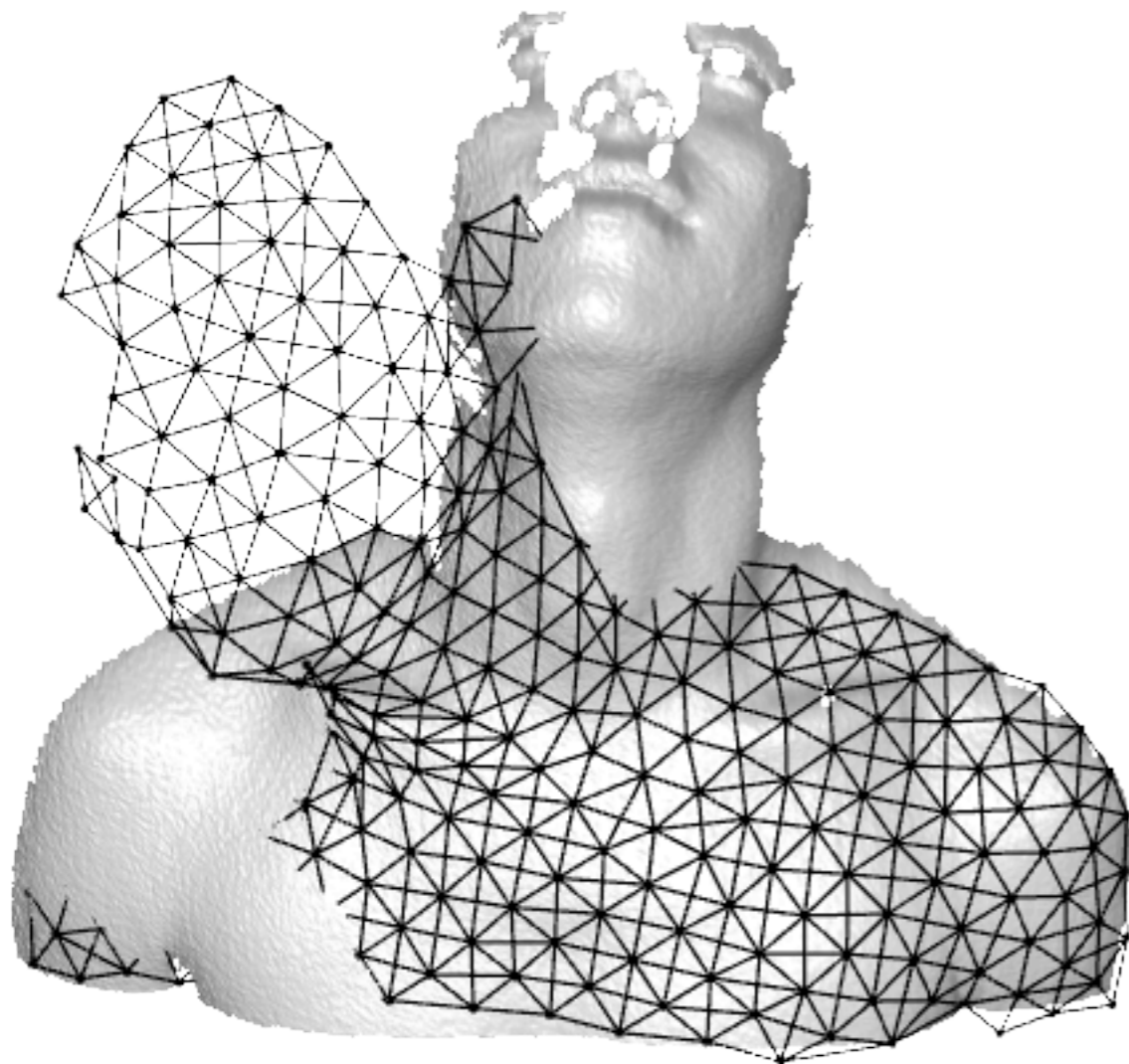
unknown warp

incomplete match

deformation  
graph

detail preservation  
global consistency

# Deformation Model



Embedded Deformation

incomplete scan

high-resolution

unknown warp

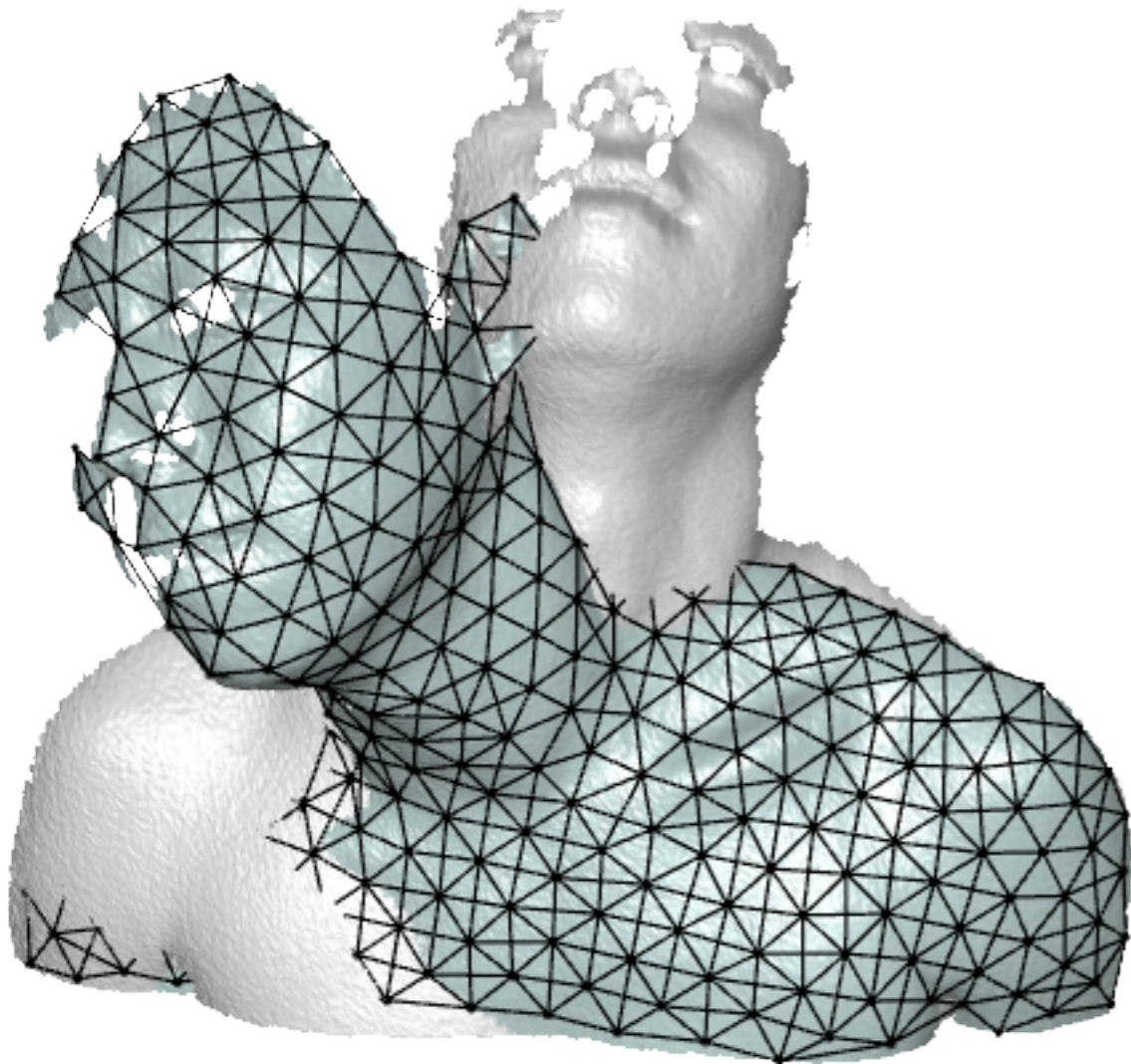
incomplete match

deformation  
graph

detail preservation  
global consistency

Linear Blend Skinning

# Deformation Model



Embedded Deformation

incomplete scan

high-resolution

unknown warp

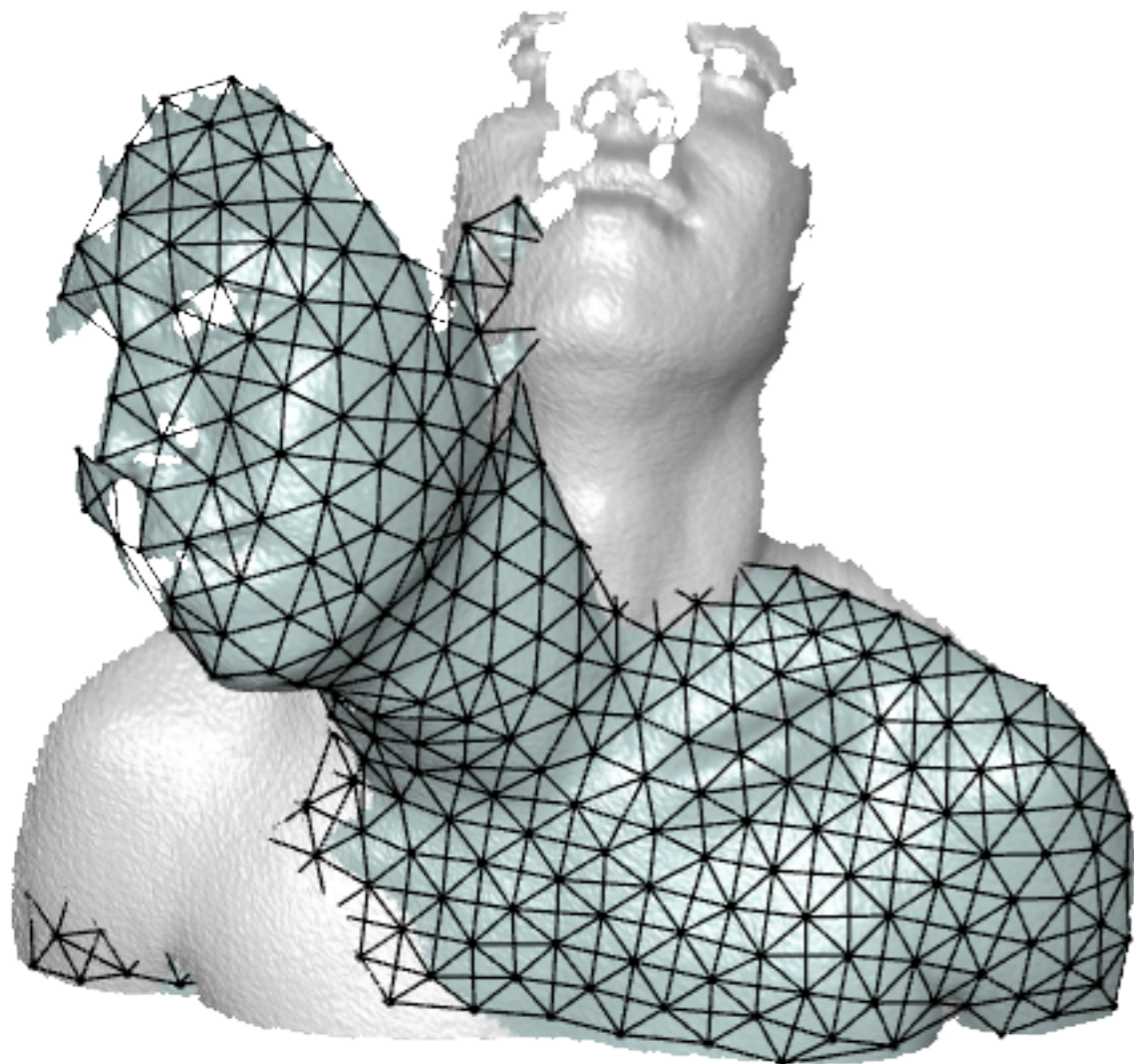
incomplete match

deformation  
graph

detail preservation  
global consistency

Linear Blend Skinning

# Deformation Model



Embedded Deformation

incomplete scan

high-resolution

unknown warp

incomplete match

deformation  
graph

detail preservation  
global consistency

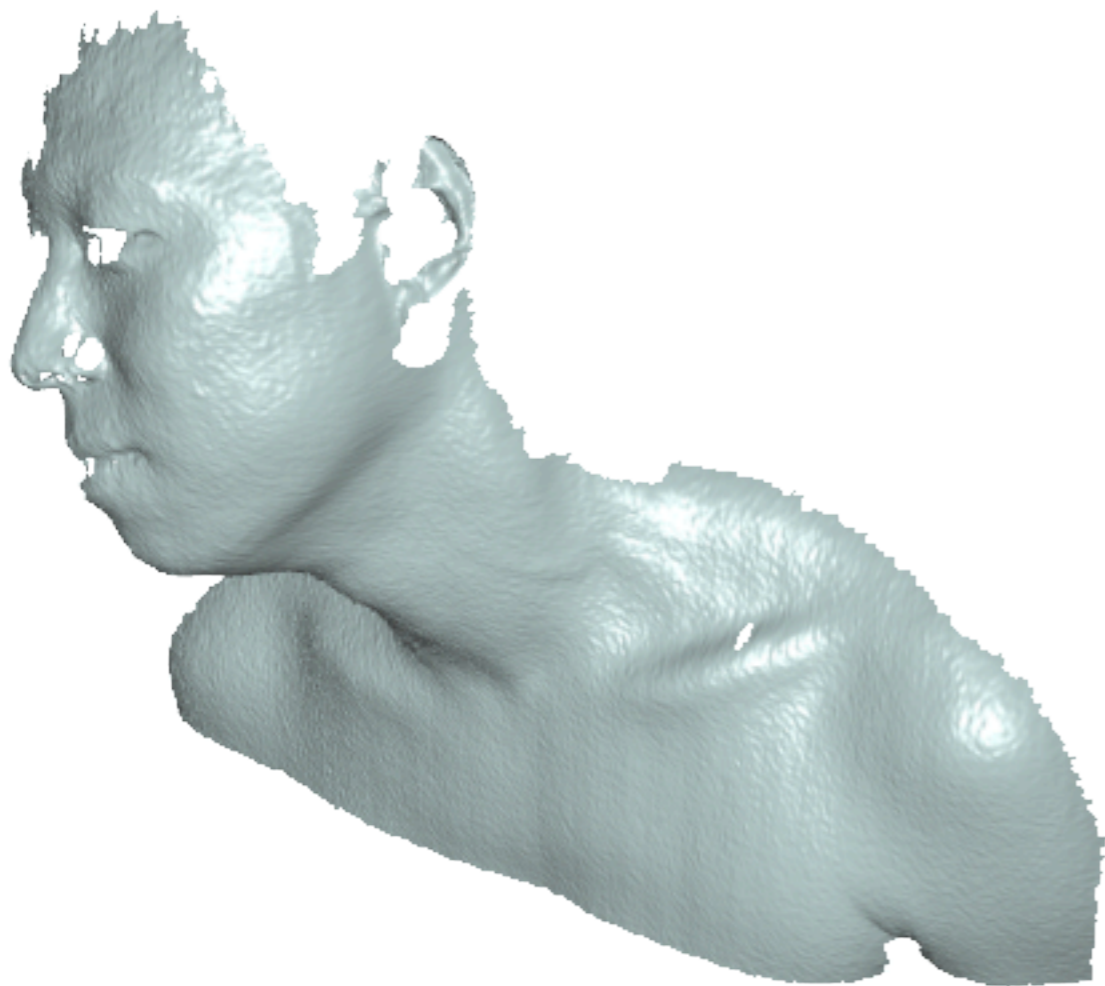
Linear Blend Skinning

# Deformation Model

detail preservation  
global consistency



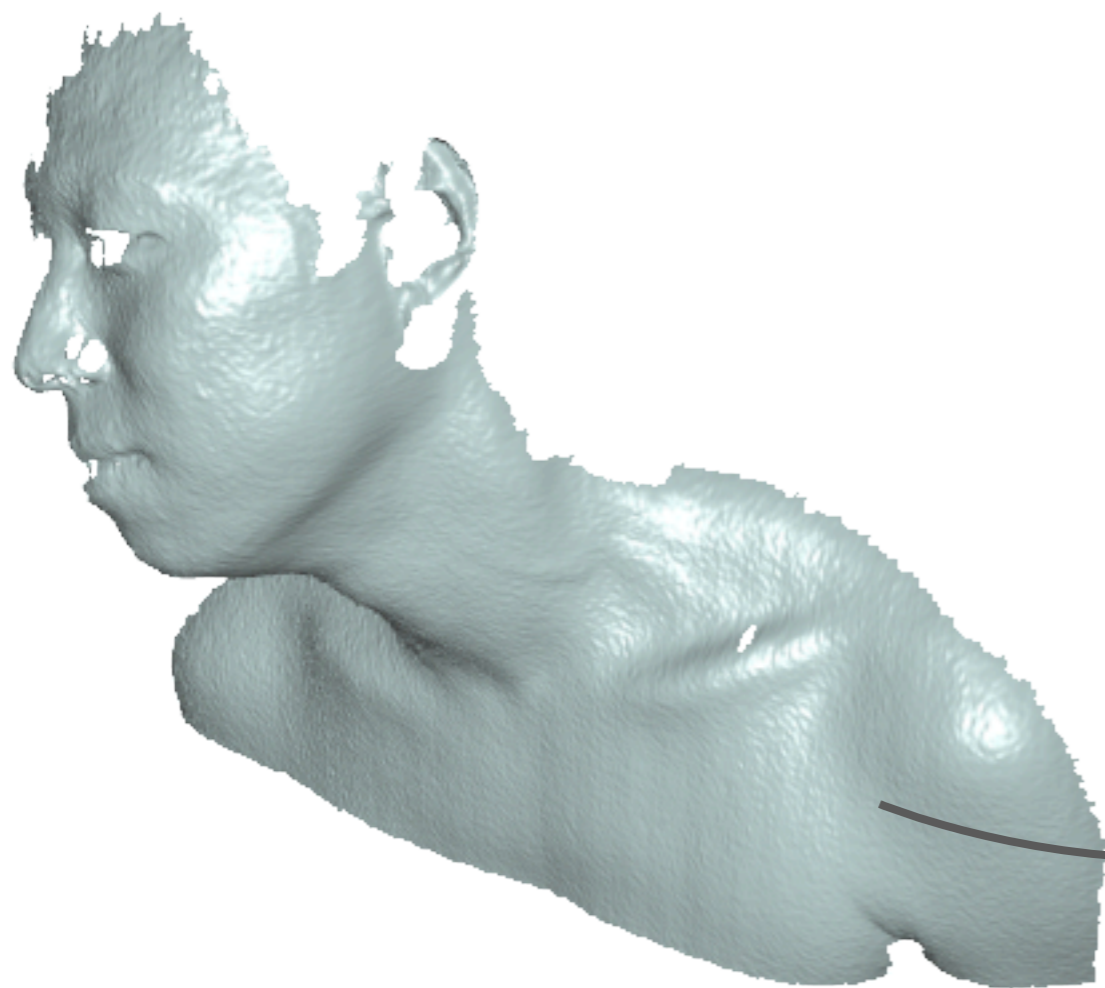
# Deformation Model



detail preservation  
global consistency

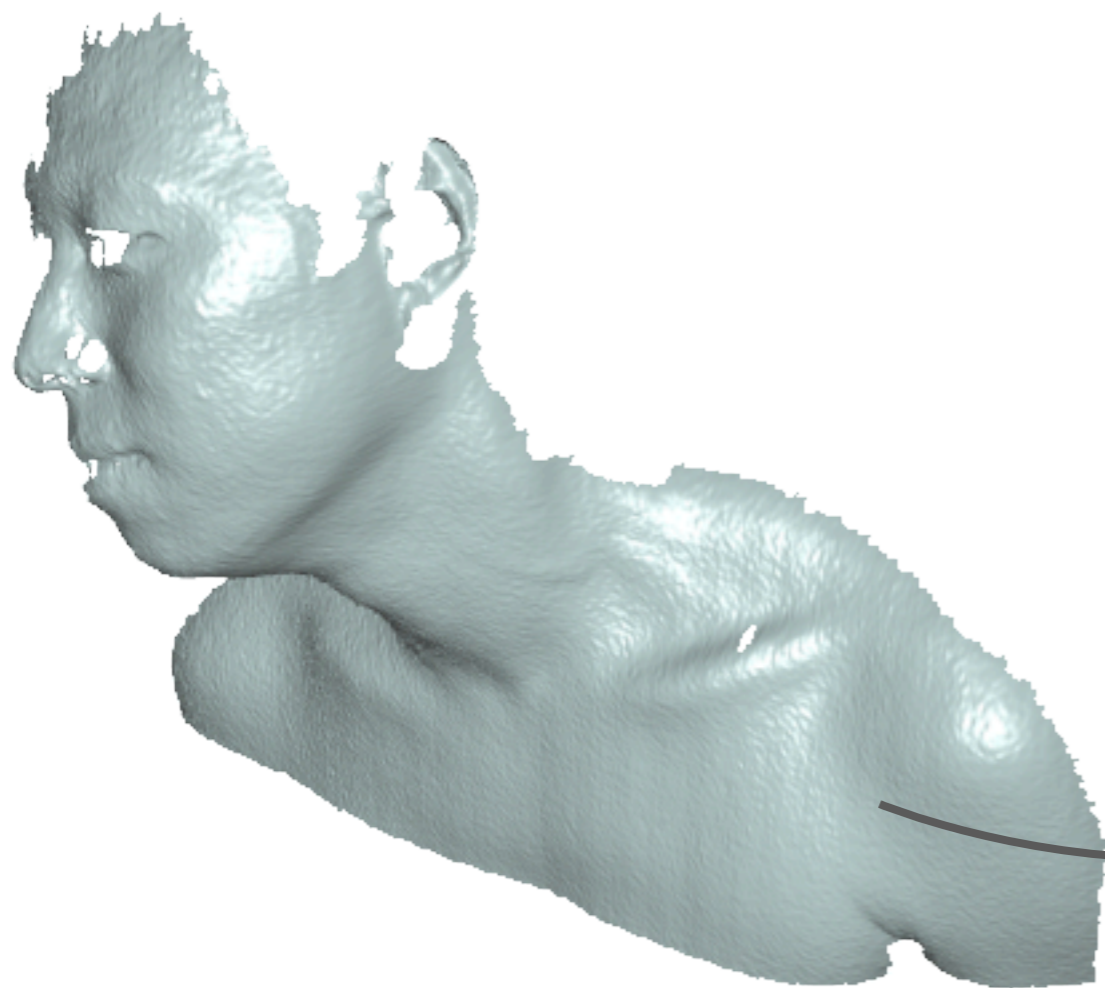


# Deformation Model



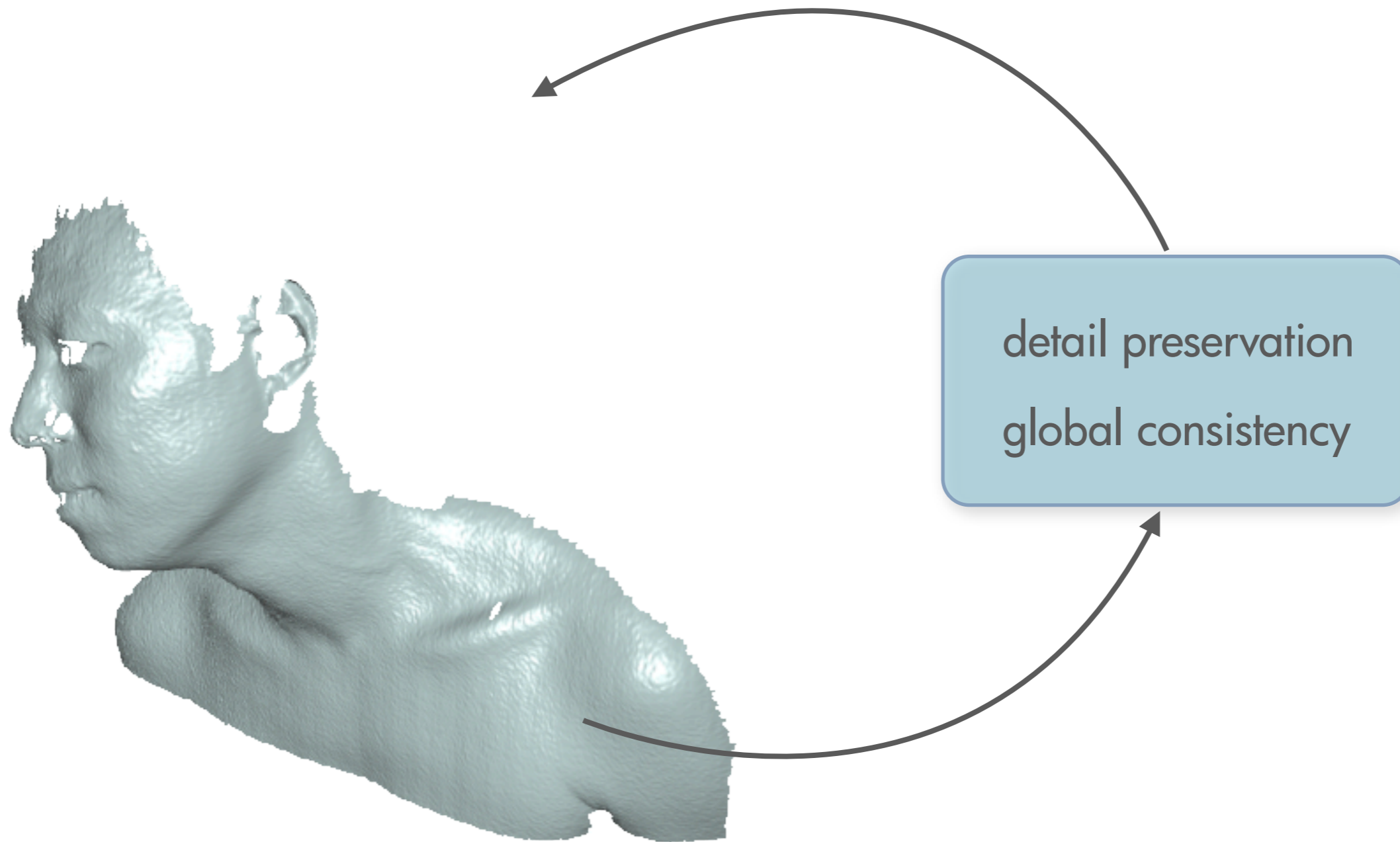
detail preservation  
global consistency

# Deformation Model

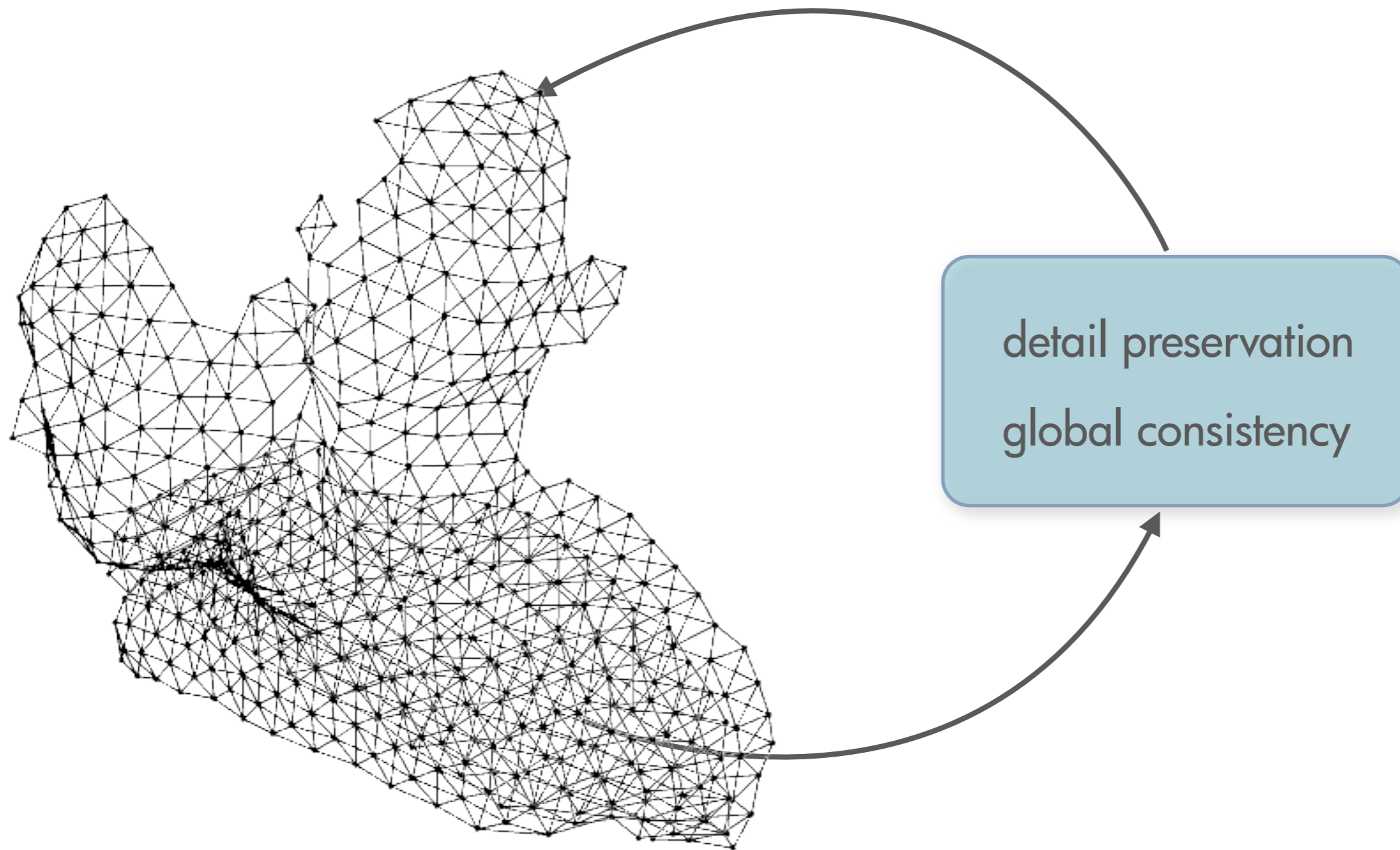


detail preservation  
global consistency

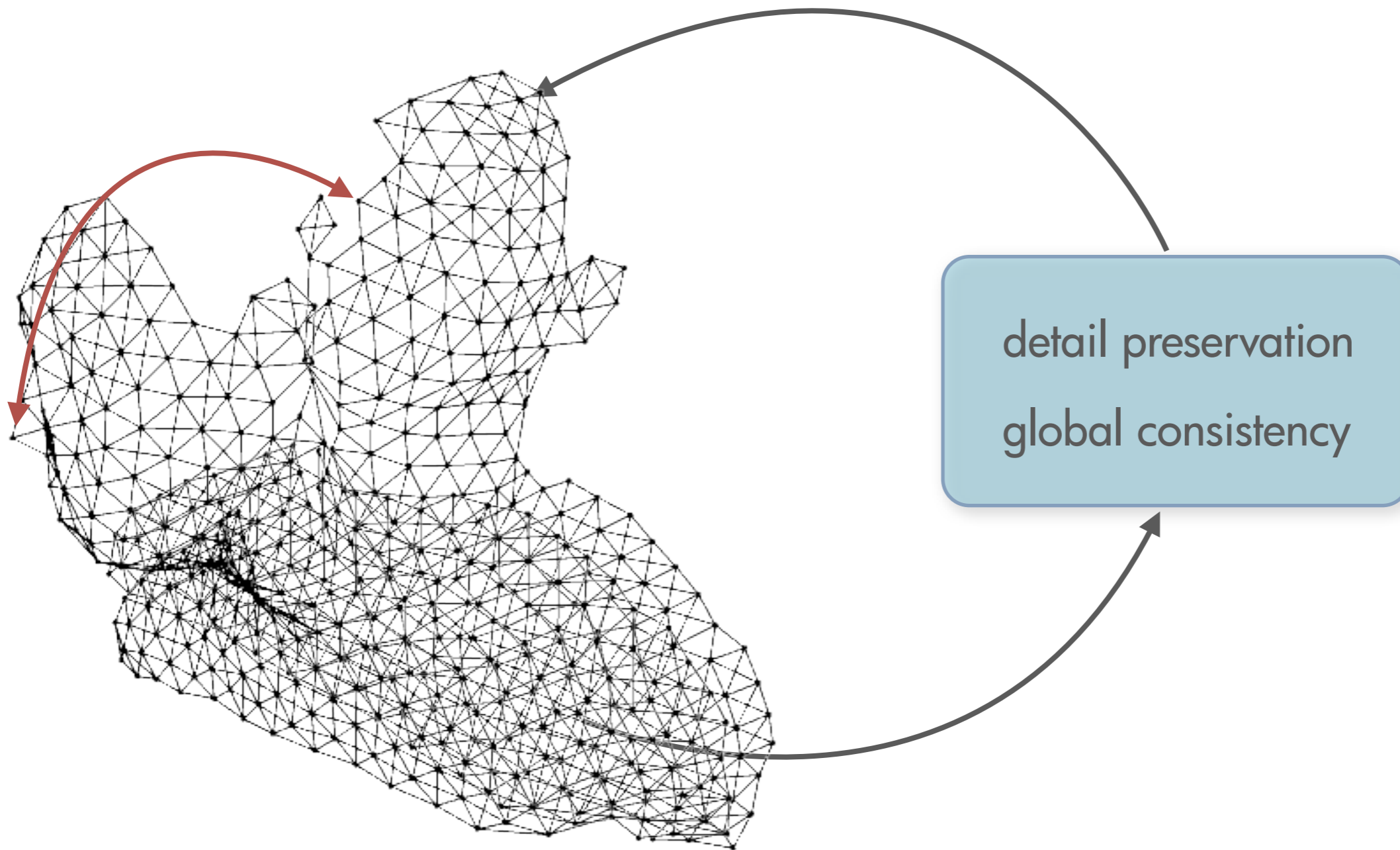
# Deformation Model



# Deformation Model

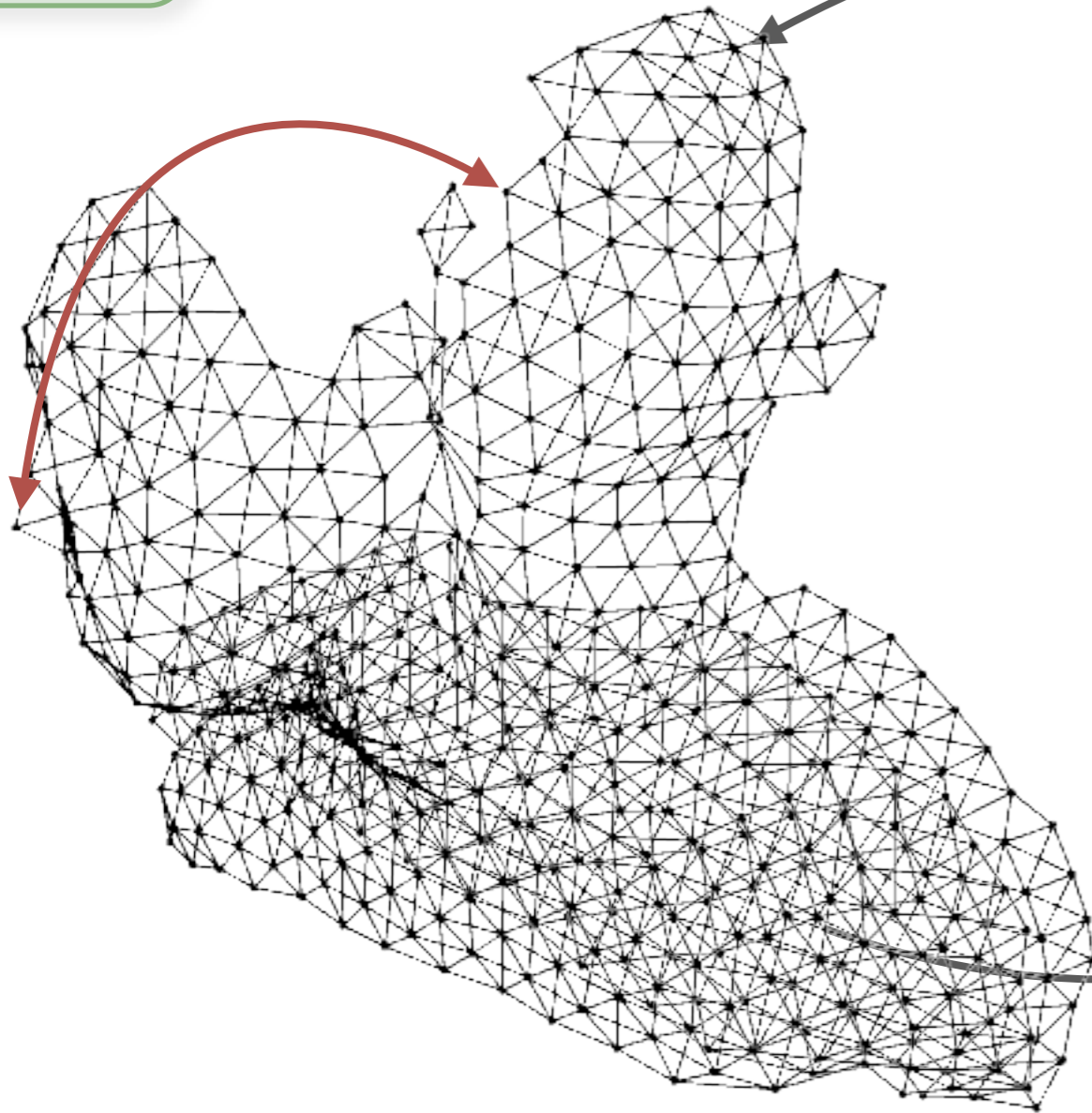


# Deformation Model



# Deformation Model

$E_{\text{rigid}}$

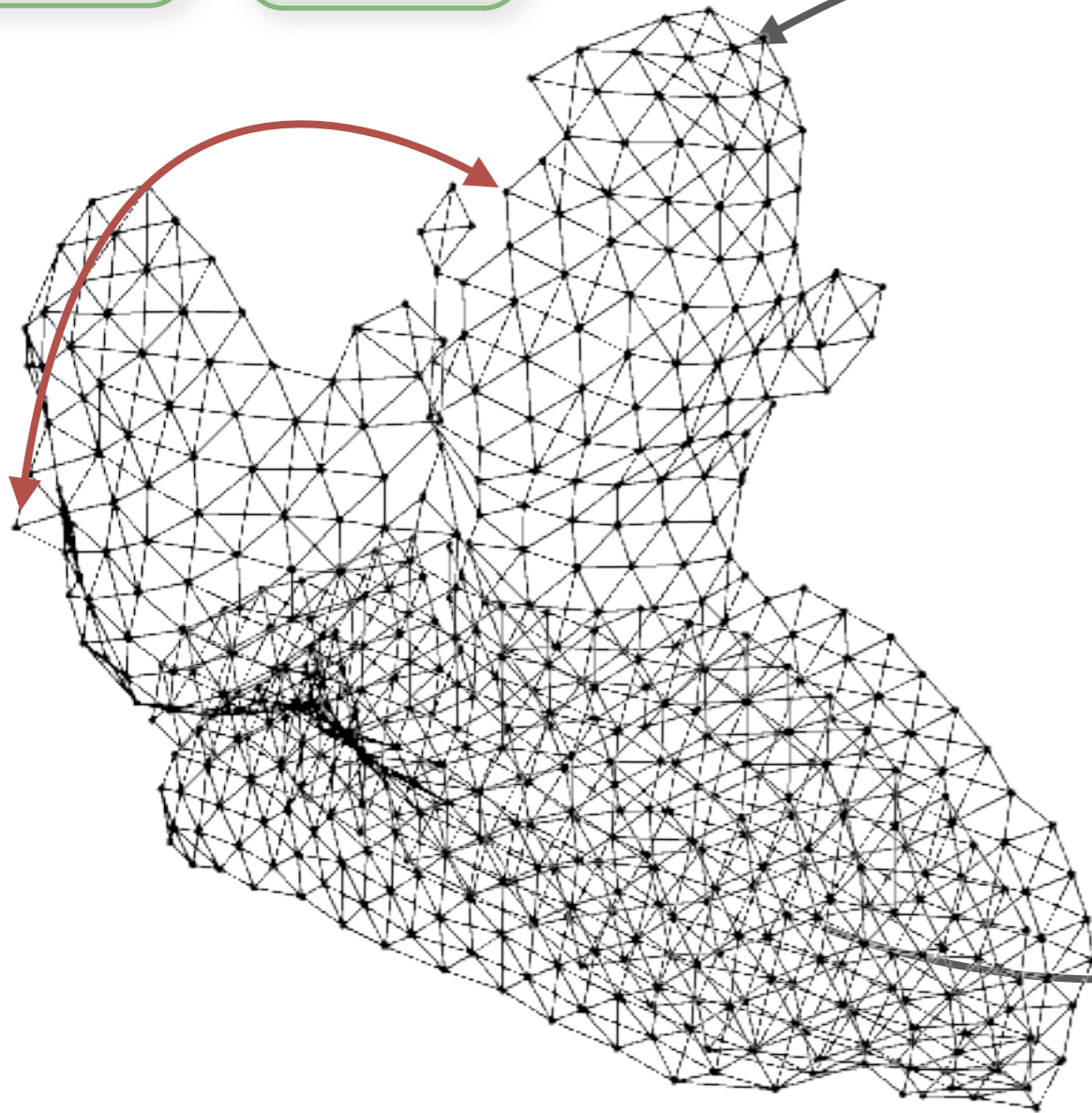


detail preservation  
global consistency

# Deformation Model

$E_{\text{rigid}}$

$E_{\text{smooth}}$



detail preservation  
global consistency

# Non-Linear Energy Minimization

$E_{\text{rigid}}$

$E_{\text{smooth}}$



# Non-Linear Energy Minimization

$E_{\text{rigid}}$

$E_{\text{smooth}}$



# Non-Linear Energy Minimization



$E_{\text{rigid}}$

$E_{\text{smooth}}$

# Non-Linear Energy Minimization



$E_{\text{rigid}}$

$E_{\text{smooth}}$

# Non-Linear Energy Minimization

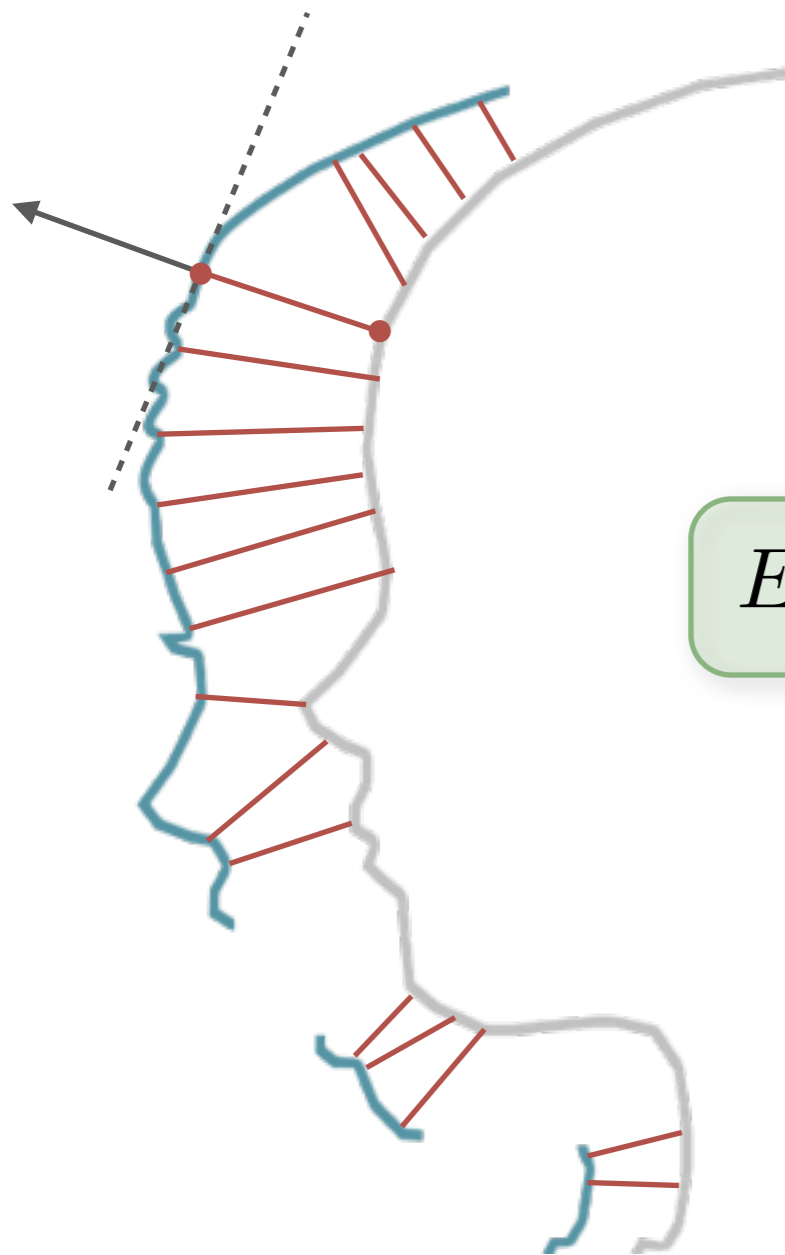


$E_{\text{point}}$

$E_{\text{rigid}}$

$E_{\text{smooth}}$

# Non-Linear Energy Minimization



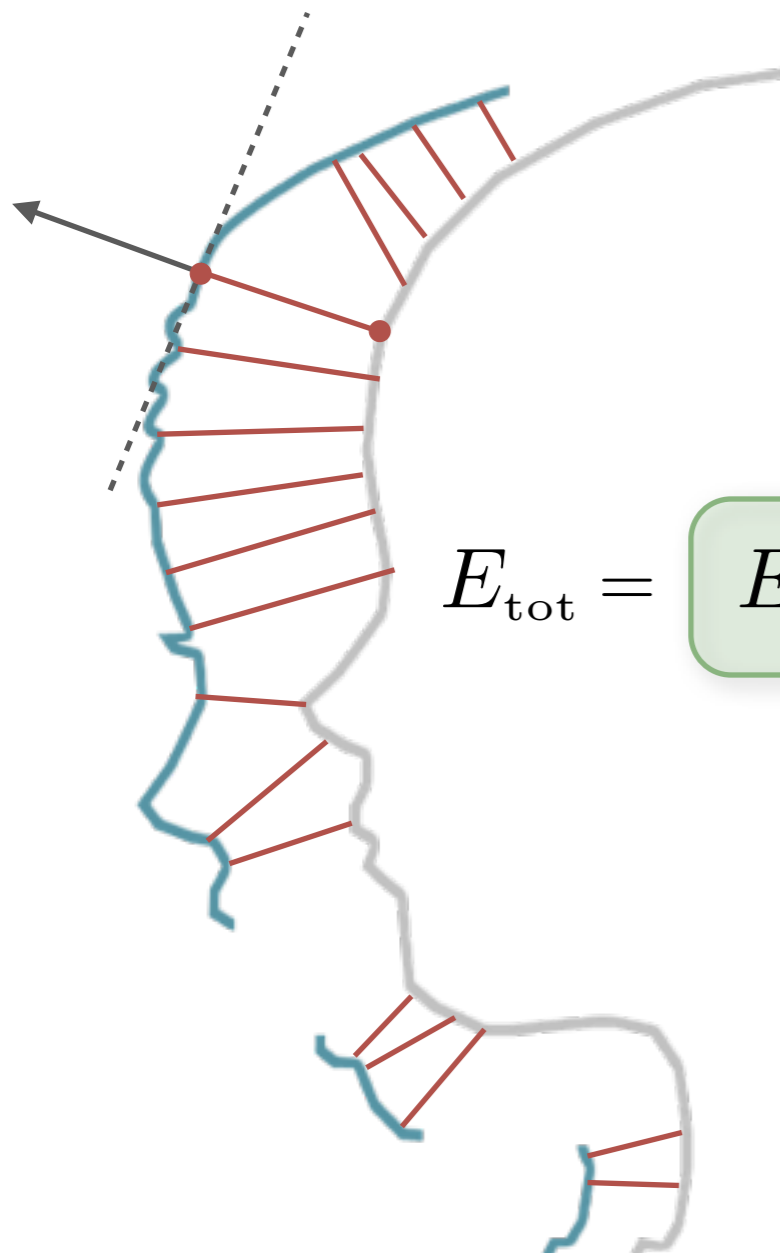
$E_{\text{plane}}$

$E_{\text{point}}$

$E_{\text{rigid}}$

$E_{\text{smooth}}$

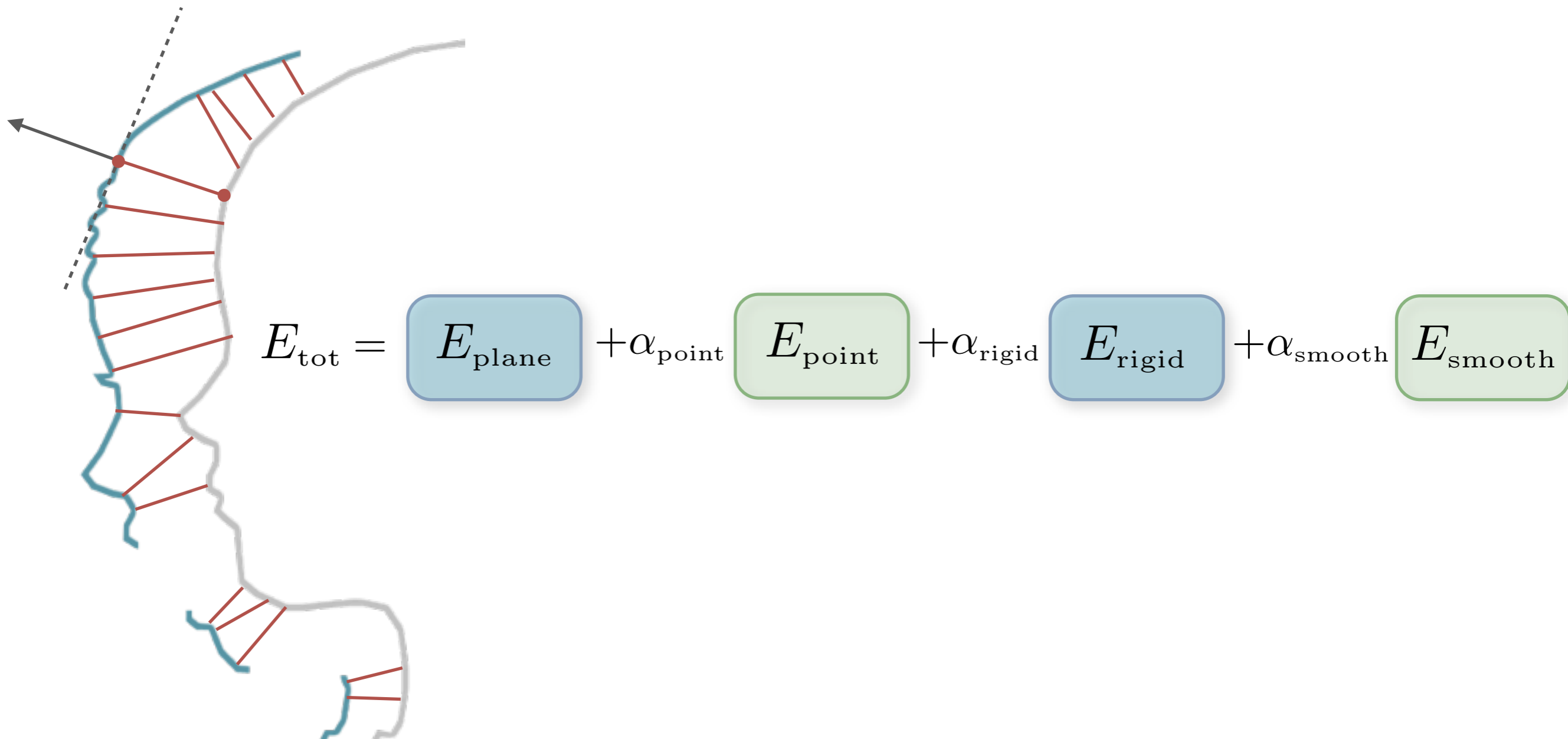
# Non-Linear Energy Minimization



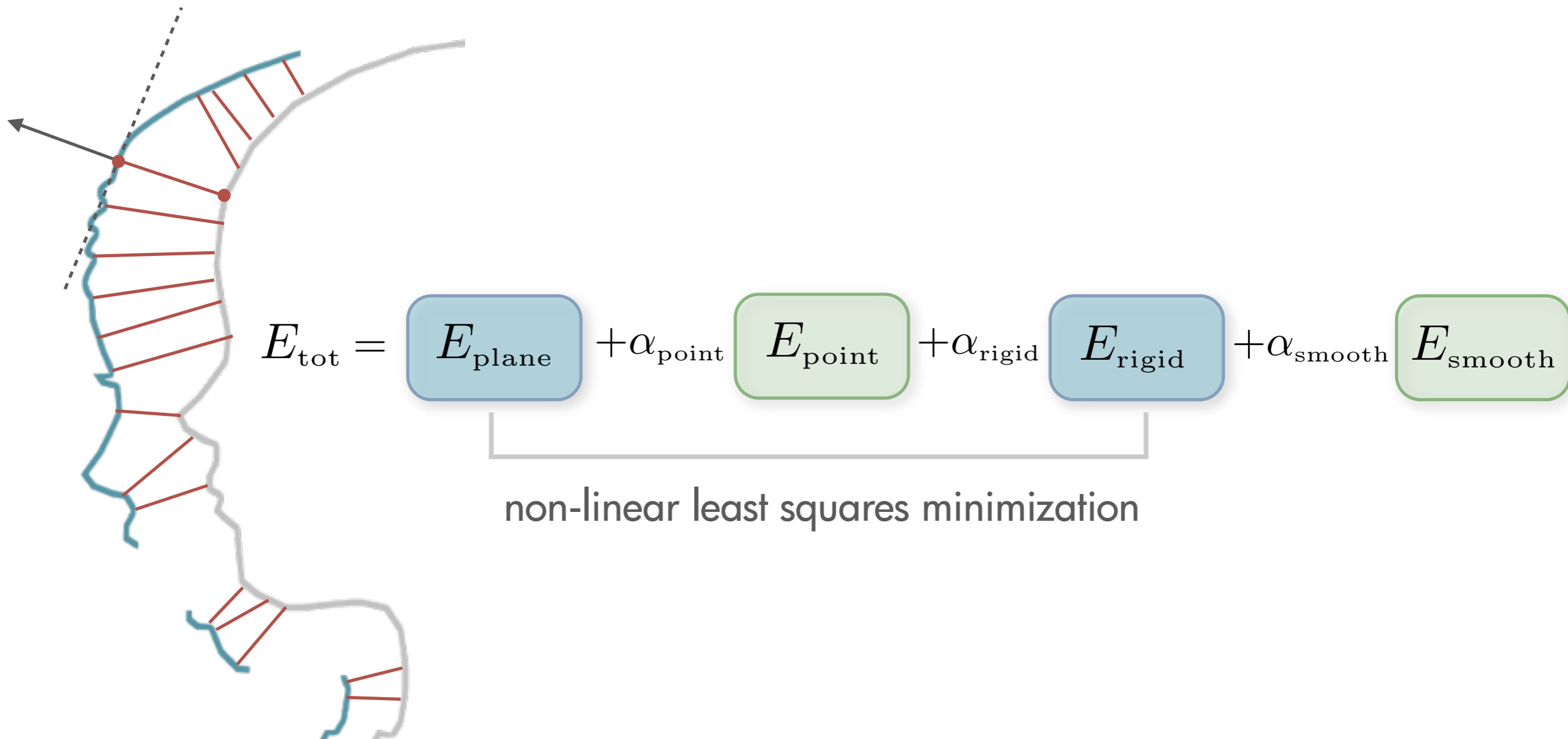
The diagram on the left shows a blue curve with several red line segments connecting points on it. A dashed line with an arrow points to one of the red segments, indicating a specific point or region of interest.

$$E_{\text{tot}} = E_{\text{plane}} + \alpha_{\text{point}} E_{\text{point}} + \alpha_{\text{rigid}} E_{\text{rigid}} + \alpha_{\text{smooth}} E_{\text{smooth}}$$

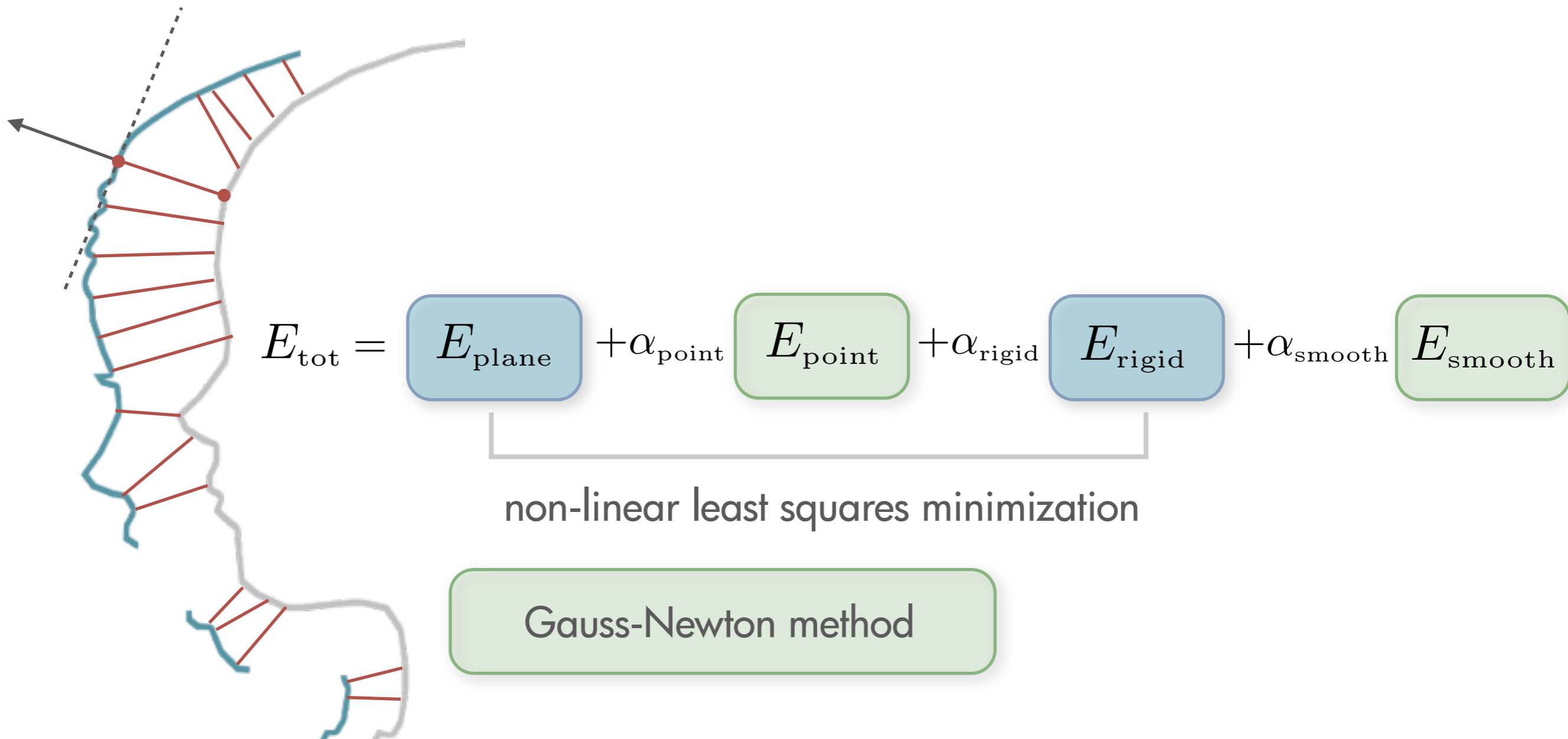
# Non-Linear Energy Minimization



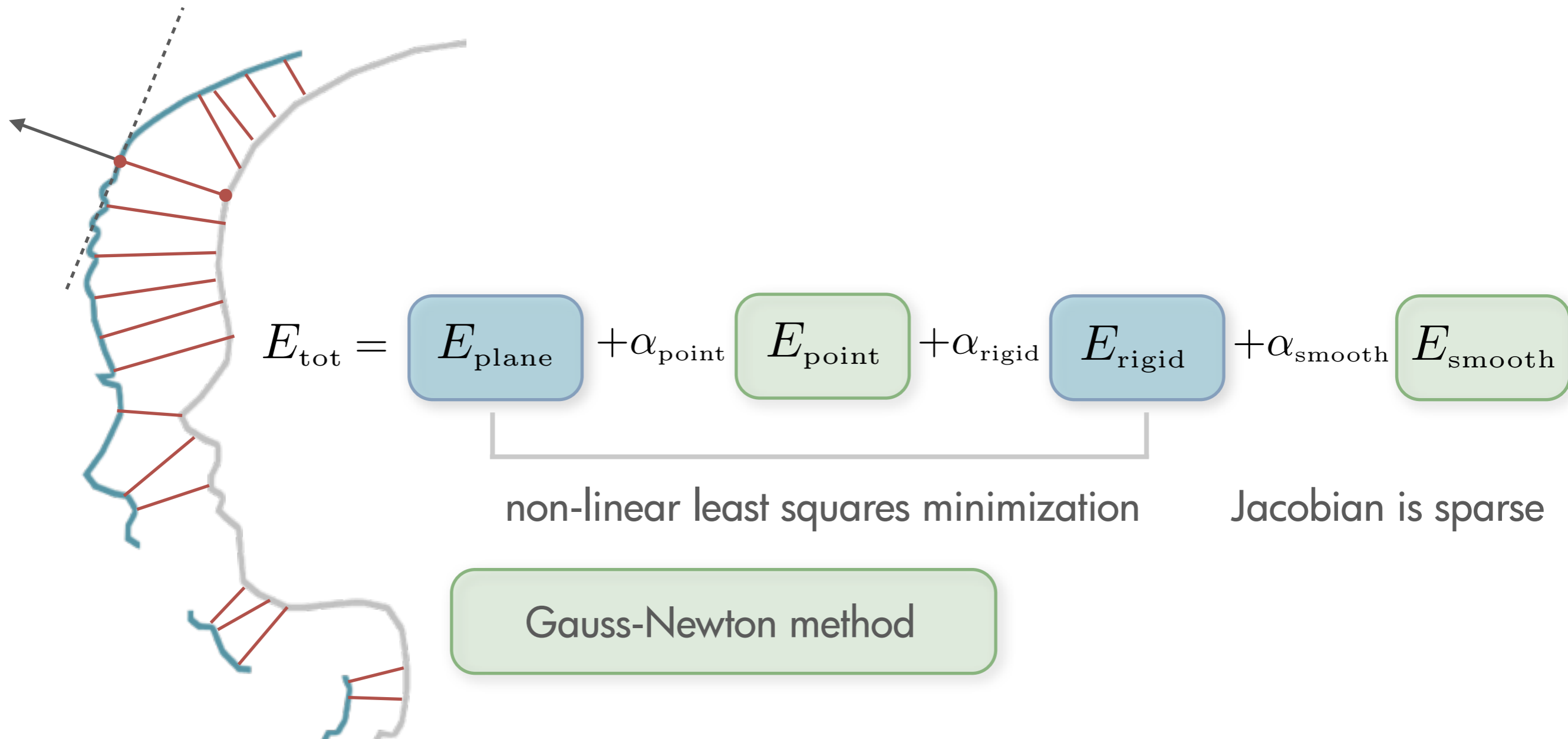
# Non-Linear Energy Minimization



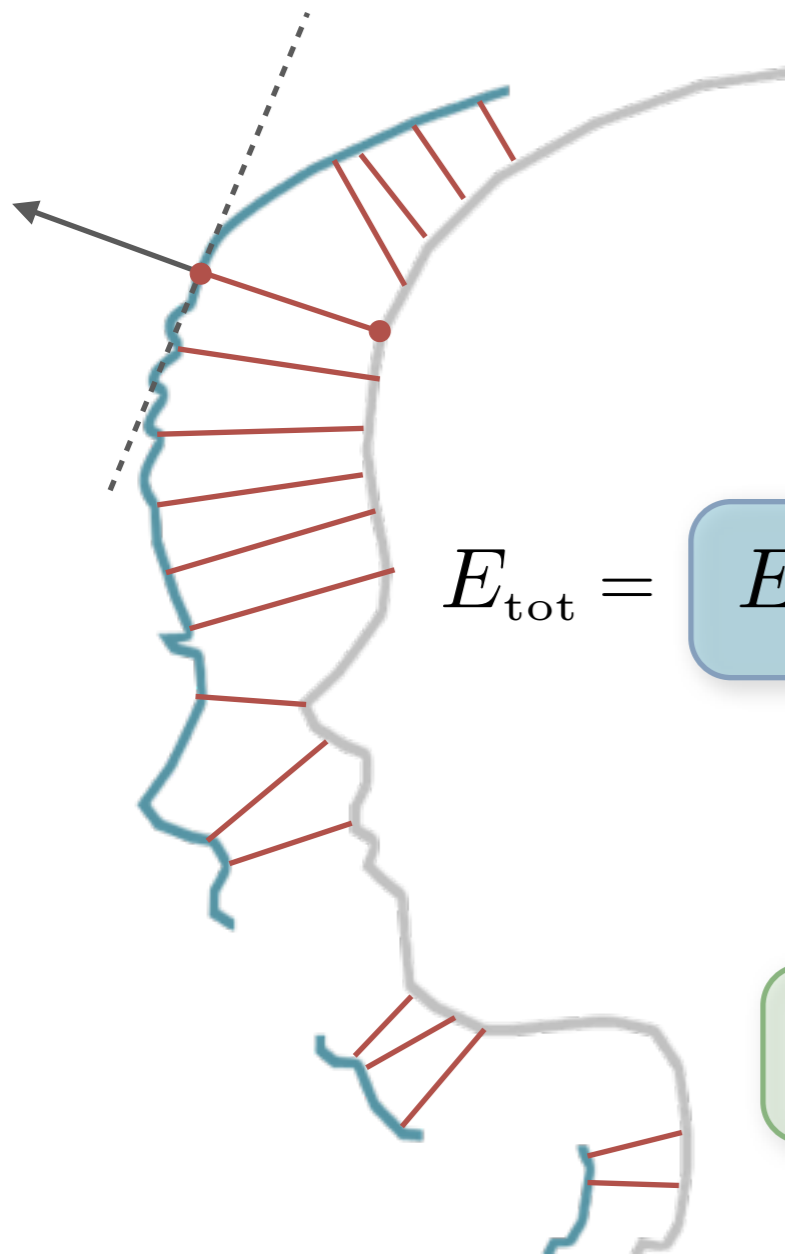
# Non-Linear Energy Minimization



# Non-Linear Energy Minimization



# Non-Linear Energy Minimization



$$E_{\text{tot}} = E_{\text{plane}} + \alpha_{\text{point}} E_{\text{point}} + \alpha_{\text{rigid}} E_{\text{rigid}} + \alpha_{\text{smooth}} E_{\text{smooth}}$$

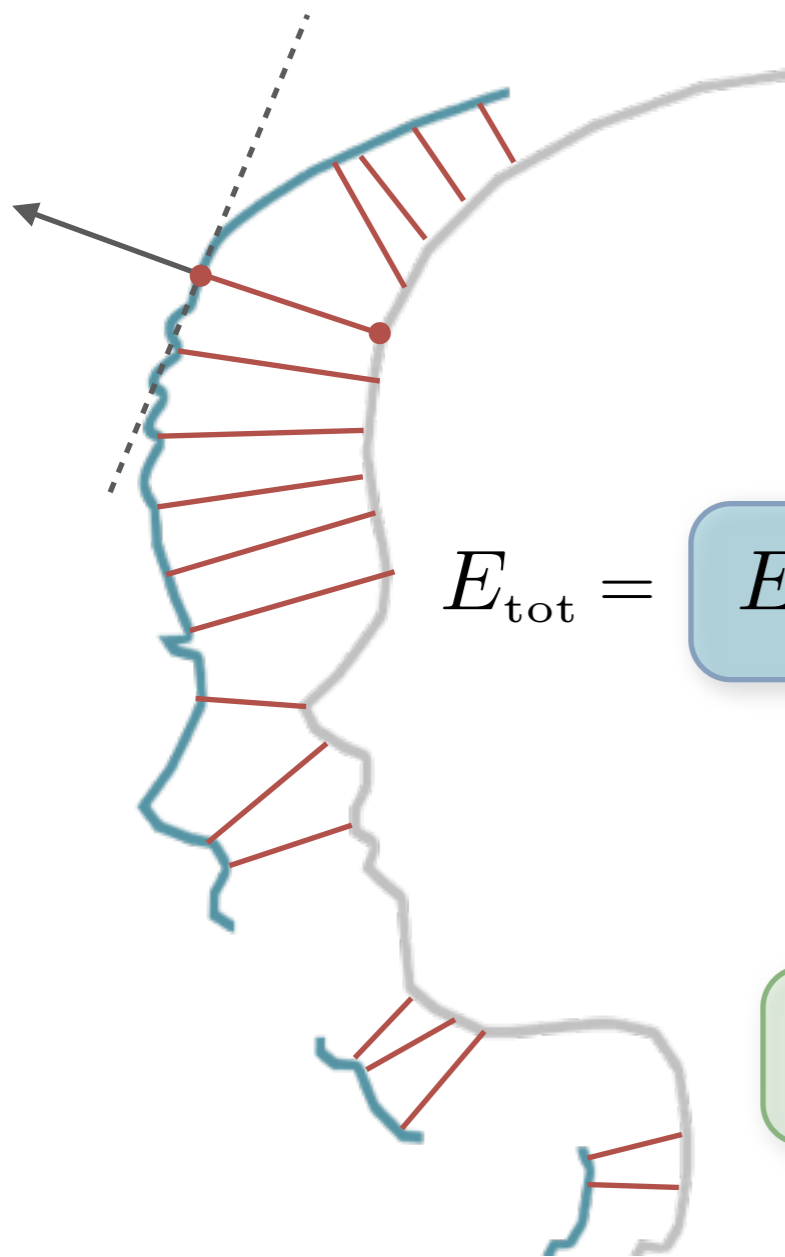
non-linear least squares minimization

Jacobian is sparse

Gauss-Newton method

sparse Cholesky factorization

# Non-Linear Energy Minimization



$$E_{\text{tot}} = E_{\text{plane}} + \alpha_{\text{point}} E_{\text{point}} + \alpha_{\text{rigid}} E_{\text{rigid}} + \alpha_{\text{smooth}} E_{\text{smooth}}$$

non-linear least squares minimization

Jacobian is sparse

Gauss-Newton method

sparse Cholesky factorization

that's it!

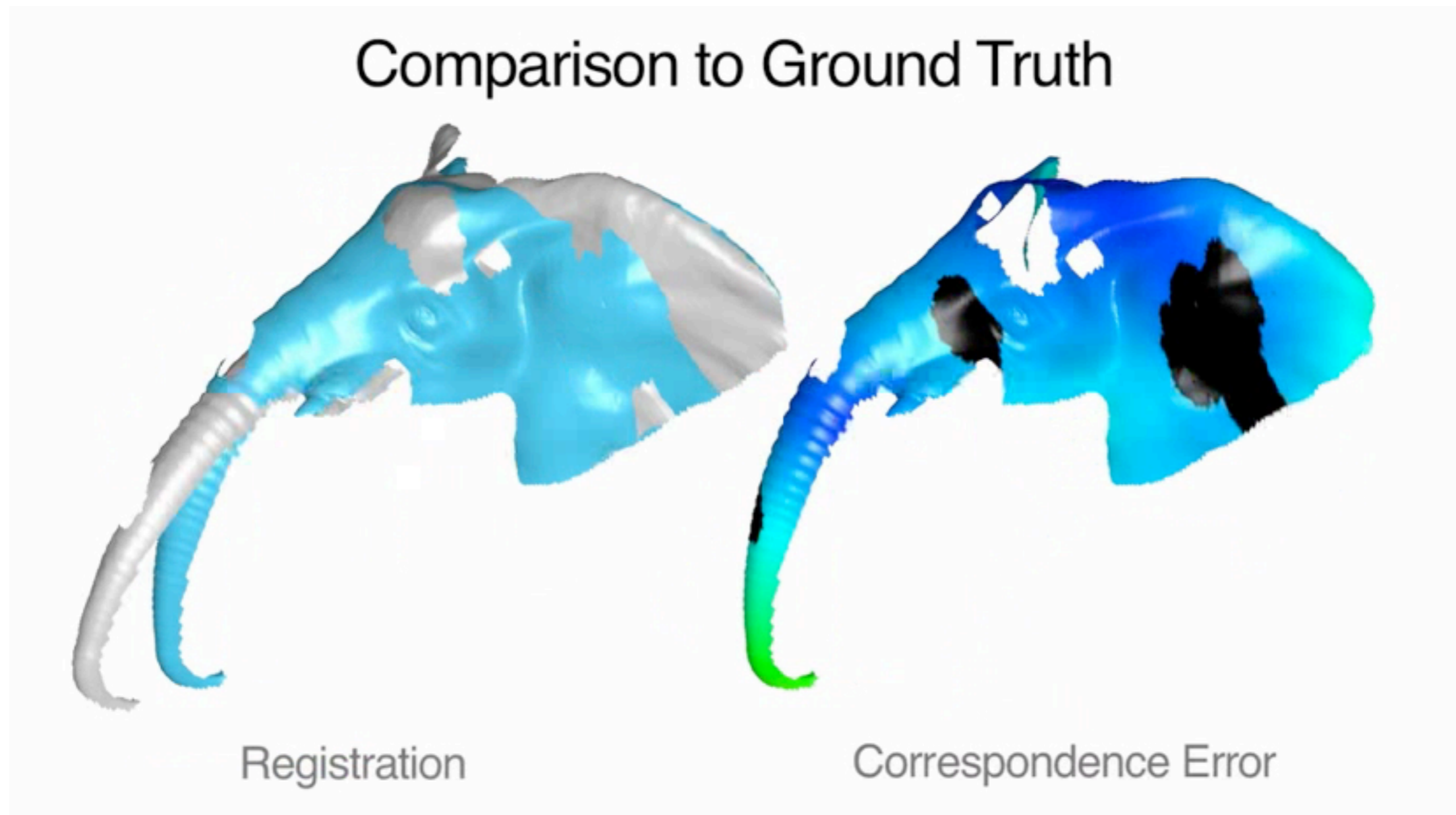
# Synthetic Model

Elephant (329 nodes, 21k vertices)



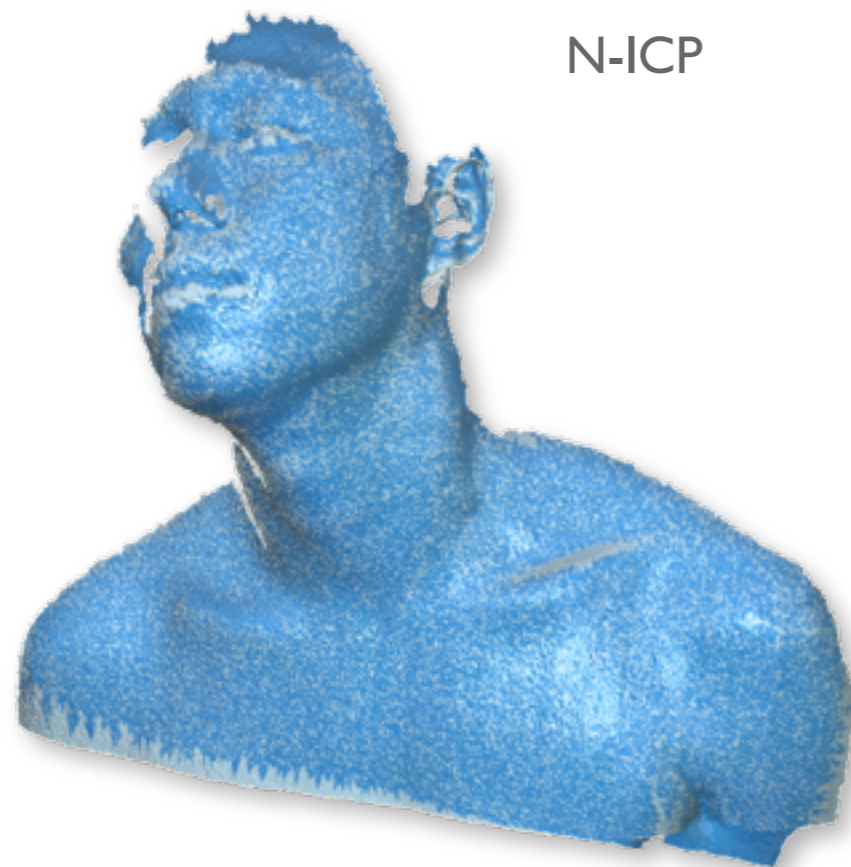
Source

# Comparison to Ground-Truth

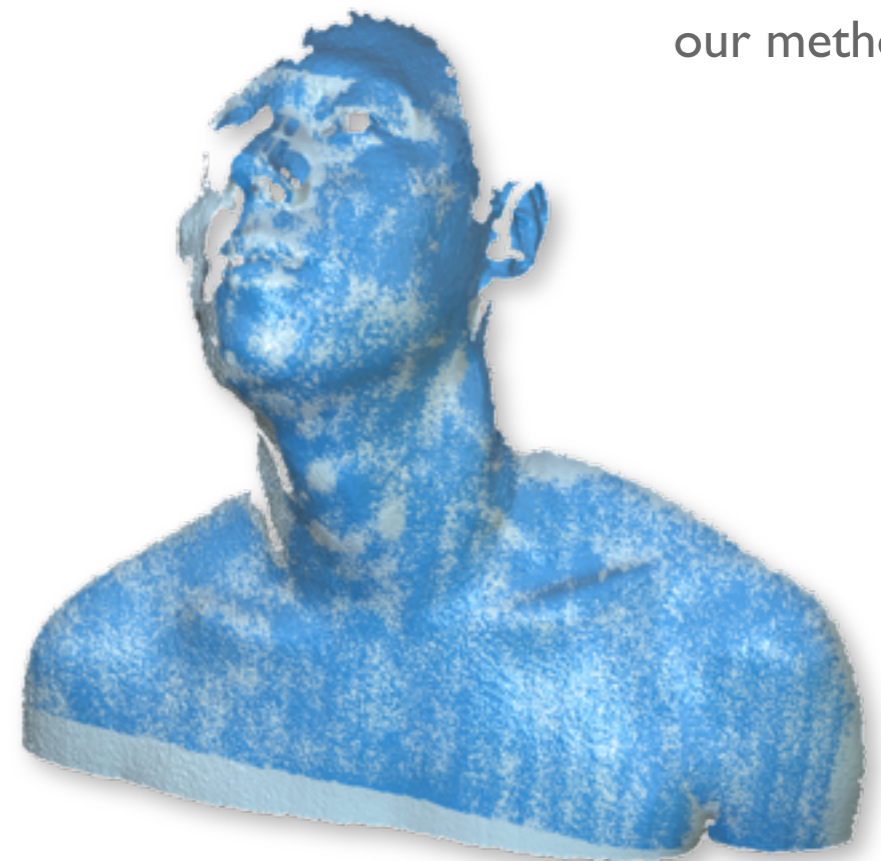


# Comparison with other N-ICP

[Pauly et al. '05] [Pottmann et al. '06]



N-ICP



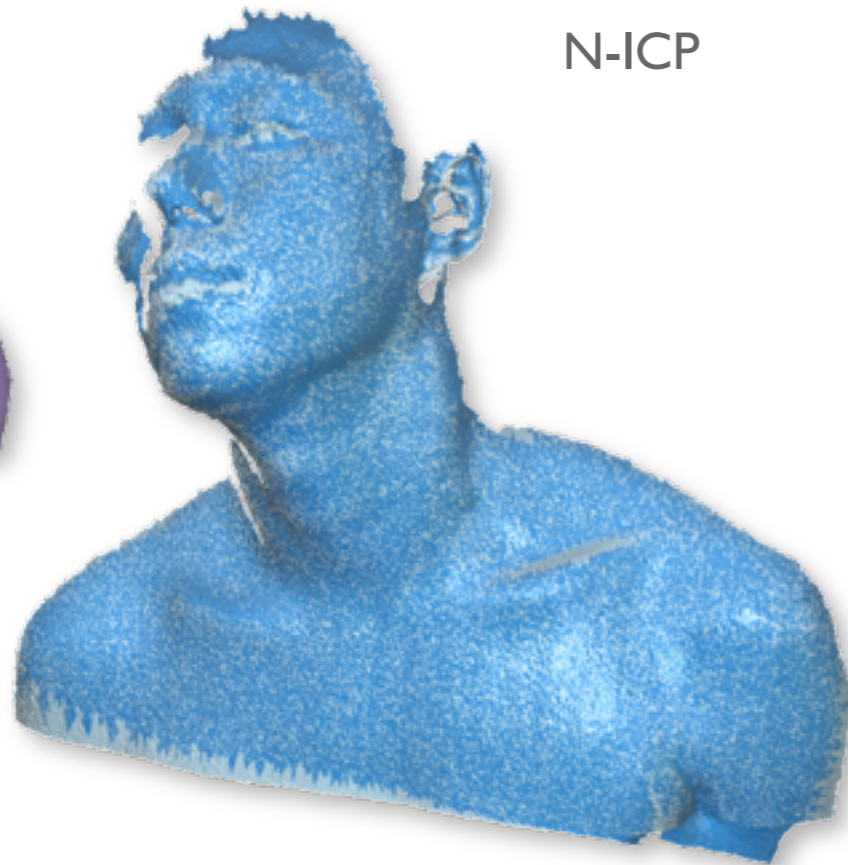
our method

# Comparison with other N-ICP

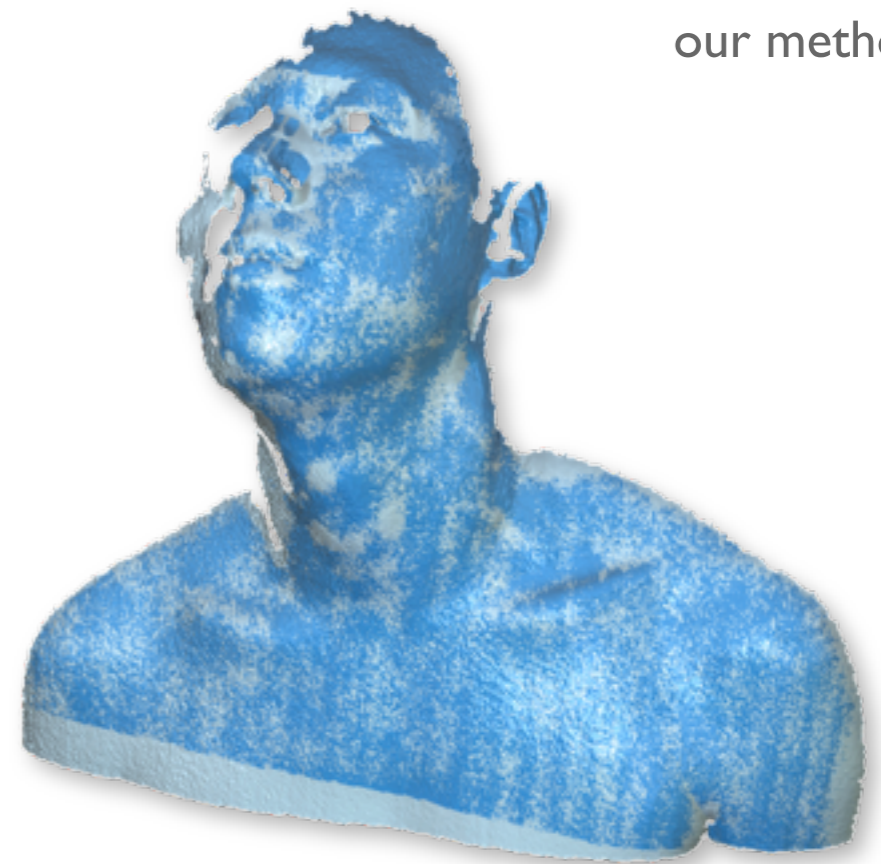
[Pauly et al. '05] [Pottmann et al. '06]



texturing



N-ICP



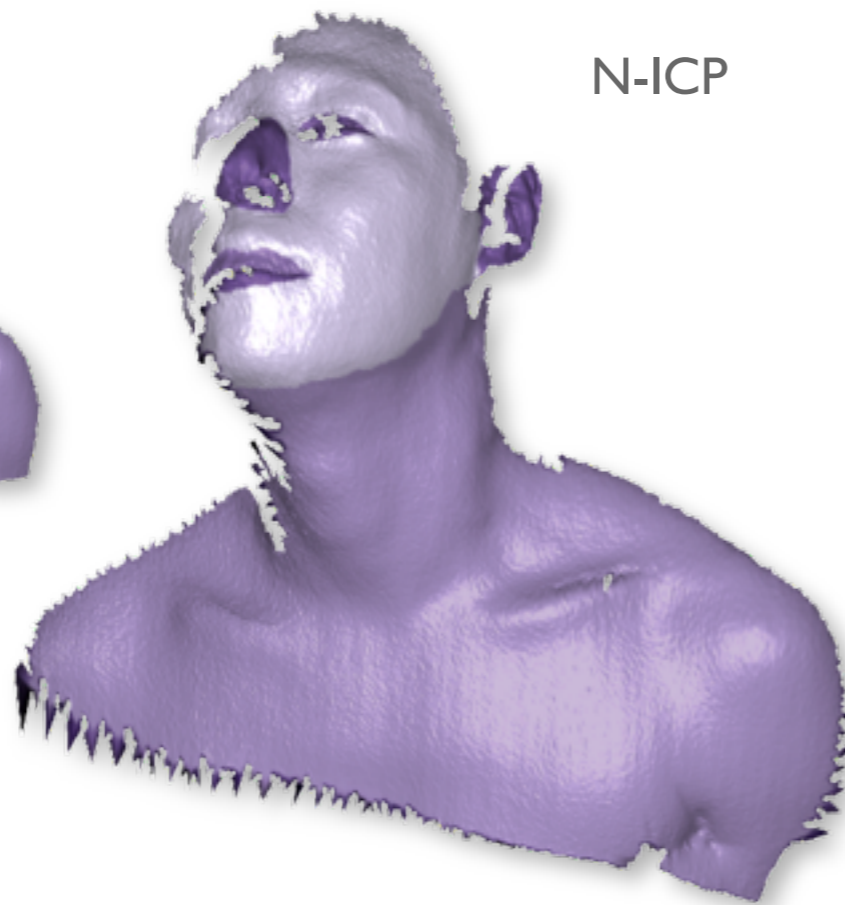
our method

# Comparison with other N-ICP

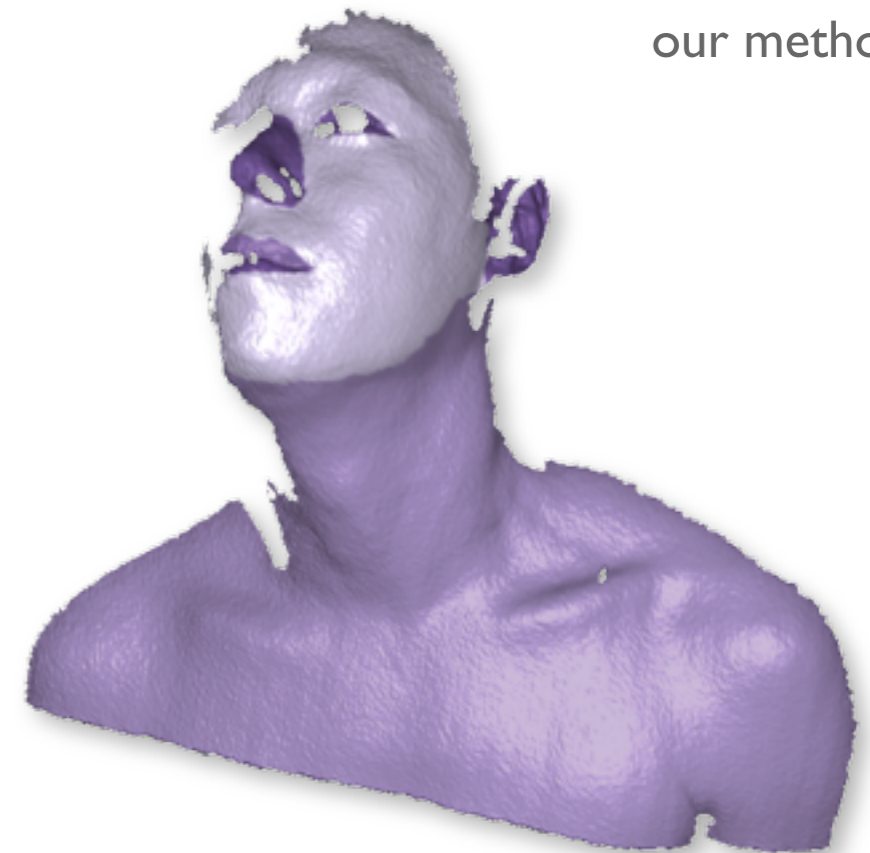
[Pauly et al. '05] [Pottmann et al. '06]



texturing



N-ICP

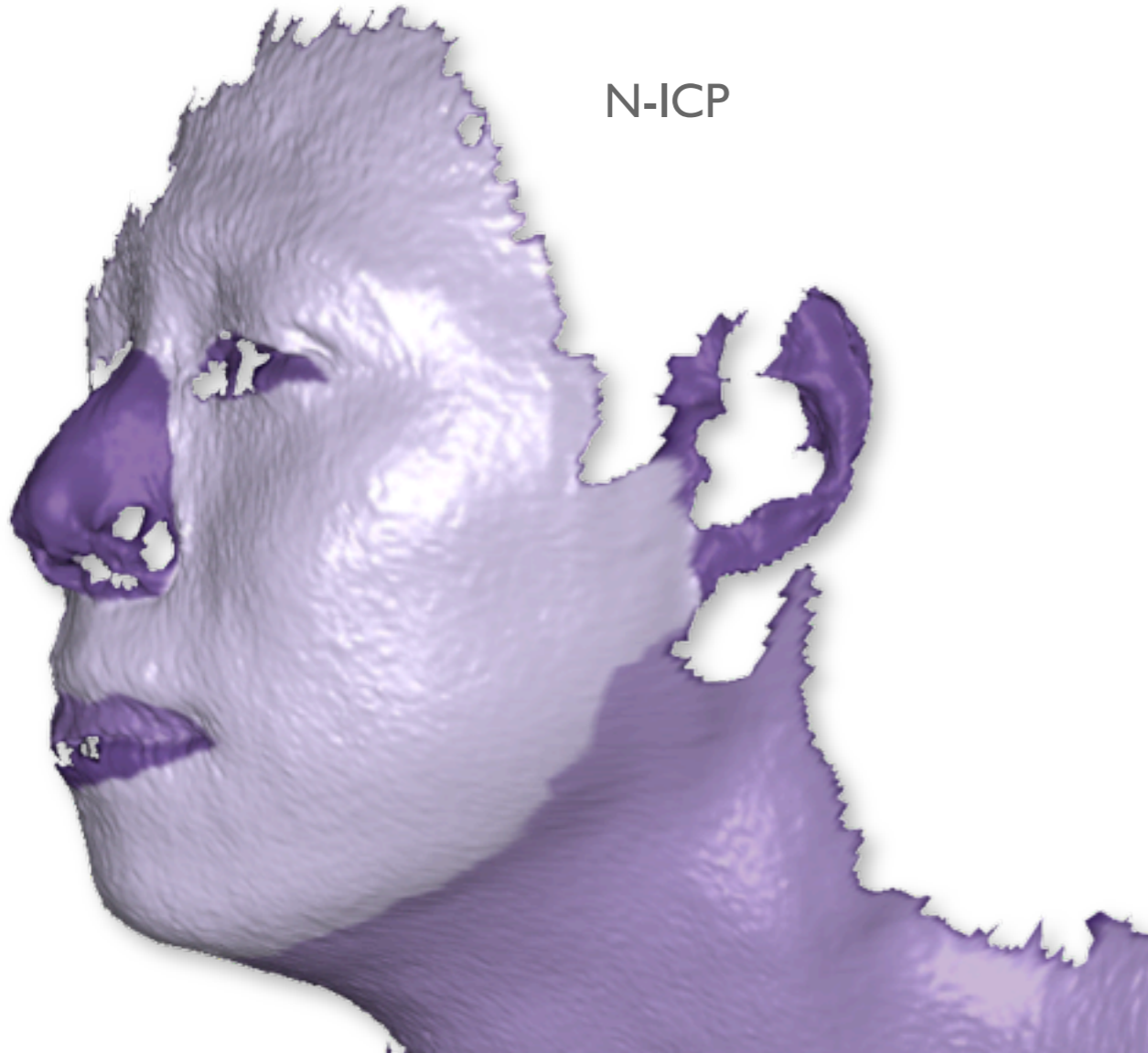


our method

# Comparison with other N-ICP

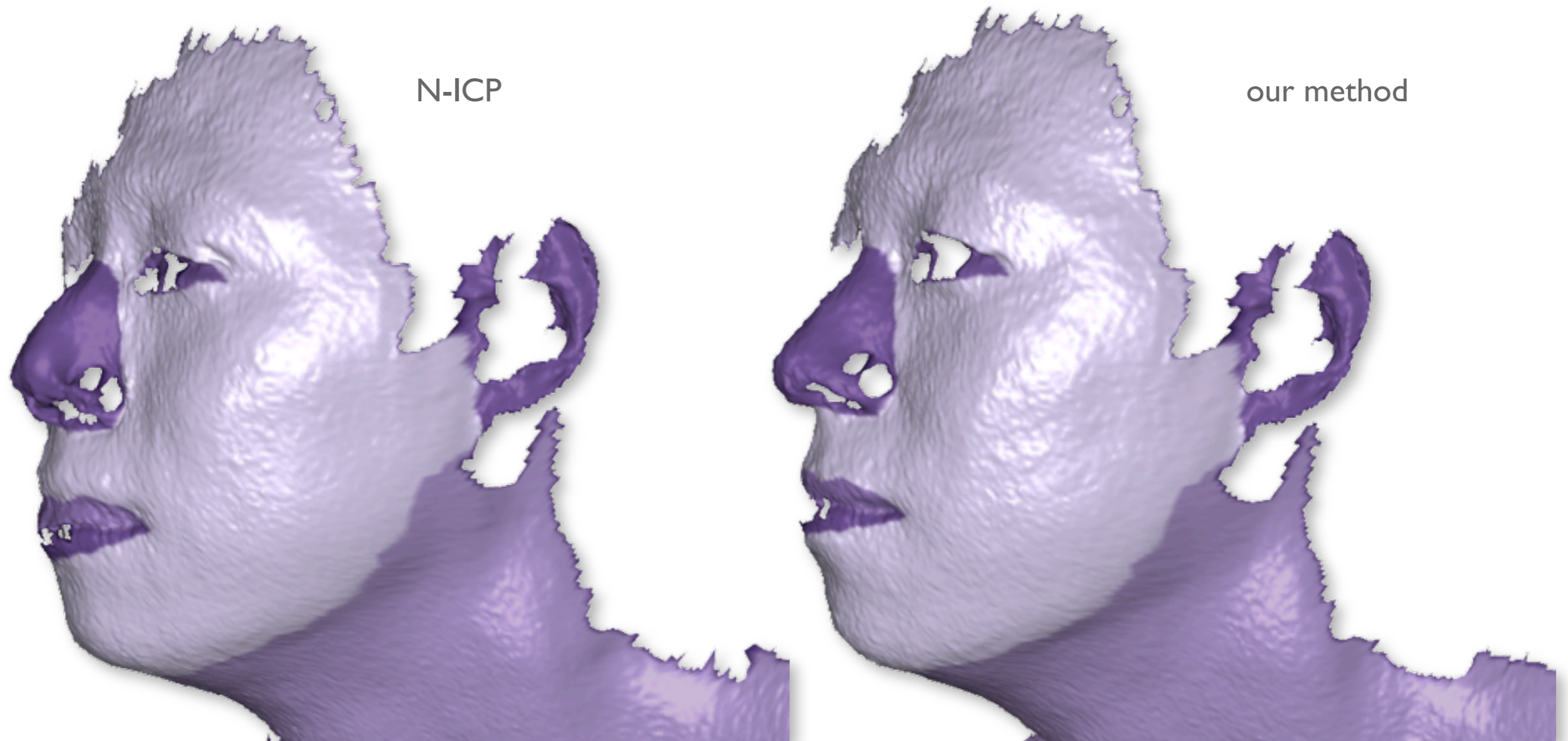
[Pauly et al. '05] [Pottmann et al. '06]

N-ICP



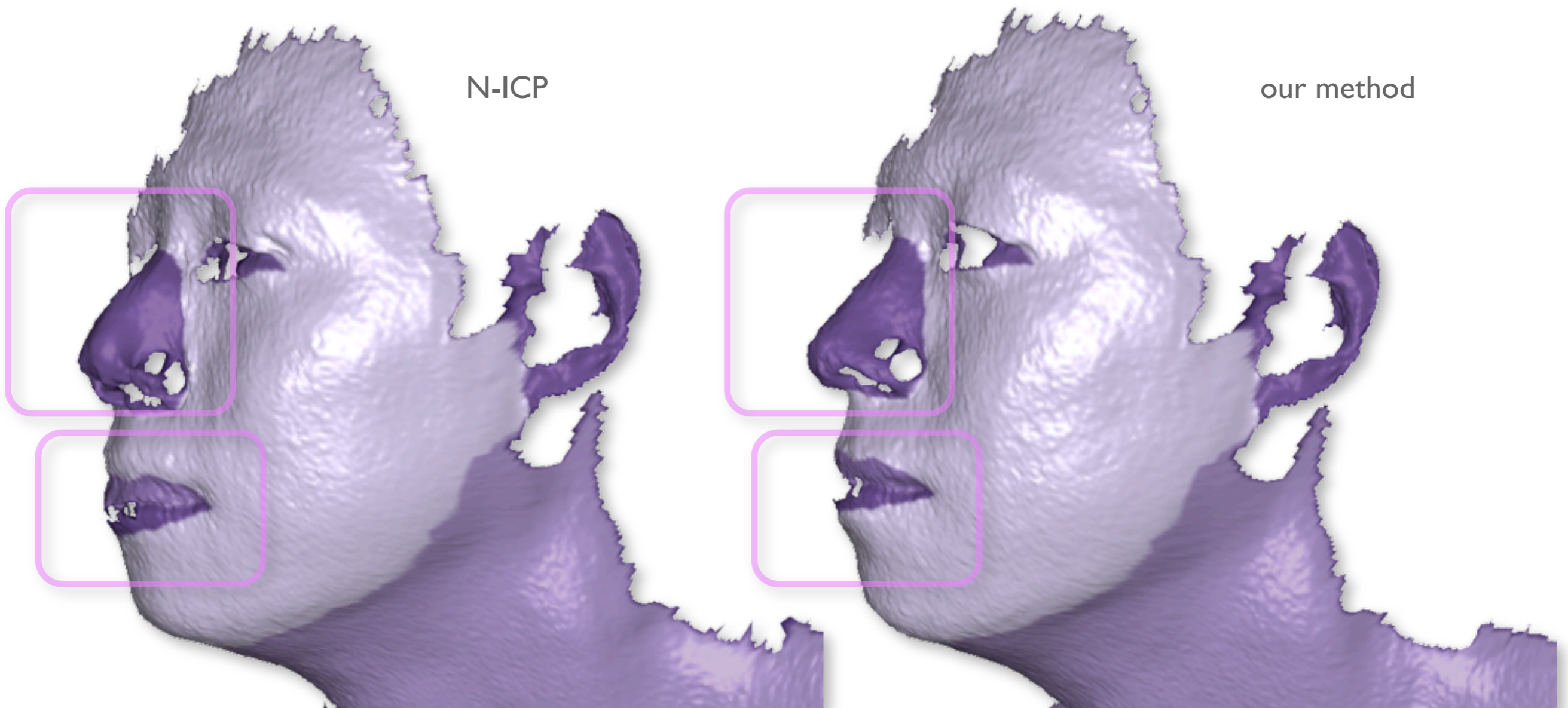
# Comparison with other N-ICP

[Pauly et al. '05] [Pottmann et al. '06]



# Comparison with other N-ICP

[Pauly et al. '05] [Pottmann et al. '06]



# Draping Table Cloth

source

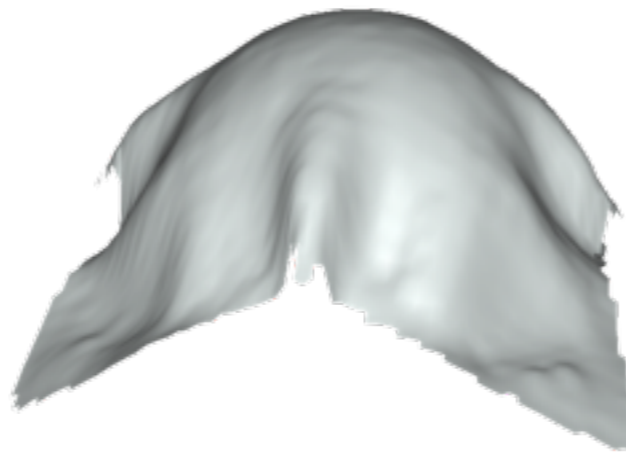


# Draping Table Cloth

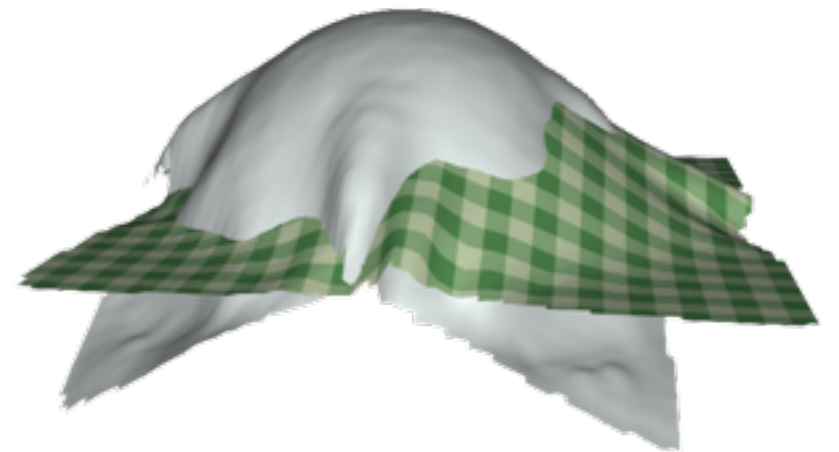
source



target



initial alignment



# Suitable for Isometric Deformations

N-ICP

our method



# Suitable for Isometric Deformations

N-ICP



our method



# Suitable for Isometric Deformations

N-ICP

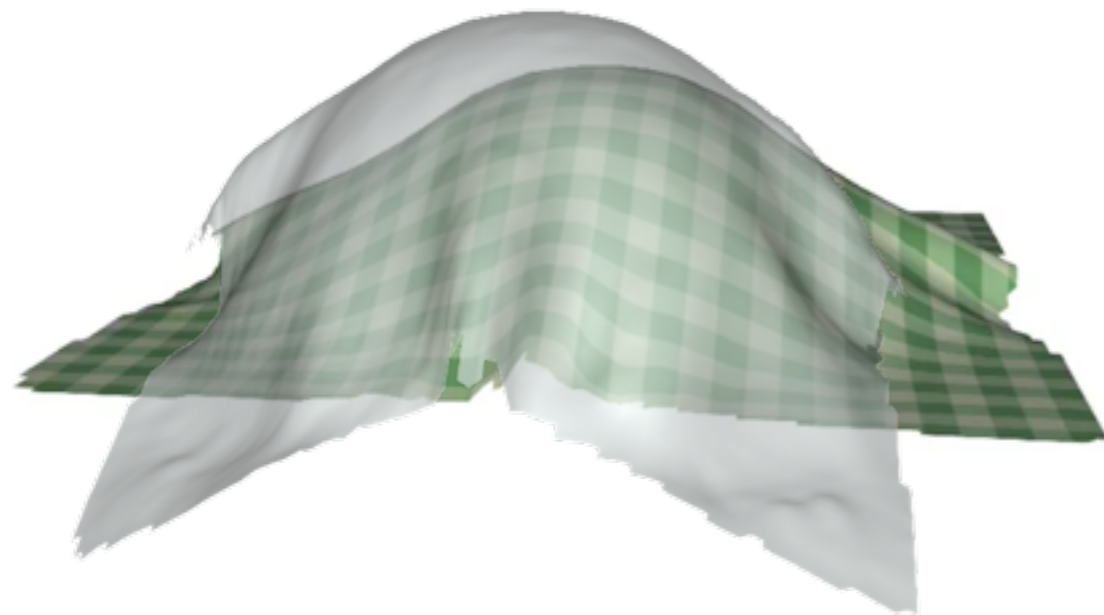


our method

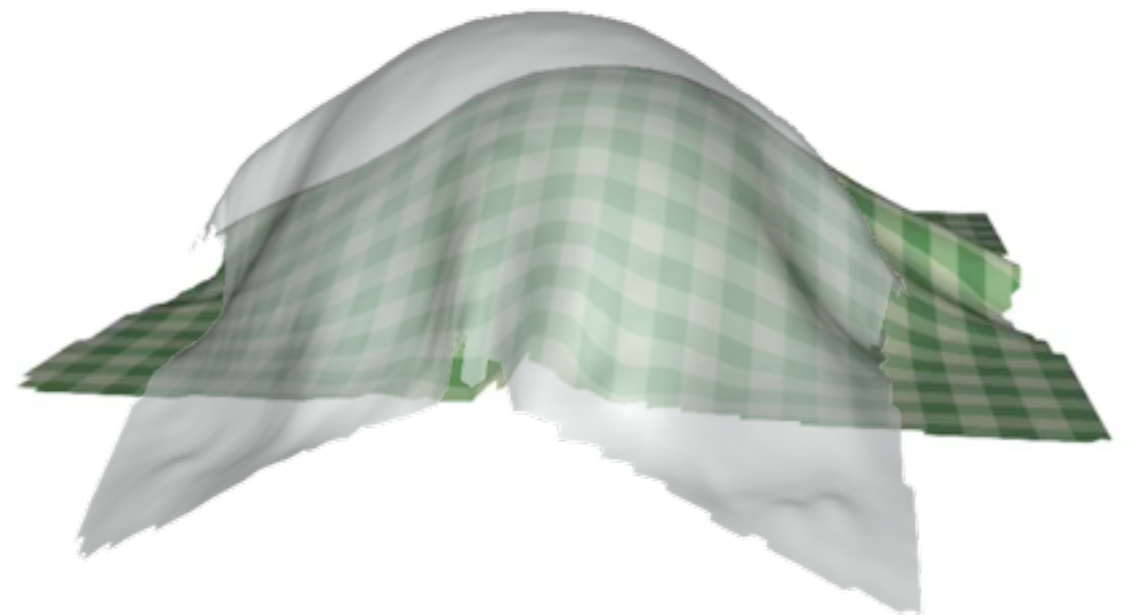


# Suitable for Isometric Deformations

N-ICP



our method



# Some Examples



# Face Capture Pipeline



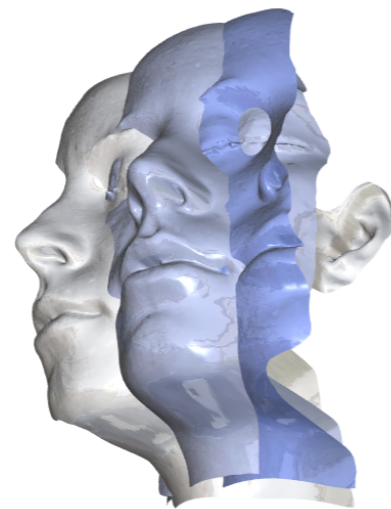
acquisition

*data provided by Paramount Pictures and Aguru Images*

# Face Capture Pipeline



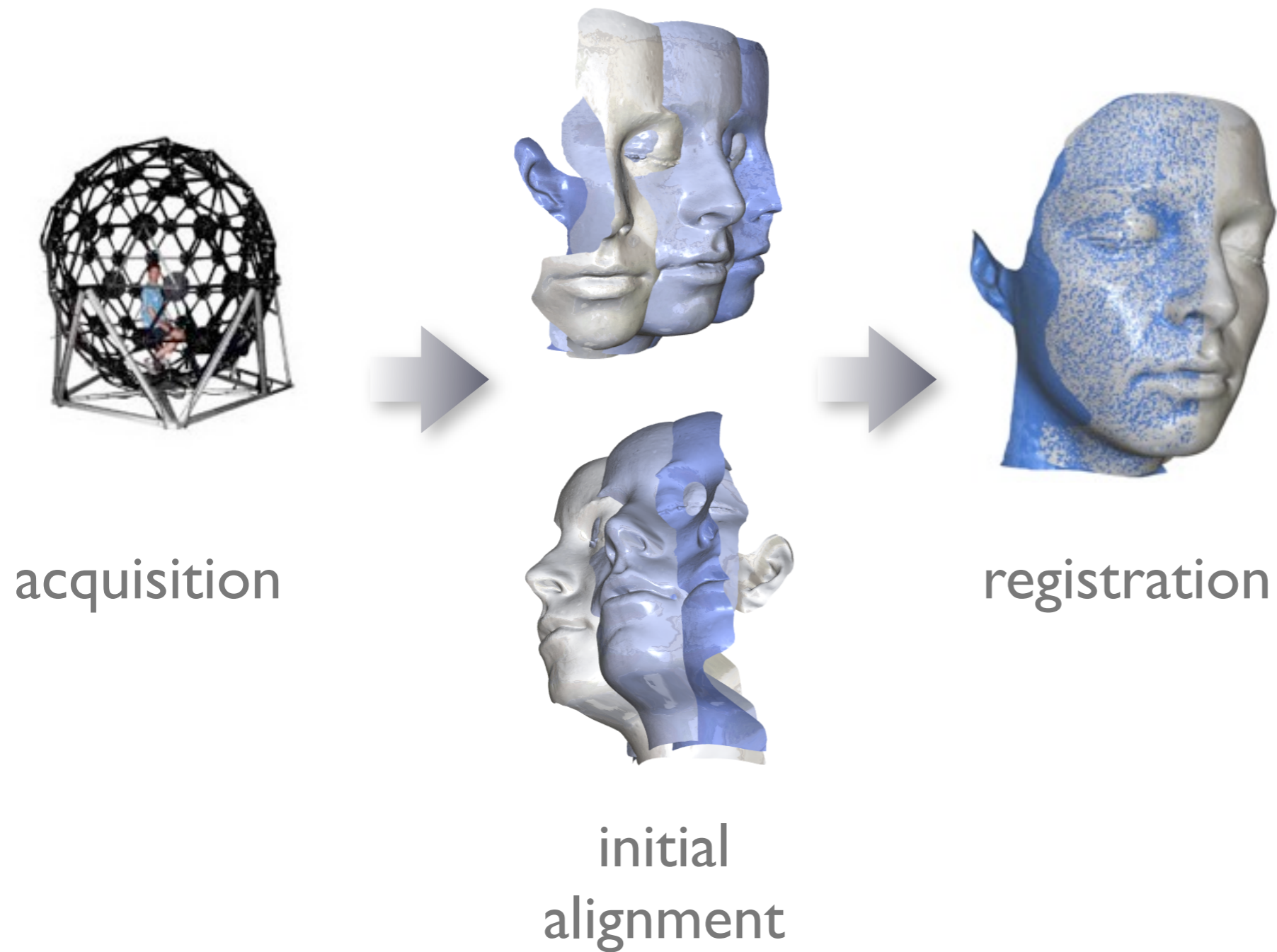
acquisition



initial  
alignment

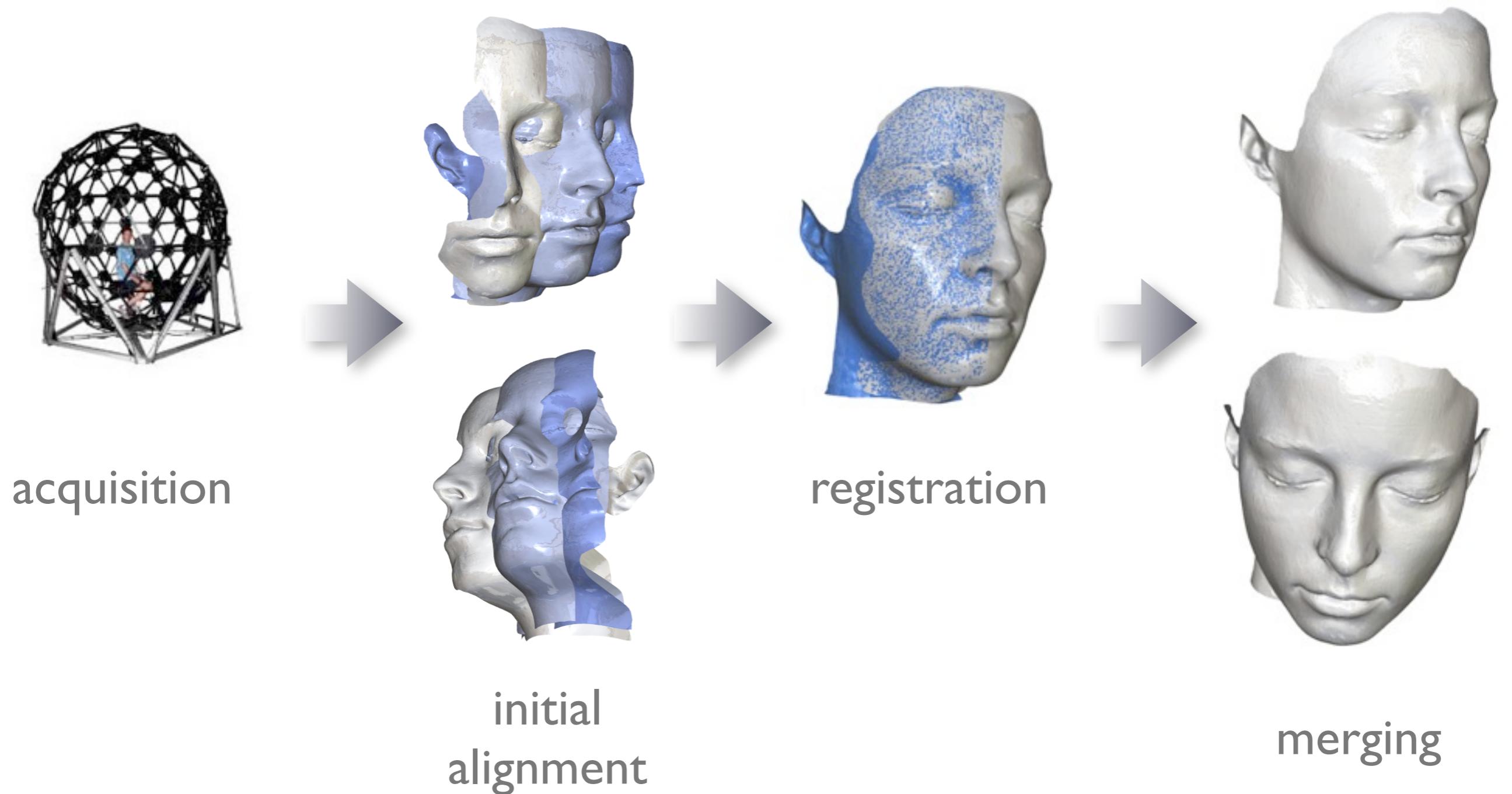
*data provided by Paramount Pictures and Aguru Images*

# Face Capture Pipeline



*data provided by Paramount Pictures and Aguru Images*

# Face Capture Pipeline

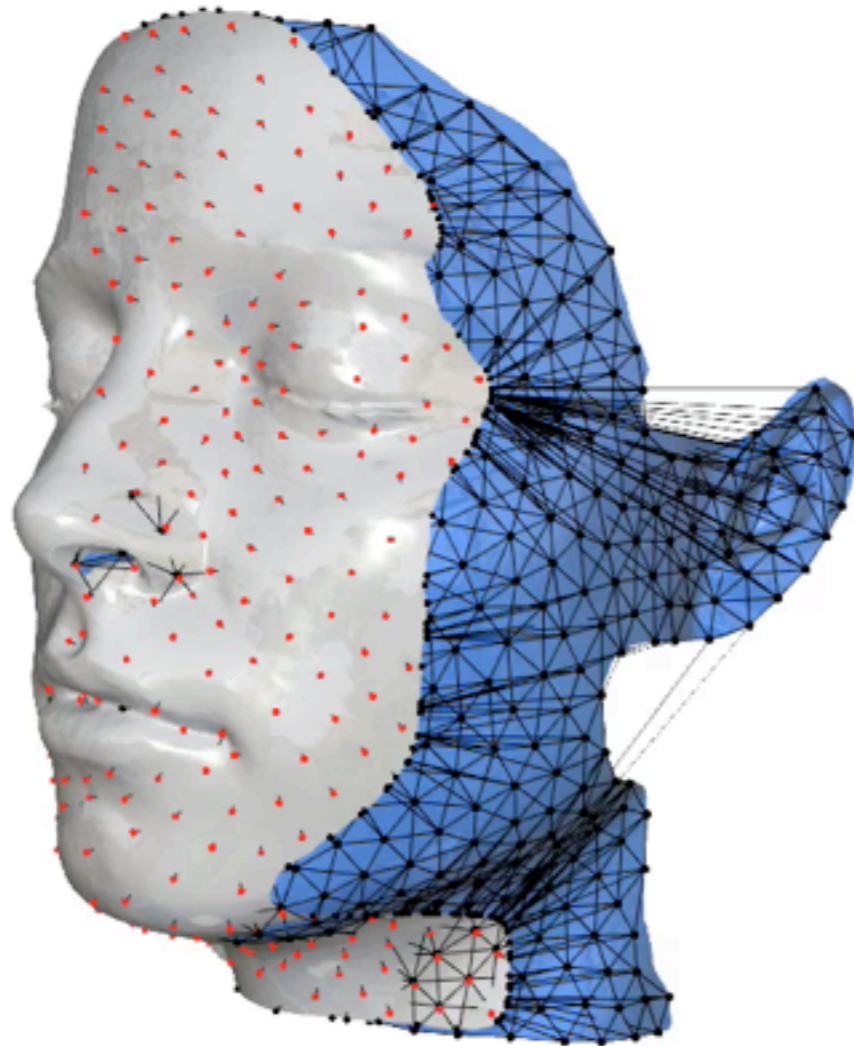


*data provided by Paramount Pictures and Aguru Images*

# Digital Face Cloning (scan-to-scan)

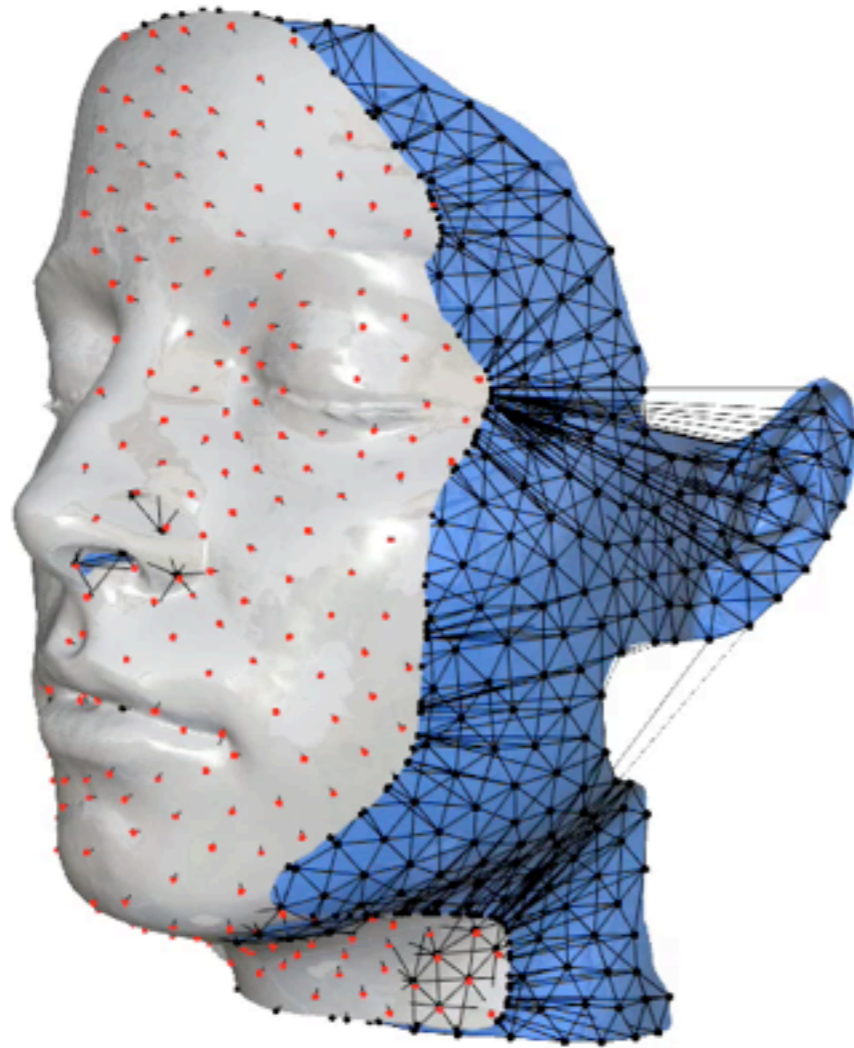
*data provided by Paramount Pictures and Aguru Images*

# Digital Face Cloning (scan-to-scan)



*data provided by Paramount Pictures and Aguru Images*

# Digital Face Cloning (scan-to-scan)

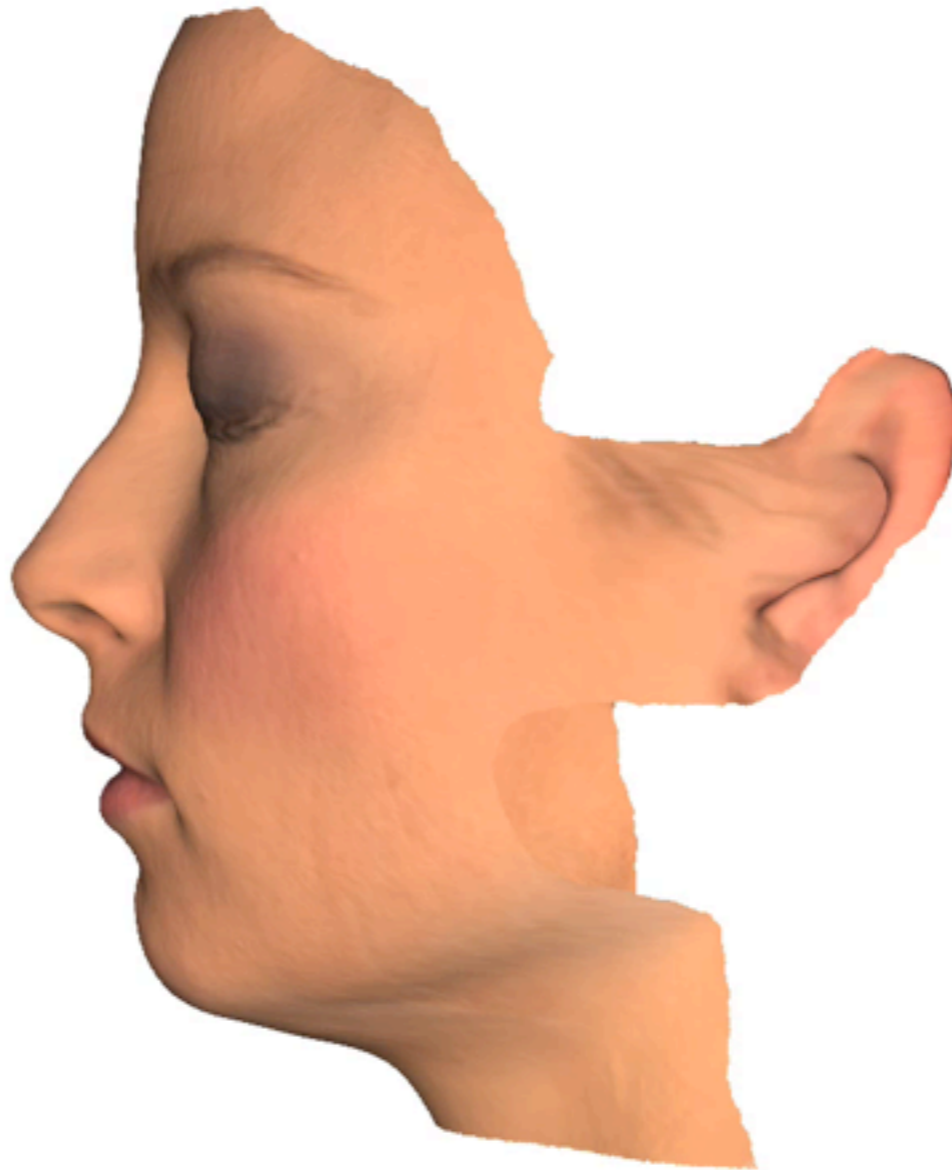


*data provided by Paramount Pictures and Aguru Images*

# Digital Face Cloning

*data provided by Paramount Pictures and Aguru Images*

# Digital Face Cloning

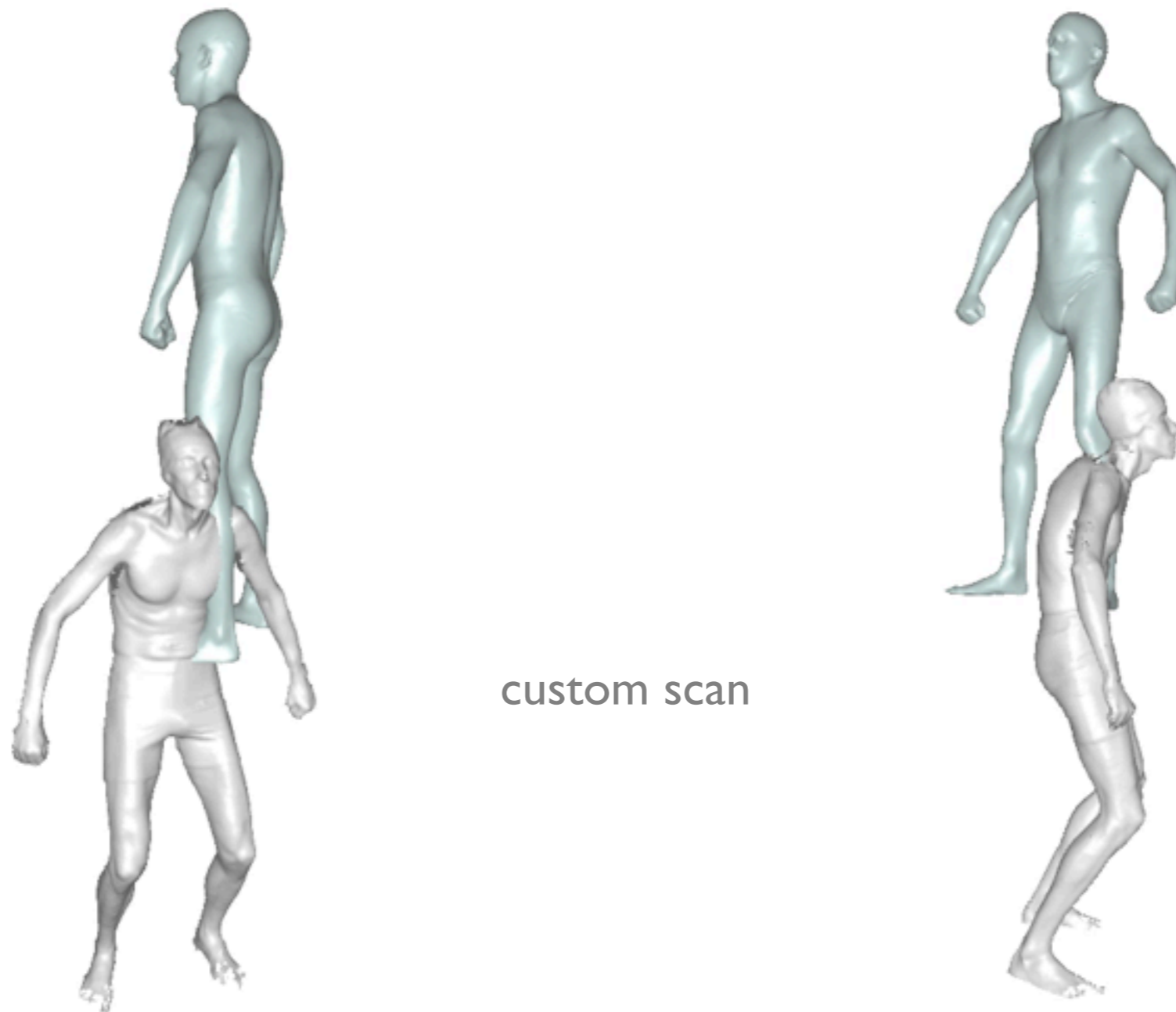


*data provided by Paramount Pictures and Aguru Images*

# It's pretty robust

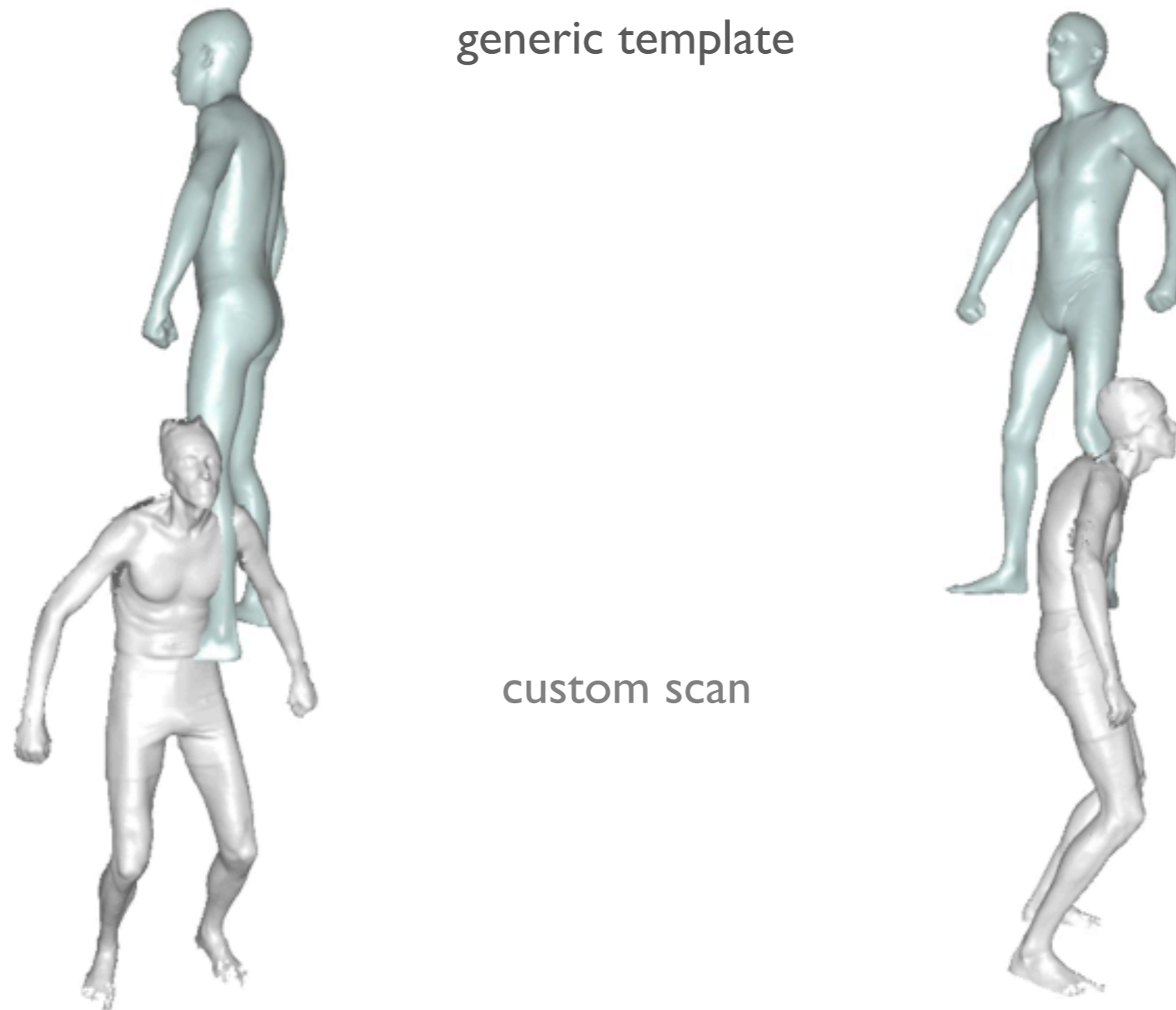
Caesar Data Set – Test with Michael Black

# It's pretty robust



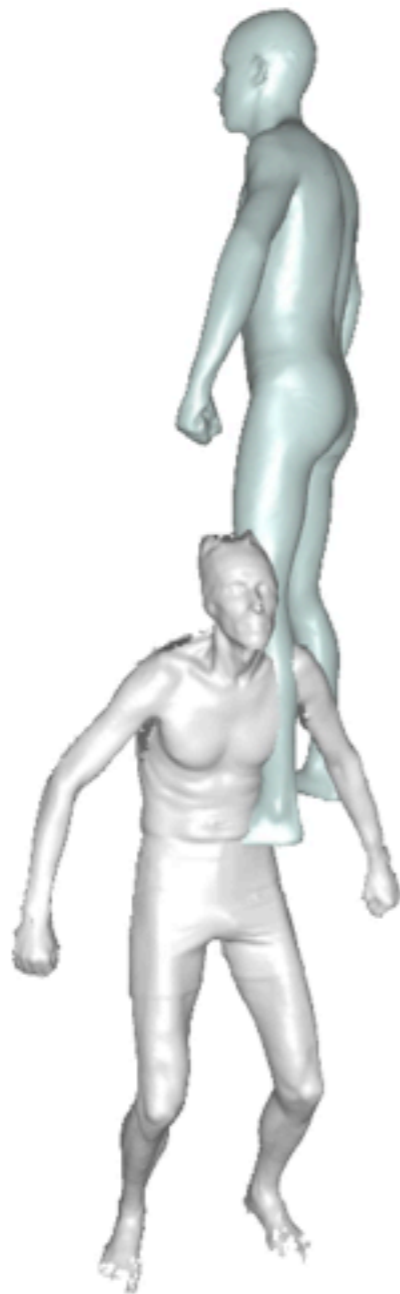
Caesar Data Set – Test with Michael Black

# It's pretty robust



Caesar Data Set – Test with Michael Black

# It's pretty robust

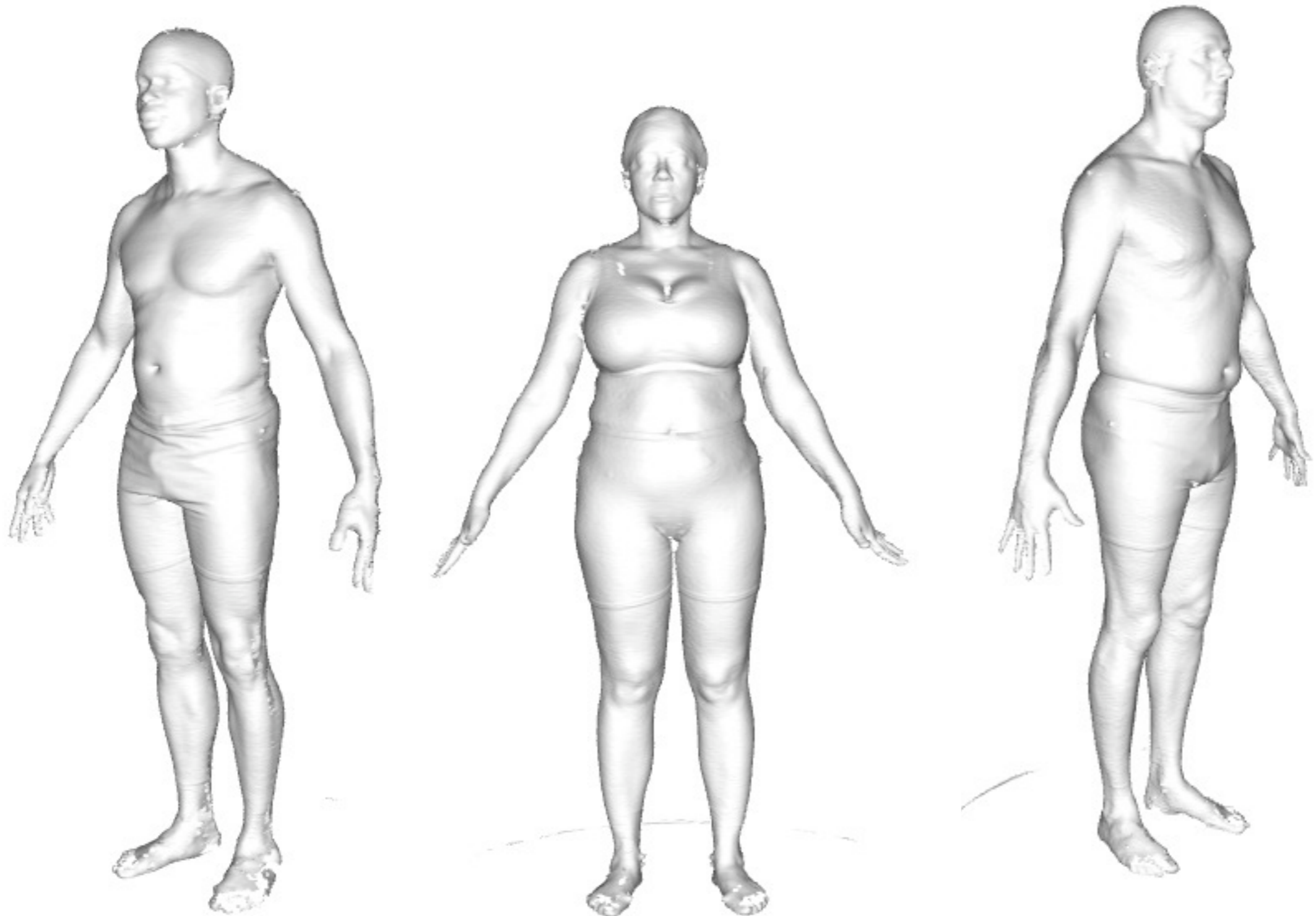


Caesar Data Set – Test with Michael Black

# Application: Statistical Database

Caesar Data Set – Test with Michael Black

# Application: Statistical Database

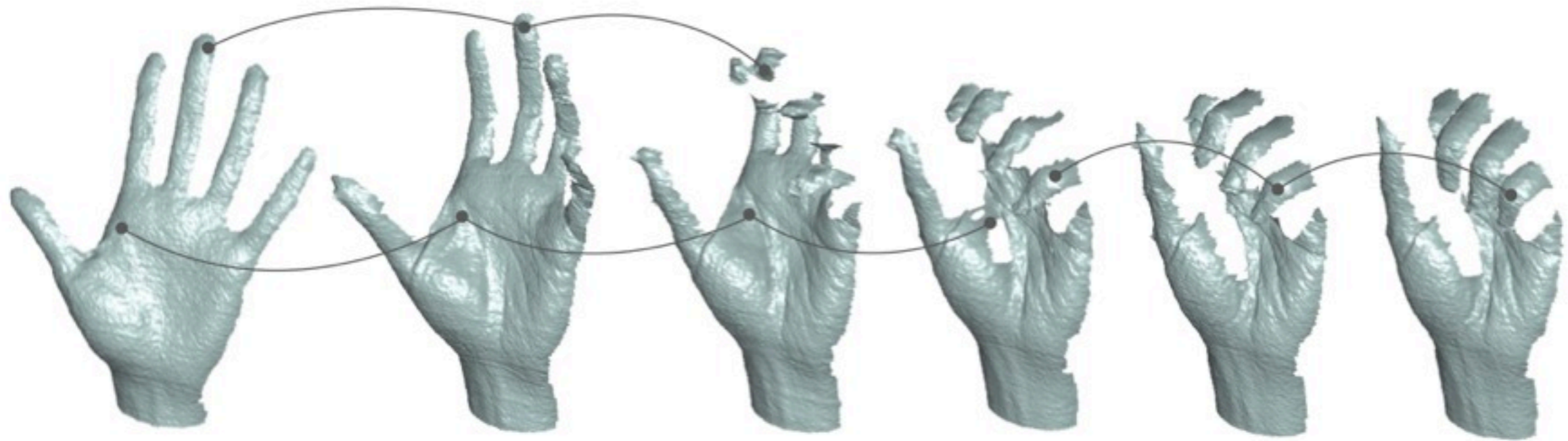


Caesar Data Set – Test with Michael Black

# What's Next?



# What's Next?



# What's Next?



[www.hao-li.com](http://www.hao-li.com)



[www.hao-li.com](http://www.hao-li.com)

