Symmetry Transforms

Motivation

Symmetry is everywhere

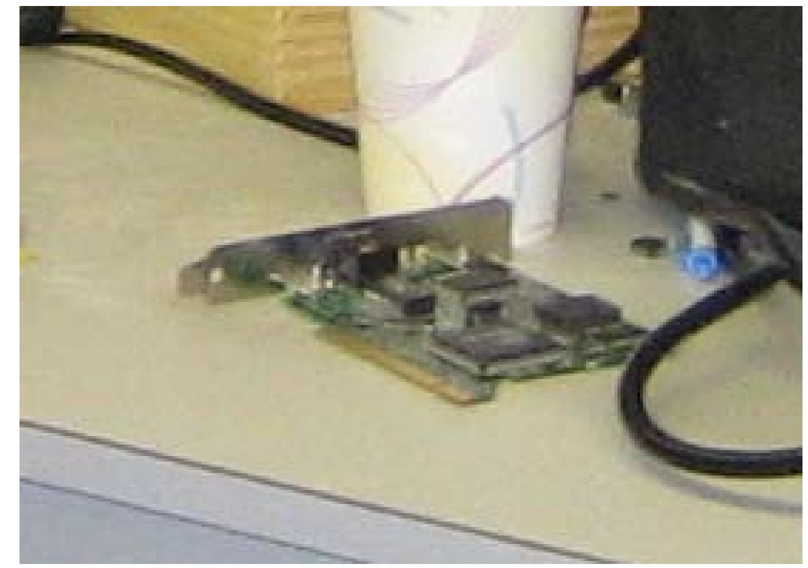
Symmetry is everywhere

Perfect Symmetry

[Blum ’64, ’67]
[Wolter ’85]
[Minovic ’97]
[Martinet ’05]

Motivation

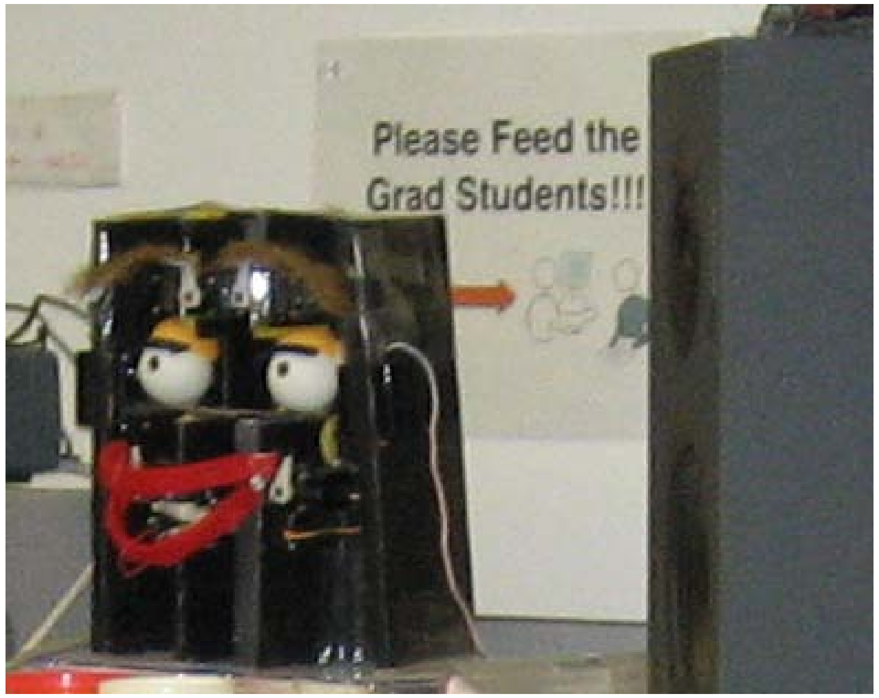
Symmetry is everywhere



Local Symmetry

[Blum ’78]
[Thrun ‘05]
[Simari ’06]

Symmetry is everywhere



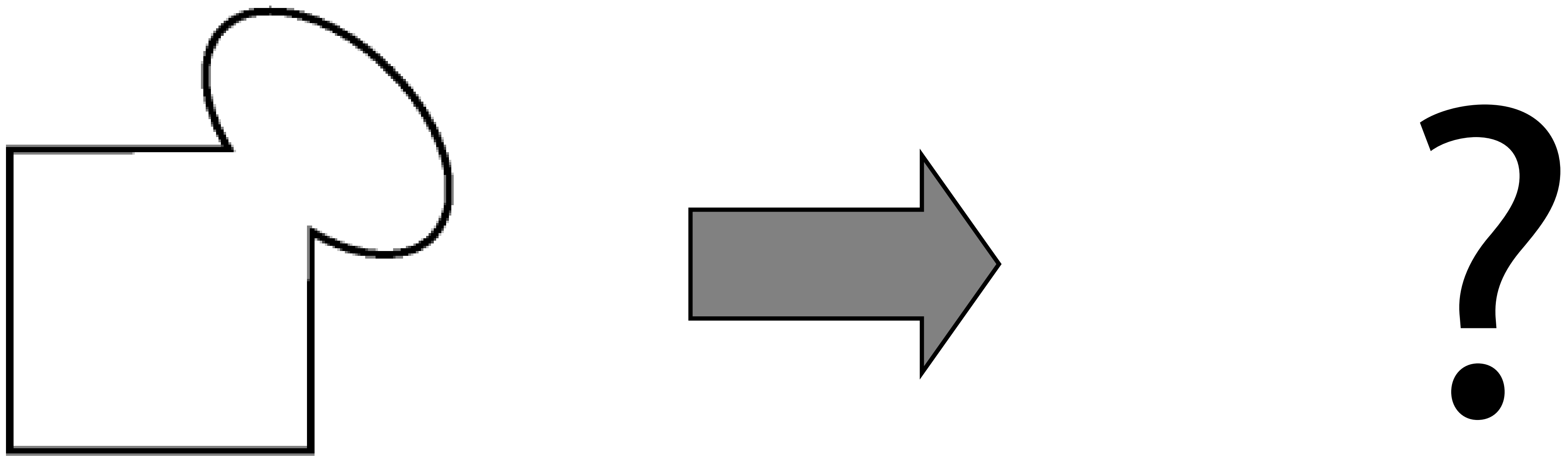
Partial Symmetry

[Zabrodsky ’95]

[Kazhdan ’03]

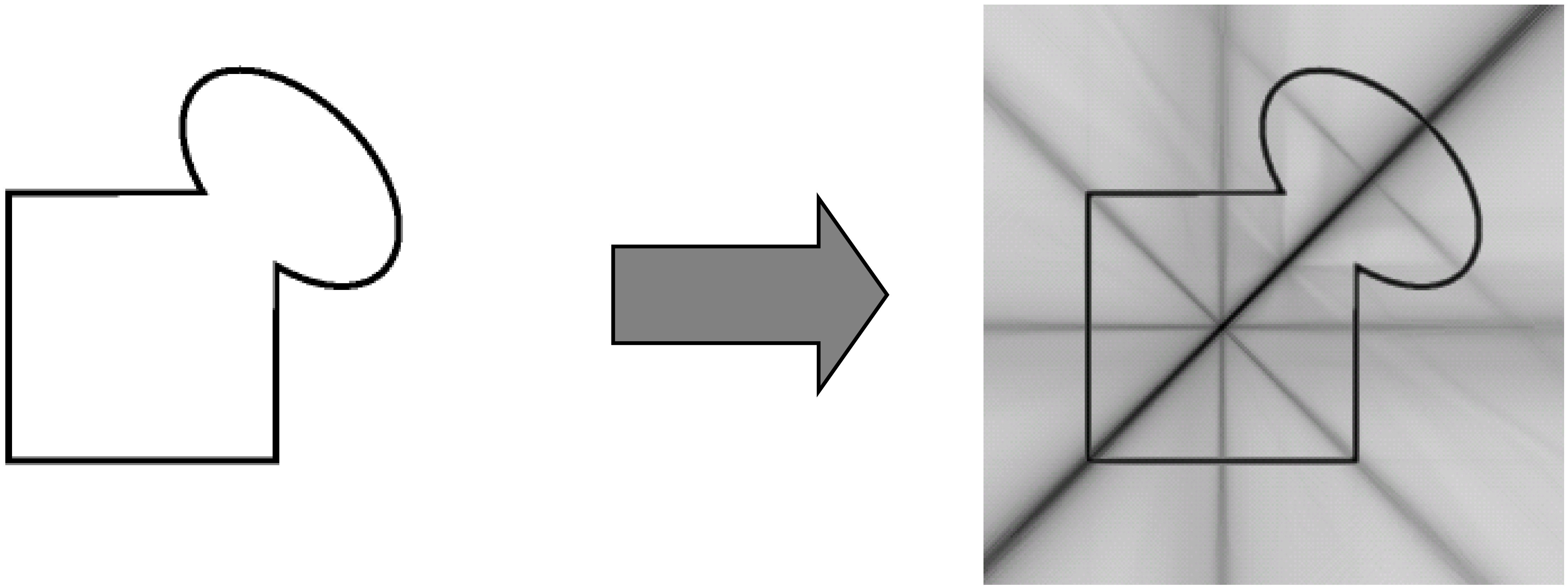
Goal

A computational representation that describes all planar symmetries of a shape



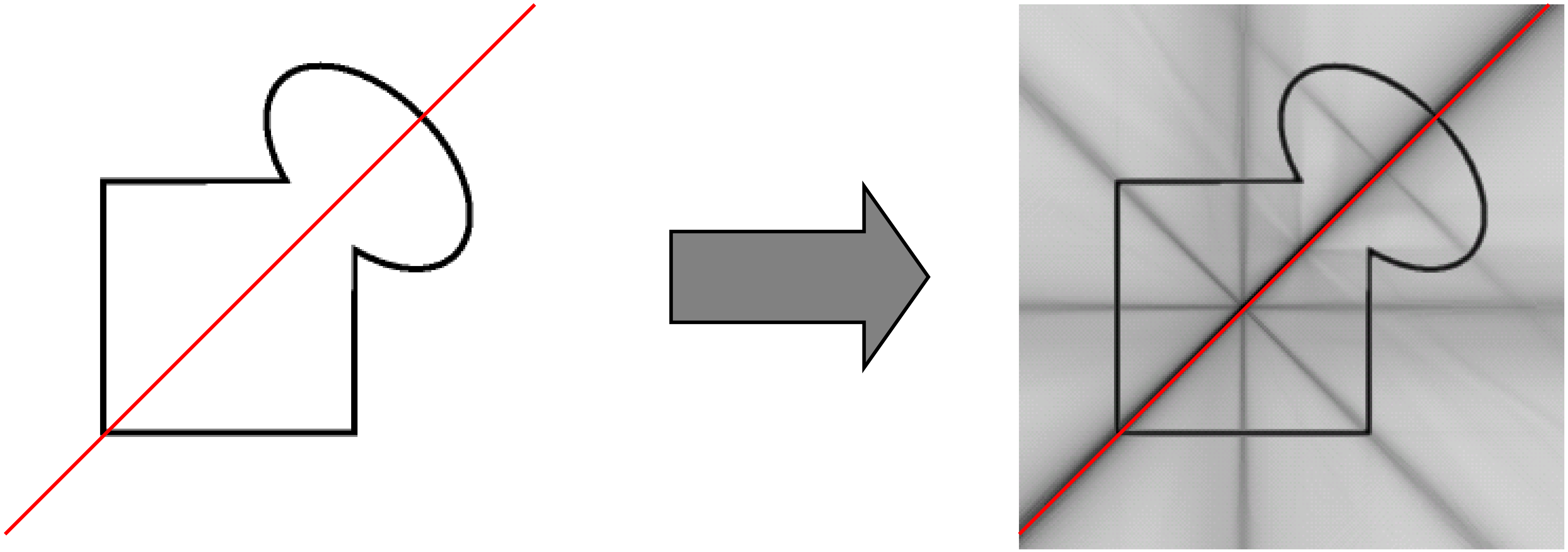
Symmetry Transform

A computational representation that describes all planar symmetries of a shape



Symmetry Transform

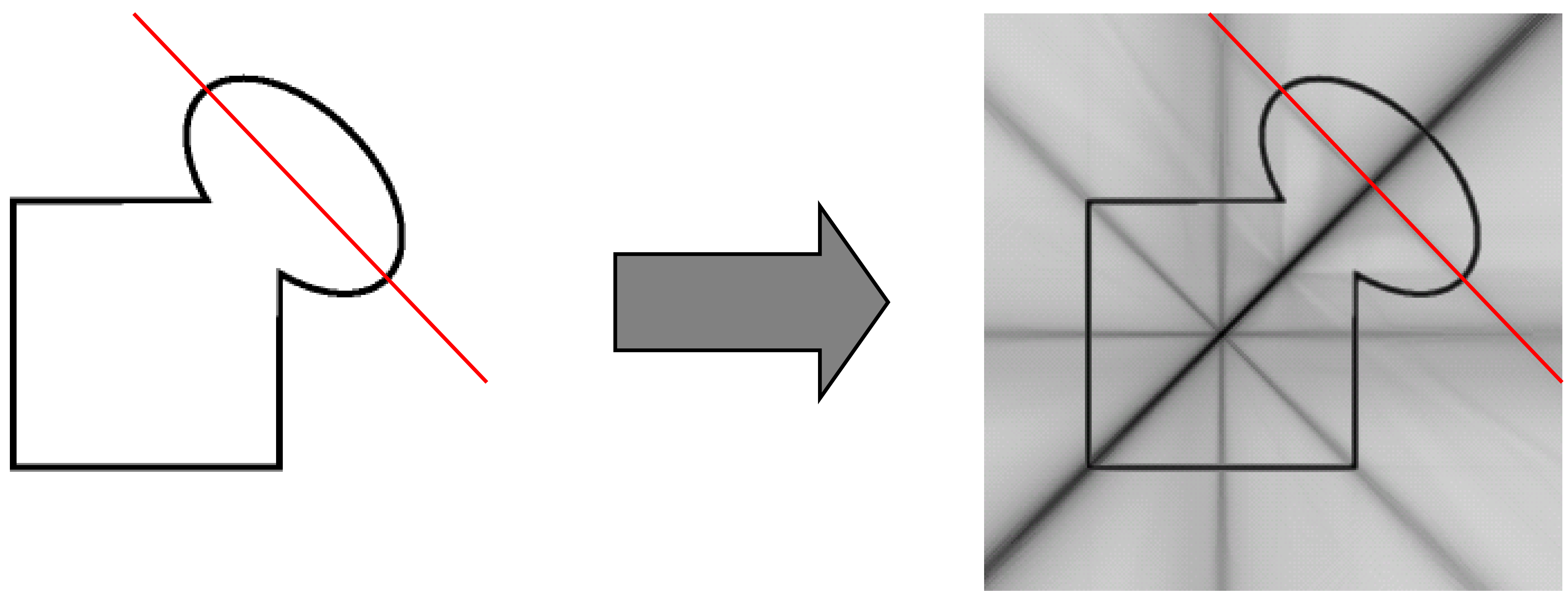
A computational representation that describes all planar symmetries of a shape



Perfect Symmetry

Symmetry = 1.0

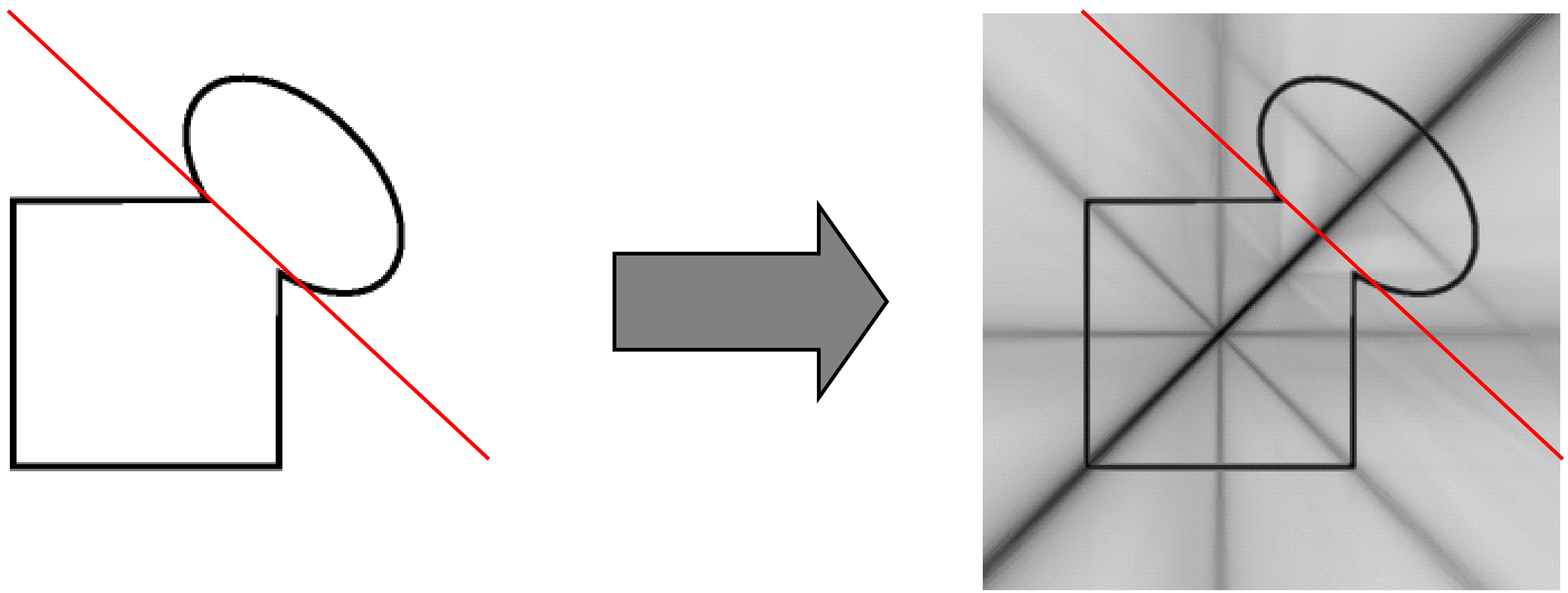
A computational representation that describes all planar symmetries of a shape



Local Symmetry

Symmetry = 0.3

A computational representation that describes all planar symmetries of a shape

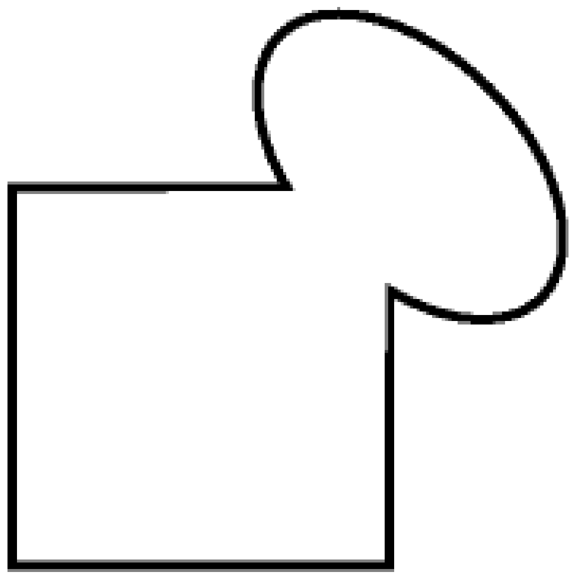


Partial Symmetry

Symmetry = 0.2

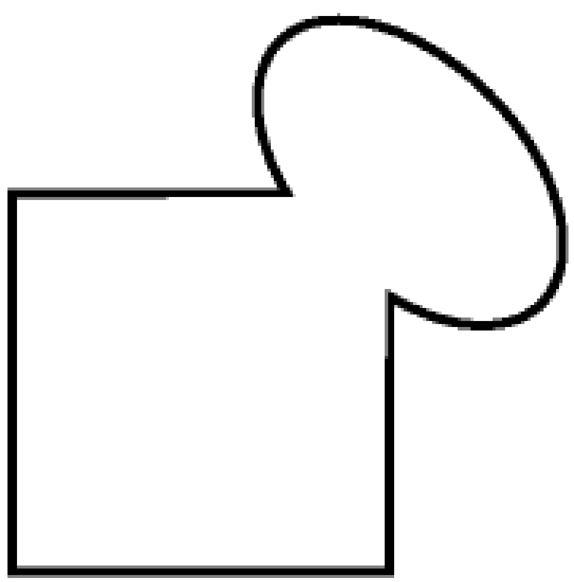
Symmetry Measure

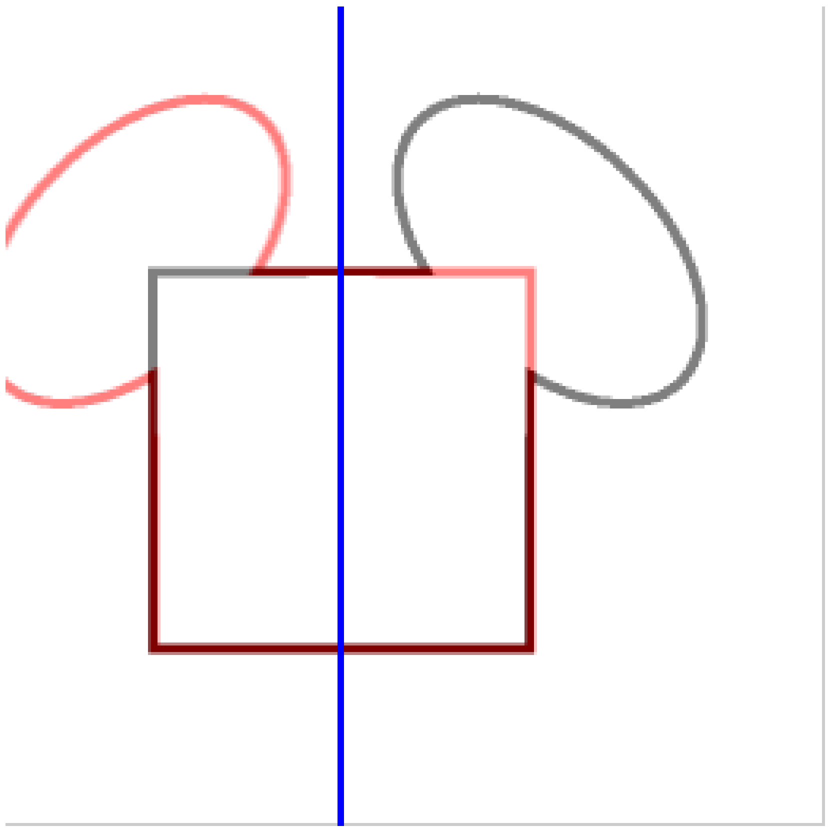
Symmetry of a shape is measured by correlation with its reflection



Symmetry Measure

Symmetry of a shape is measured by correlation with its reflection

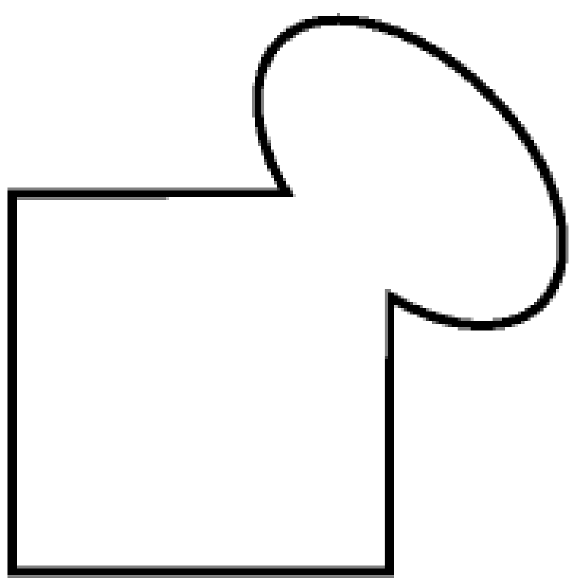


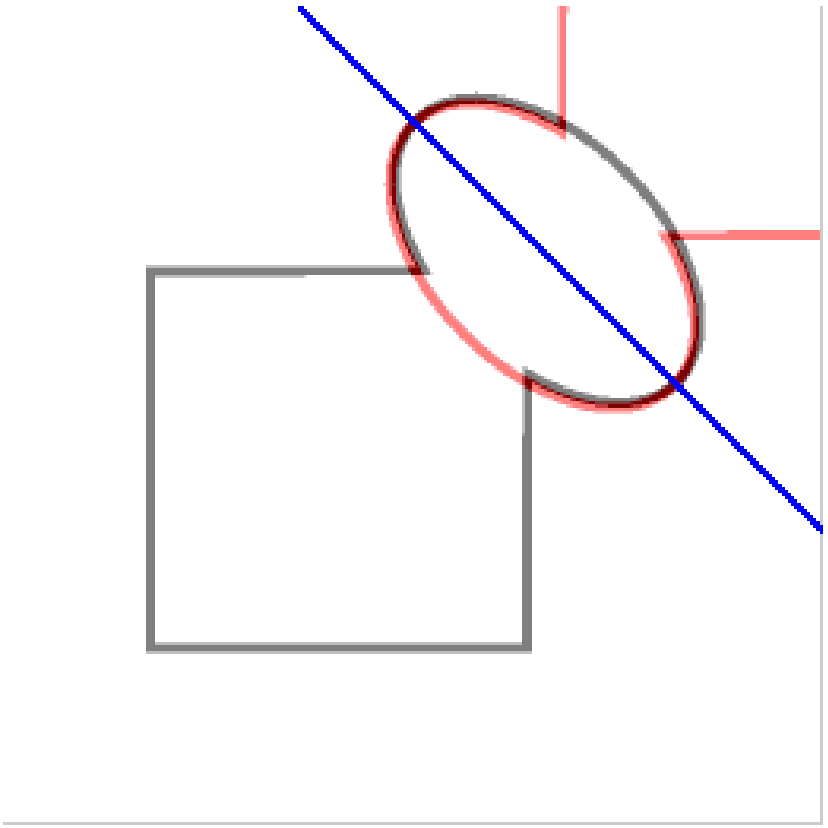


$$D(f,\gamma) = f \cdot \gamma(f)$$

Symmetry = 0.7

Symmetry of a shape is measured by correlation with its reflection

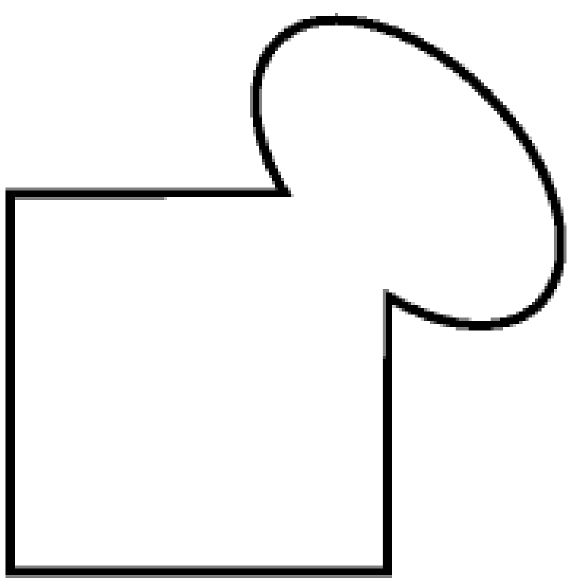


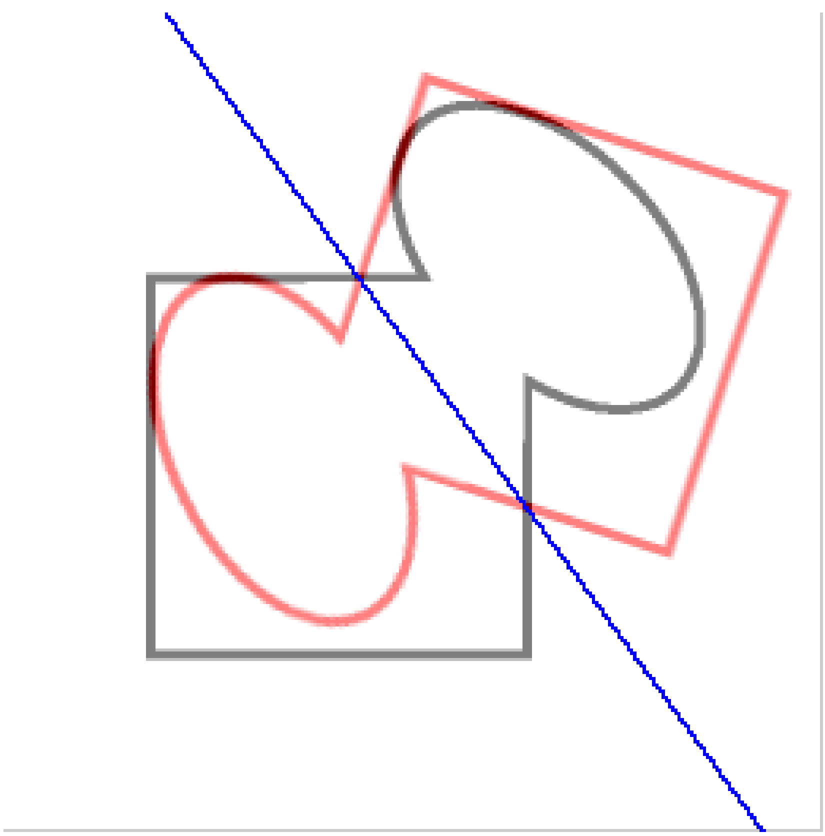


$$D(f,\gamma) = f \cdot \gamma(f)$$

Symmetry = 0.3

Symmetry of a shape is measured by correlation with its reflection

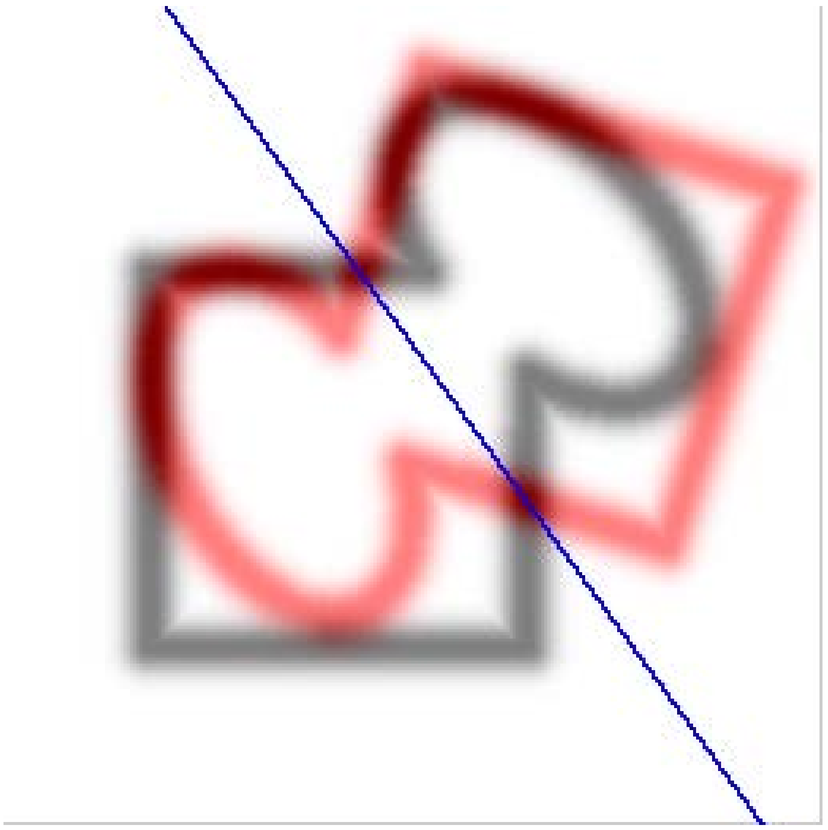




$$D(f,\gamma) = f \cdot \gamma(f)$$

Symmetry Measure

Symmetry of a shape is measured by correlation with its reflection



Symmetry = 0.1

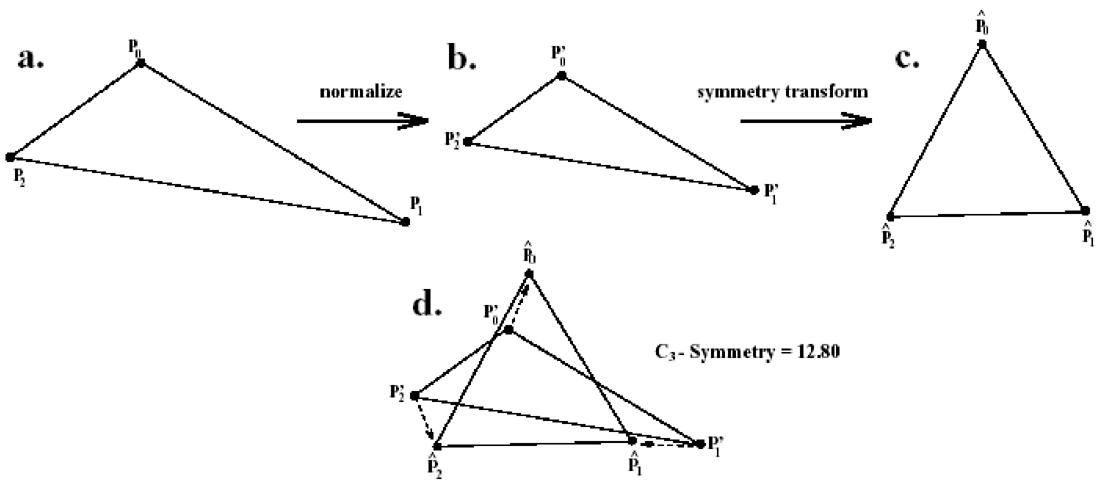
Previous Work

Zabrodsky '95

Kazhdan '03

Thrun '05

Martinet '05



Symmetry Distance

Define the *Symmetry Distance* of a function *f* with respect to any transformation γ as the L^2 distance between *f* and the nearest function invariant to γ

Can show that Symmetry Measure $D(f,\gamma) = f \cdot \gamma(f)$ is related to symmetry distance by

$$D(f,\gamma) = -2SD^2 + \|f\|^2$$

Previous Work

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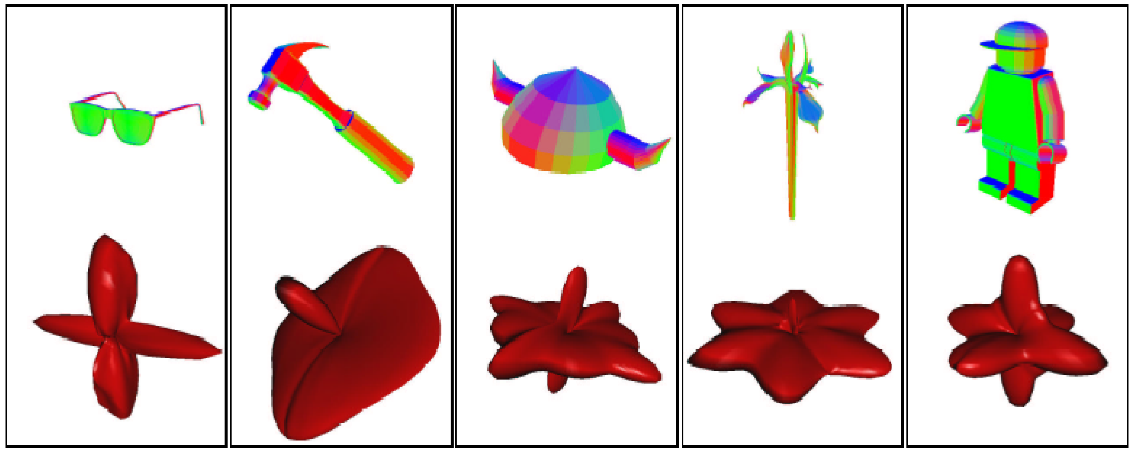

Previous Work

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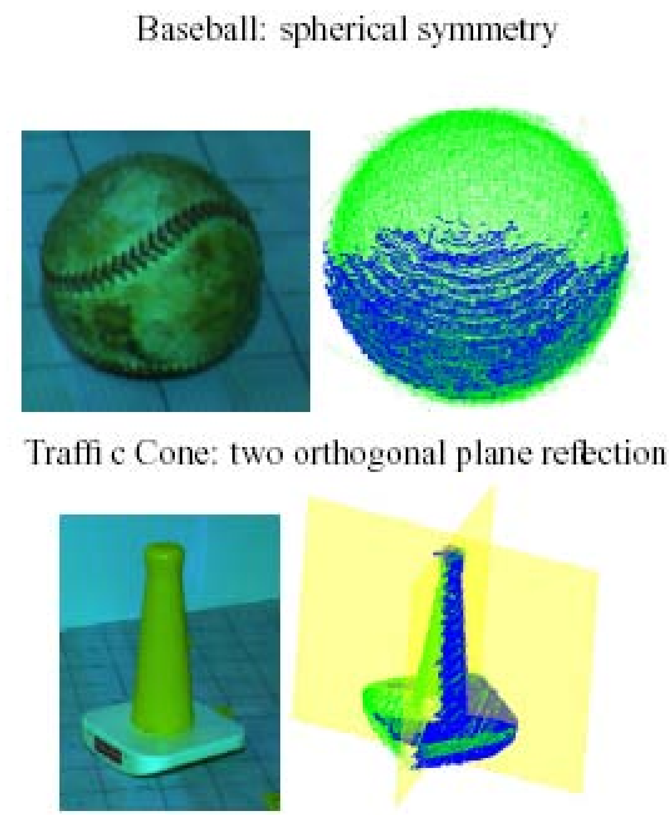

Previous Work

Zabrodsky ‘95

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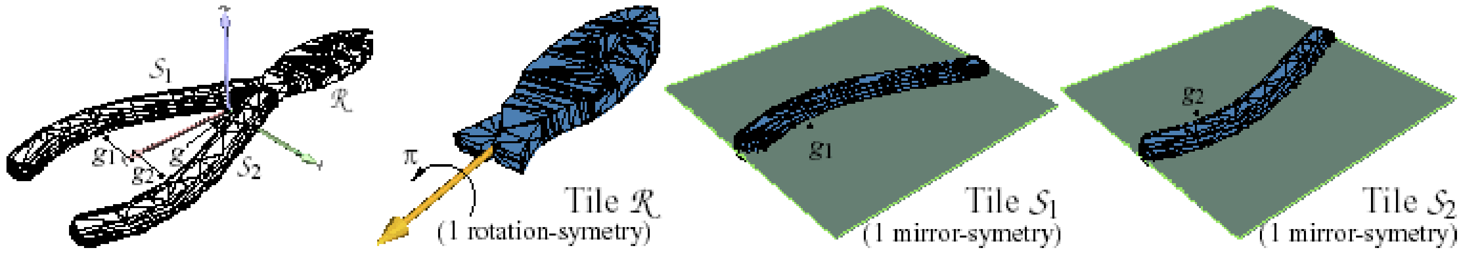


Brute Force $O(n^6)$

Convolution

Monte-Carlo

$O(n^3)$ planes

X

$O(n^3)$ dot product

Computing Discrete Transform

Brute Force $O(n^6)$
Convolution $O(n^5 \text{Log } n)$
Monte-Carlo

$O(n^2)$ normal directions

X

$O(n^3 \log n)$ per direction

Computing Discrete Transform

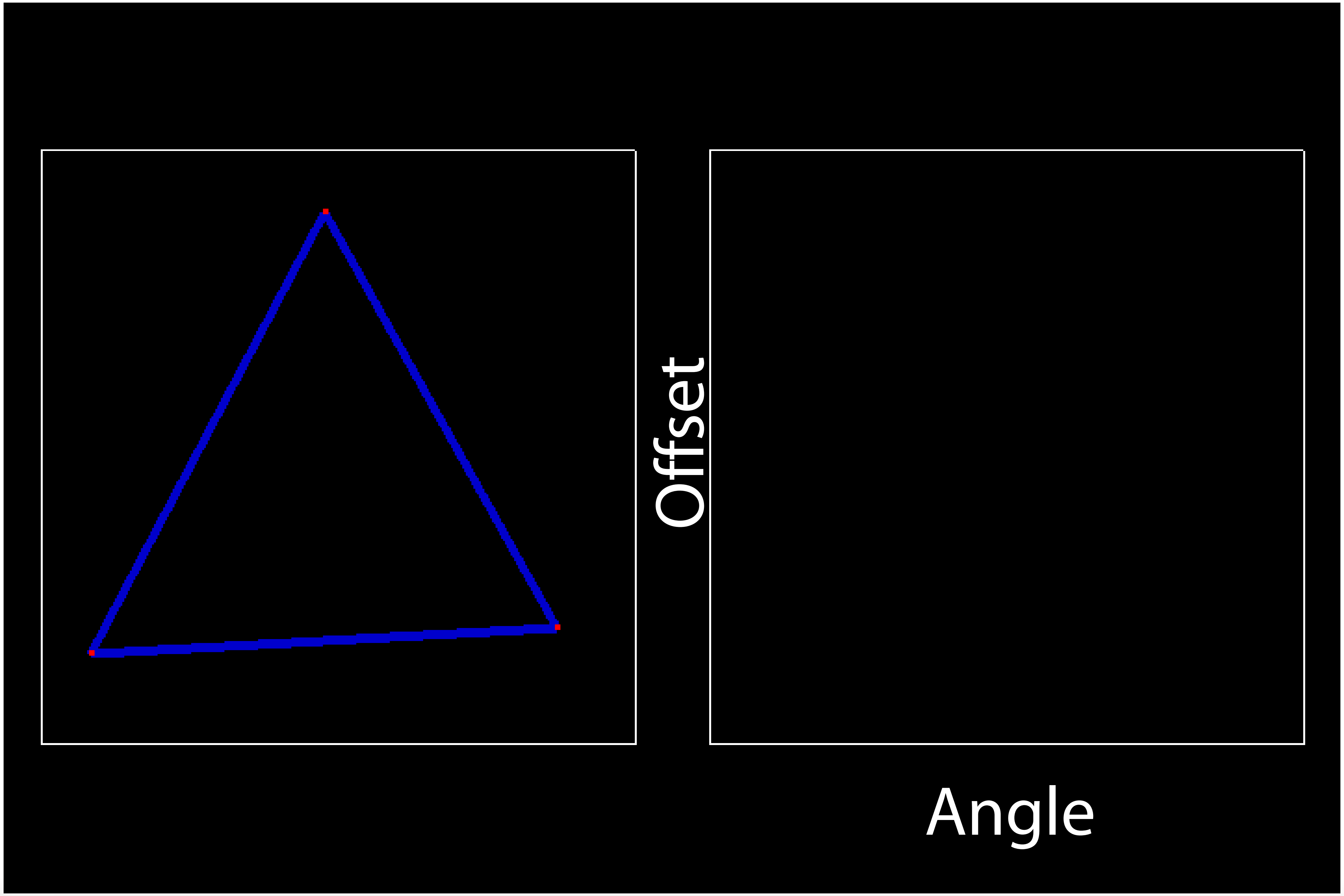
Brute Force $O(n^6)$

Convolution $O(n^5 \text{Log } n)$

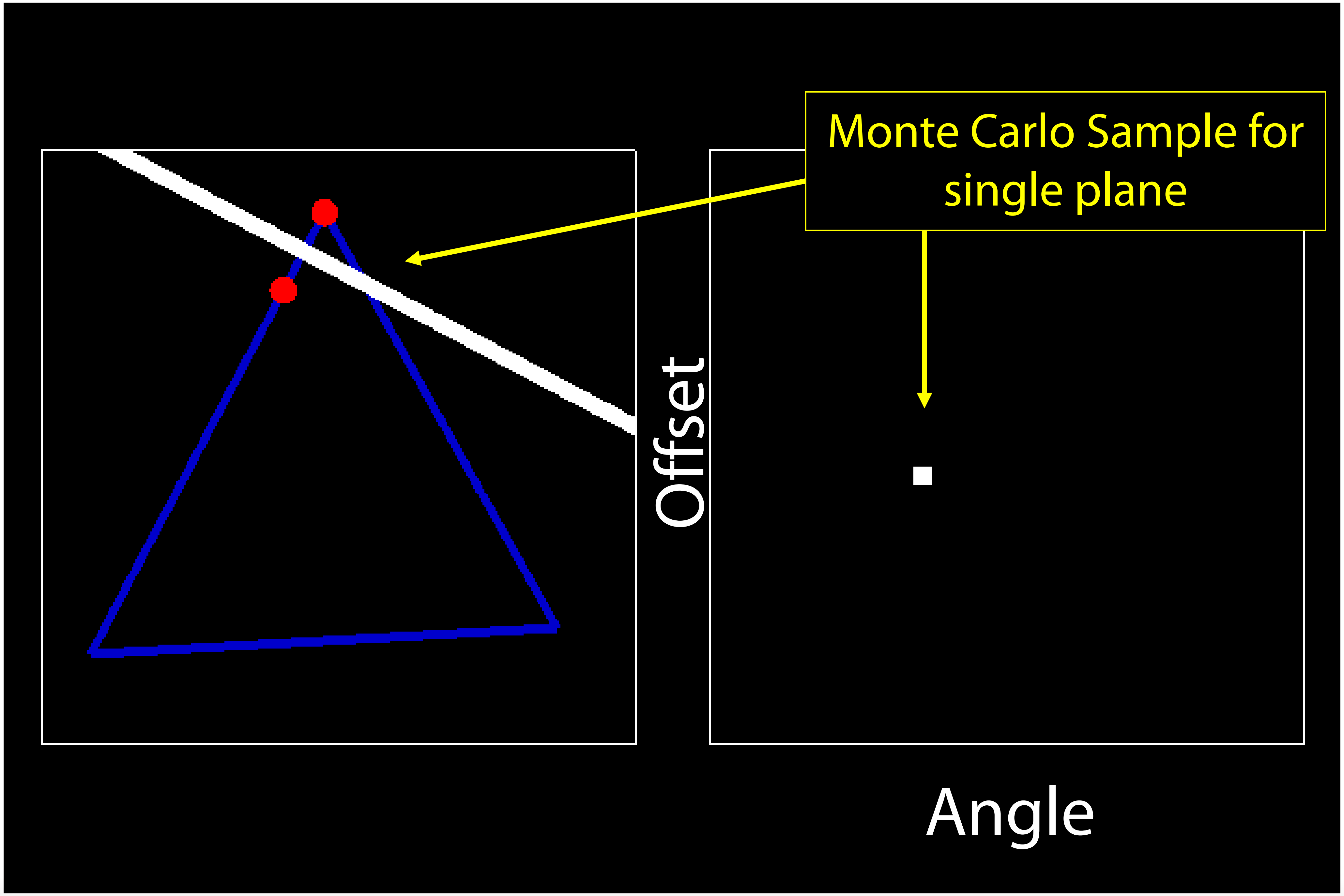
Monte-Carlo $O(n^4)$ For 3D meshes

- Most of the dot product contains zeros.
- Use Monte-Carlo Importance Sampling.

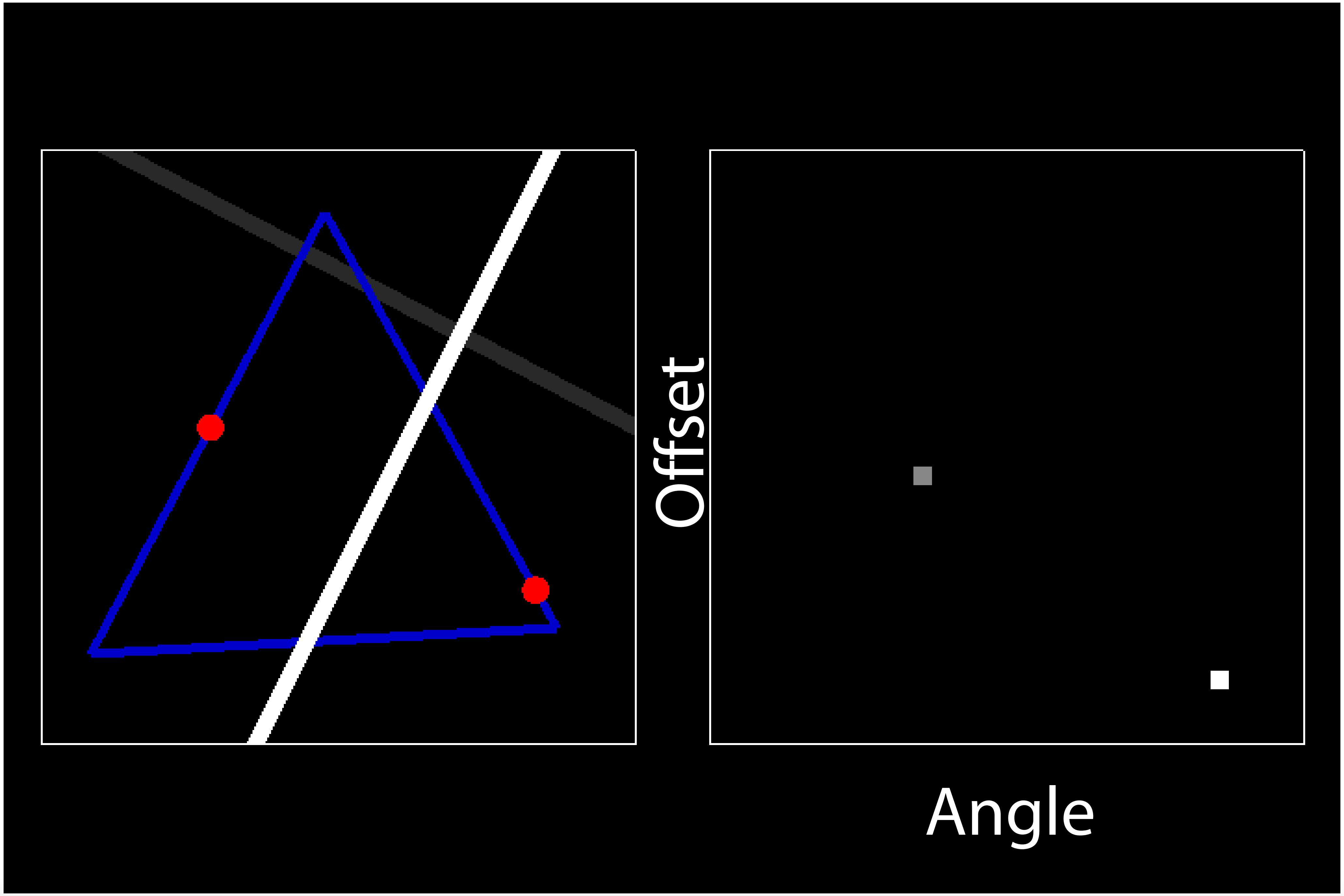
Monte Carlo



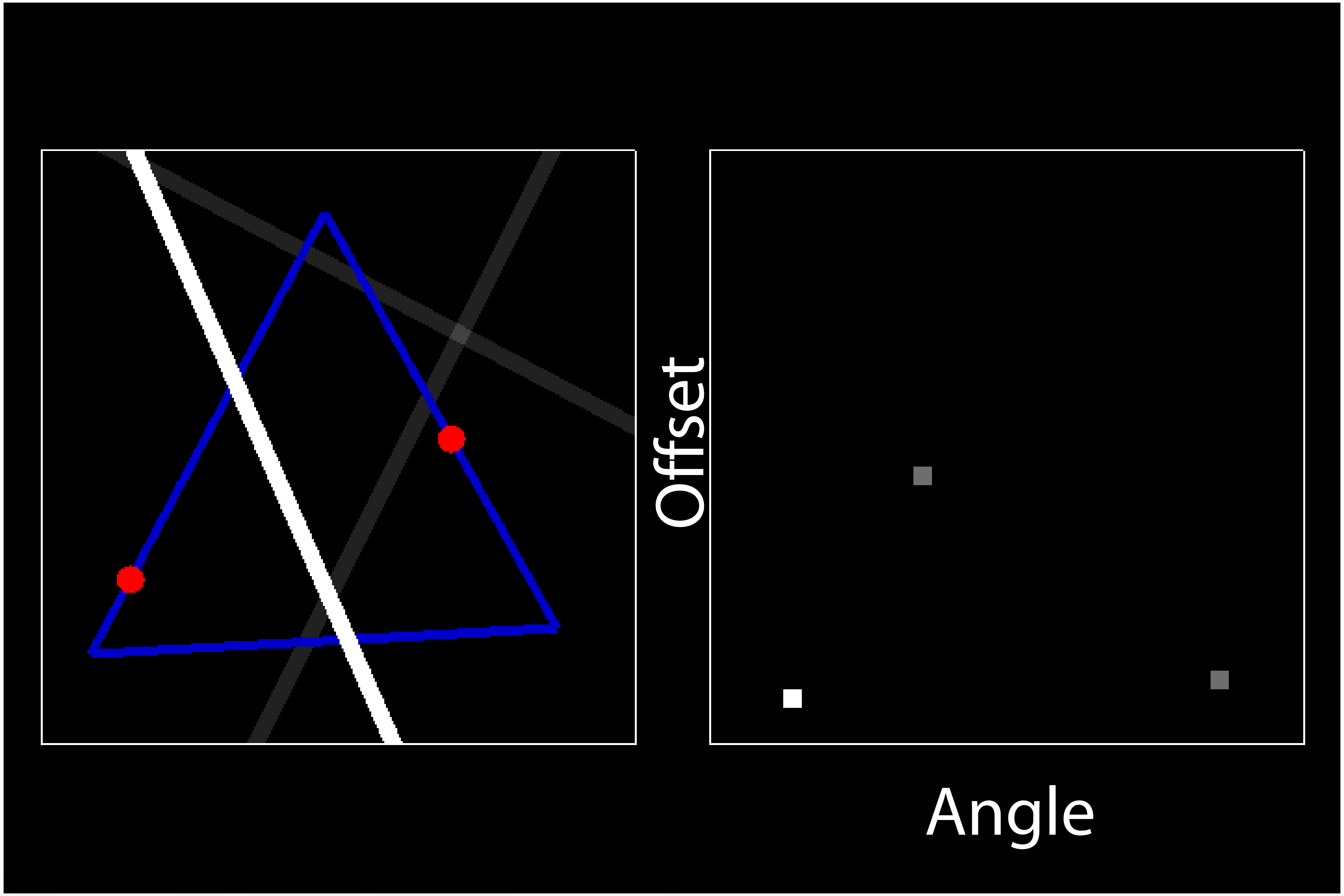
Monte Carlo



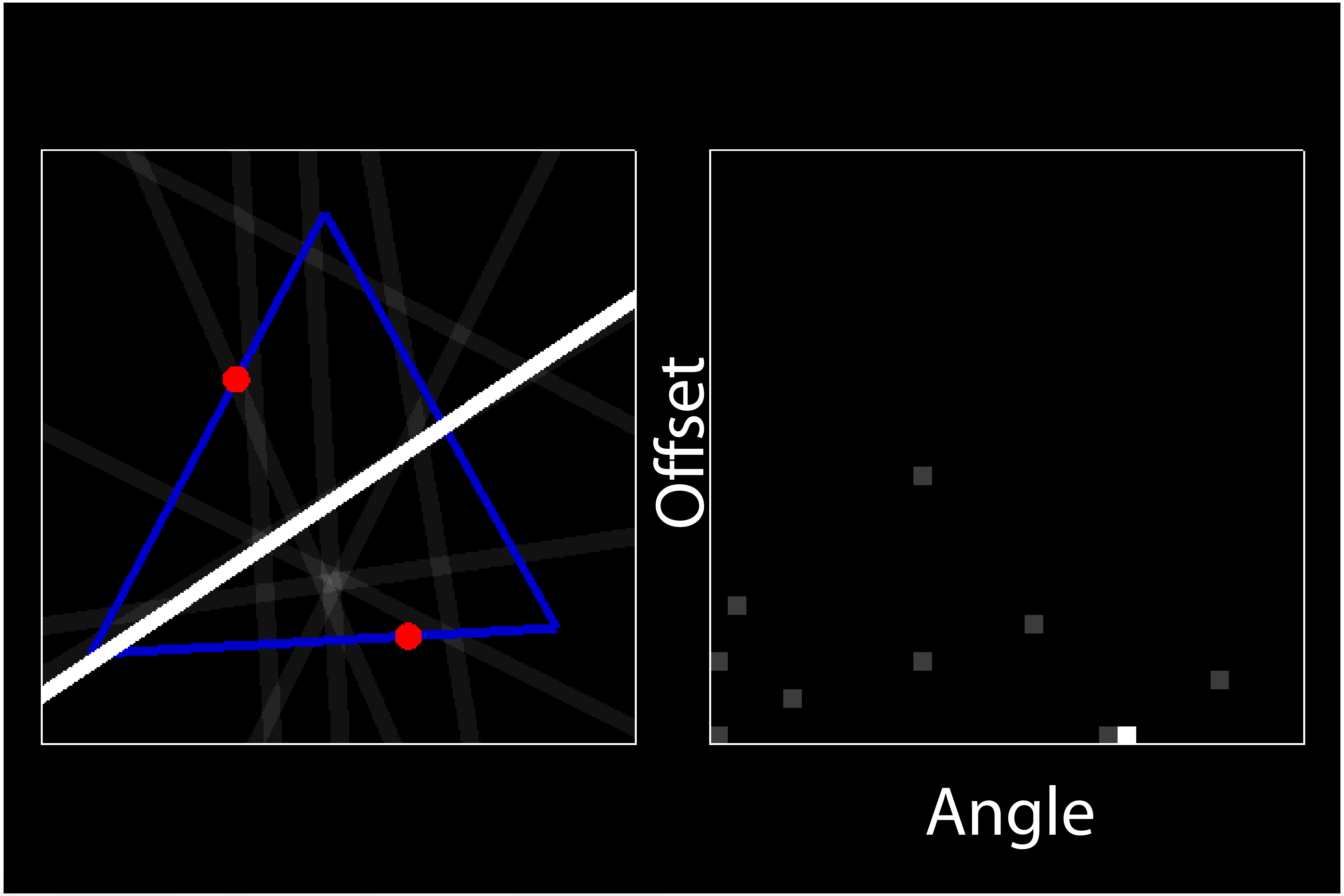
Monte Carlo



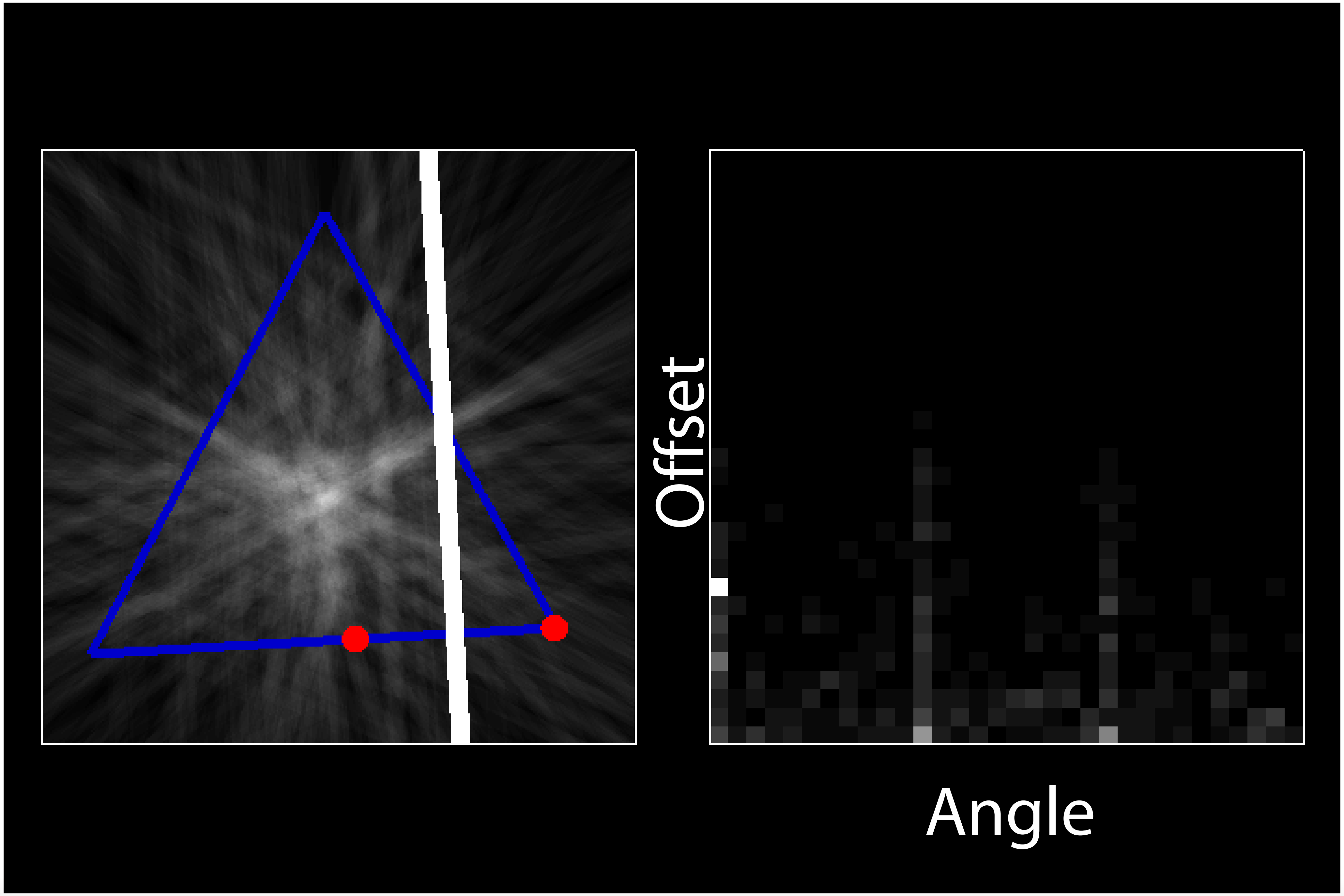
Monte Carlo



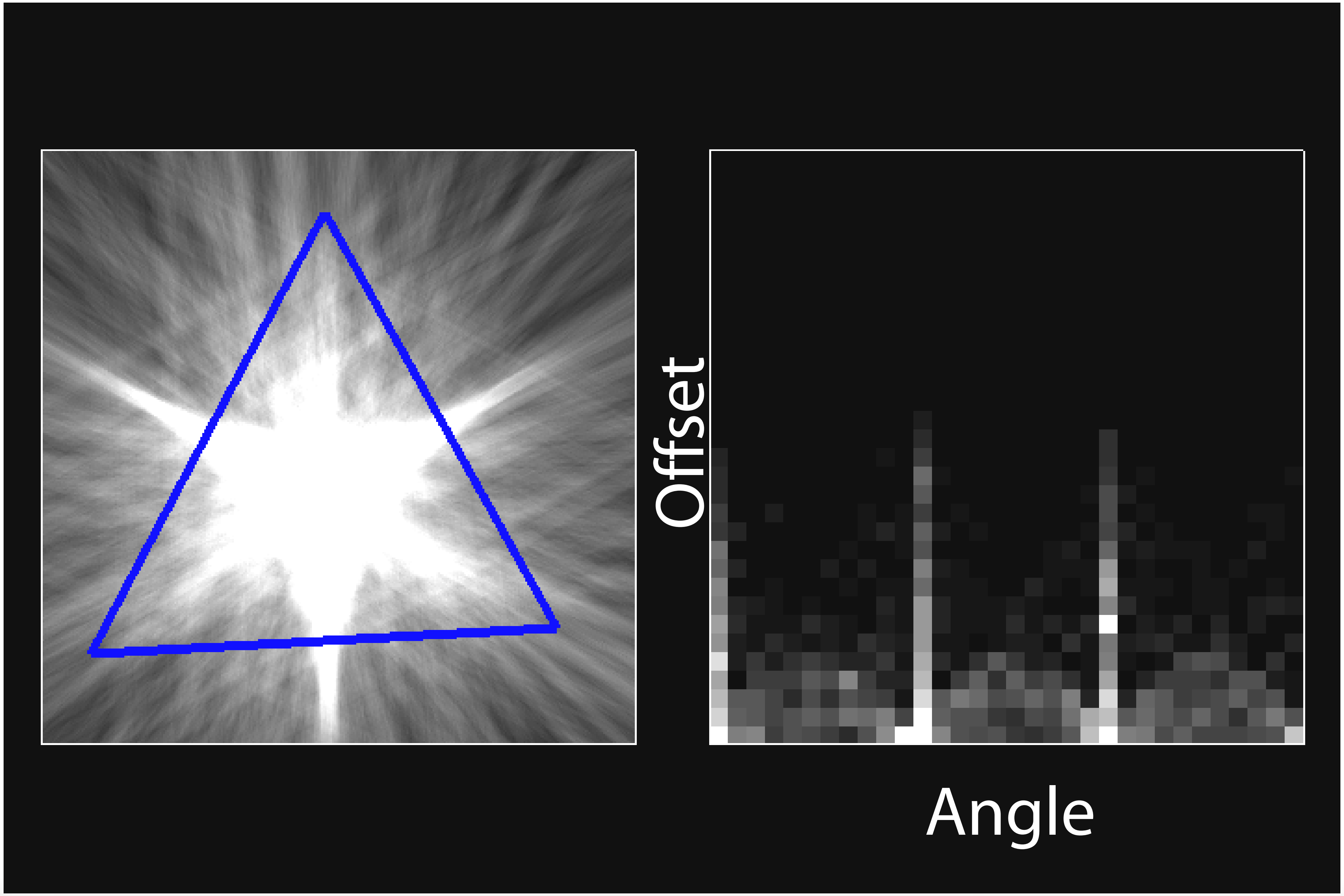
Monte Carlo



Monte Carlo

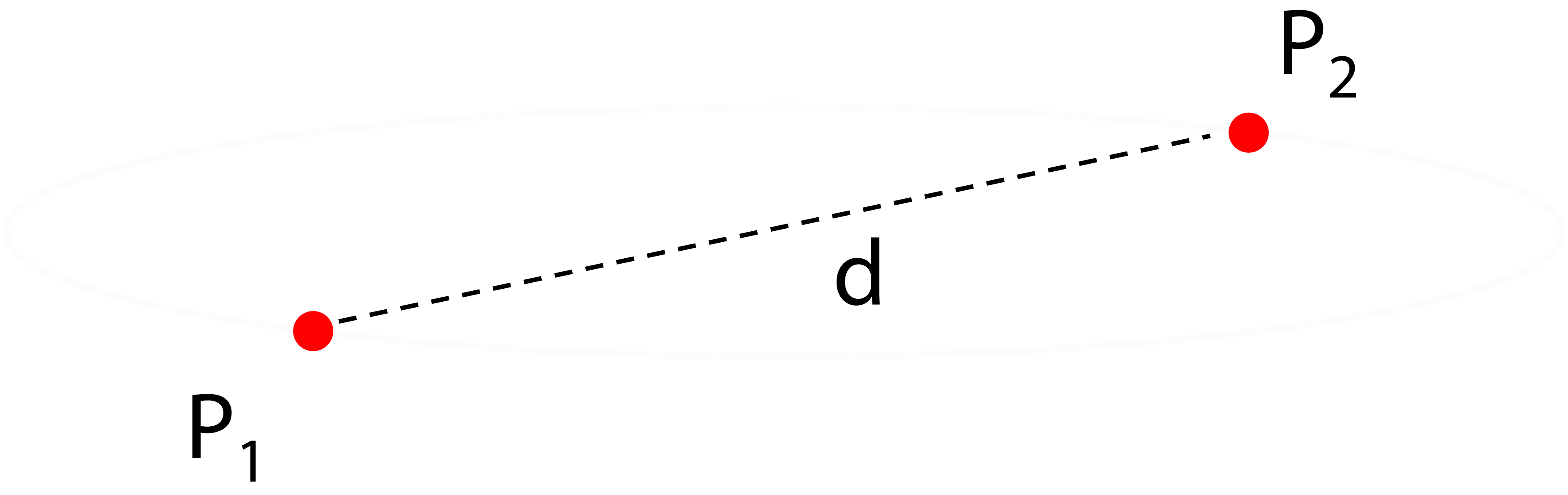


Monte Carlo



Weighting Samples

Need to weight sample pairs by the inverse of the distance between them



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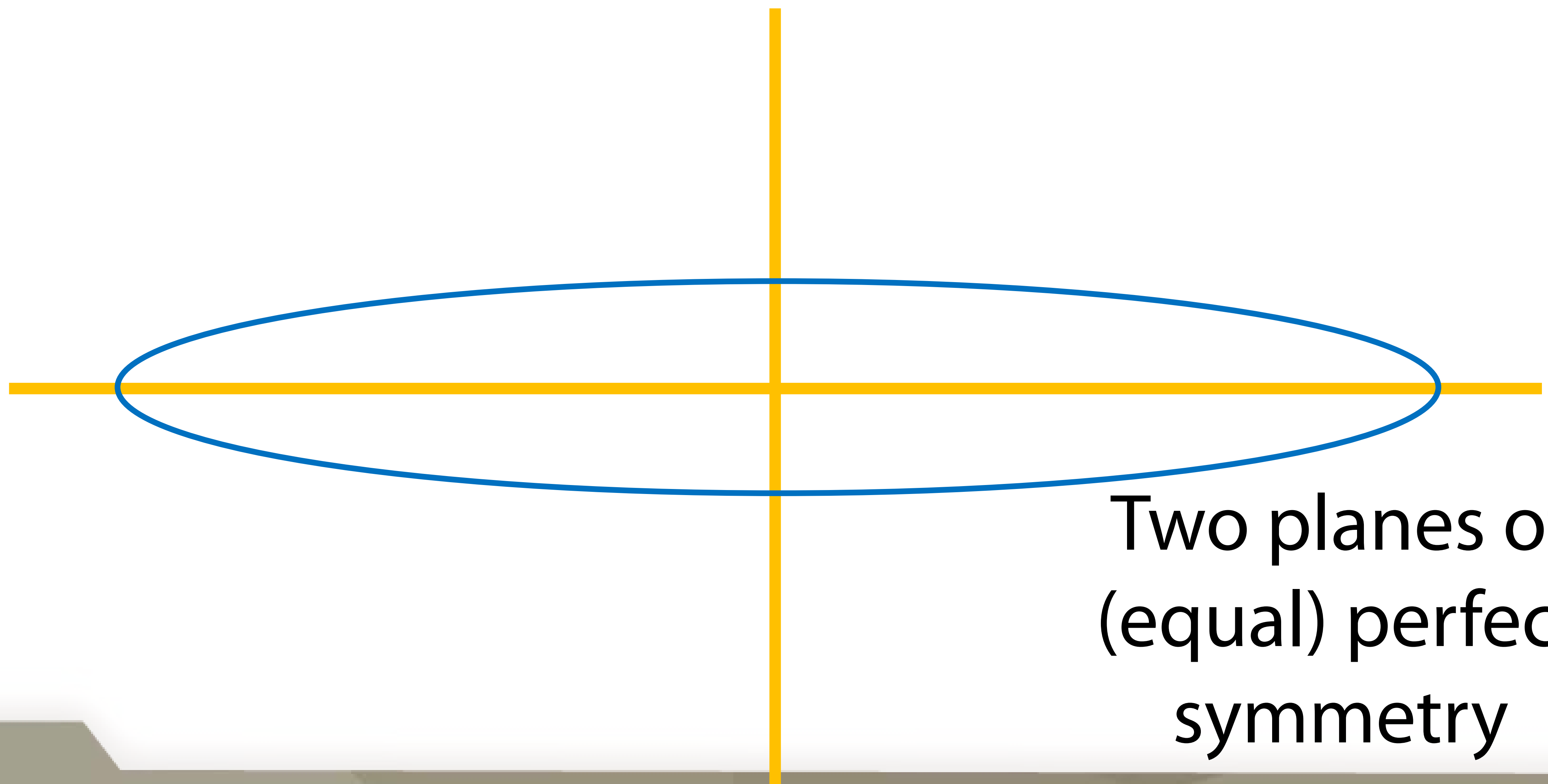
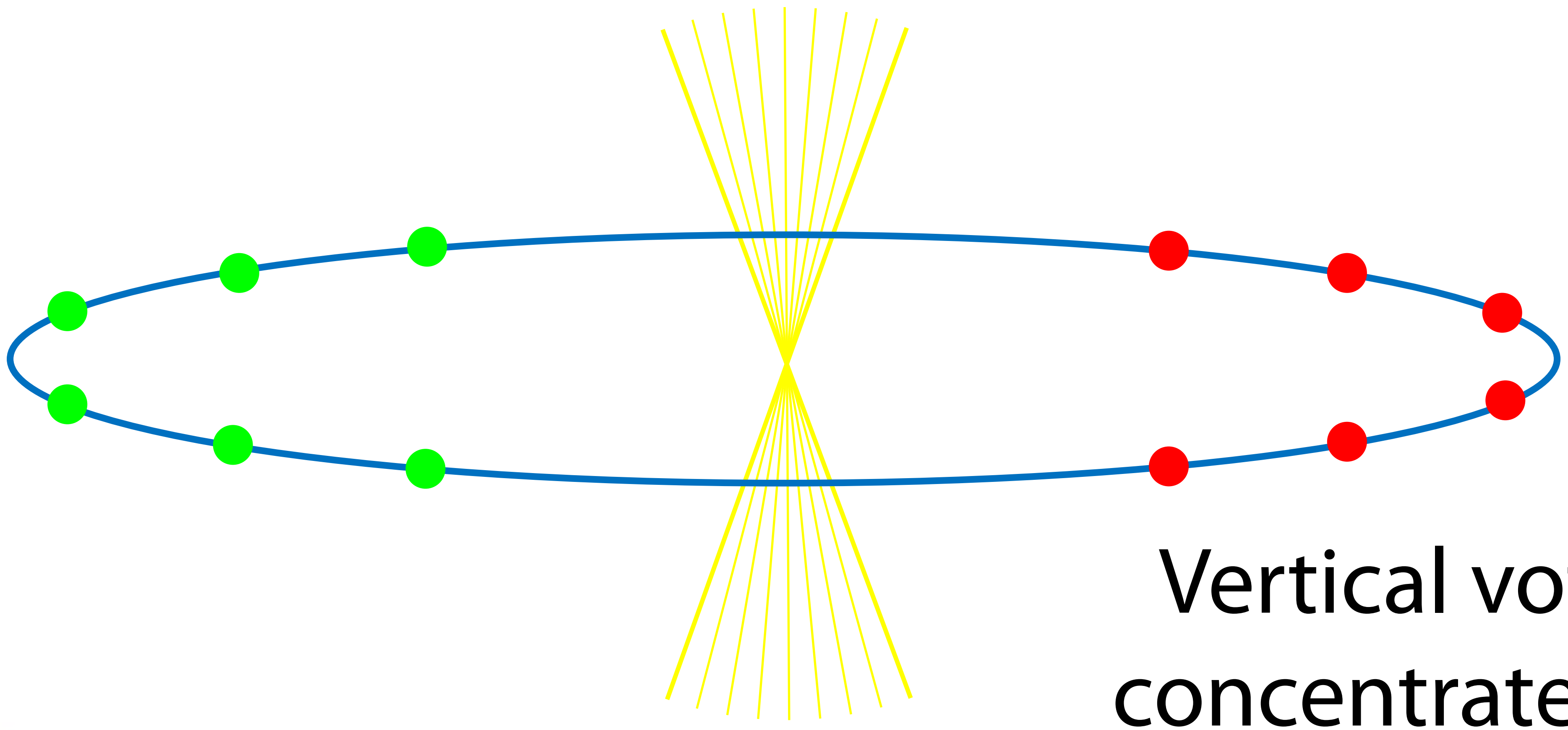


Two planes of (equal) perfect symmetry

Weighting Samples

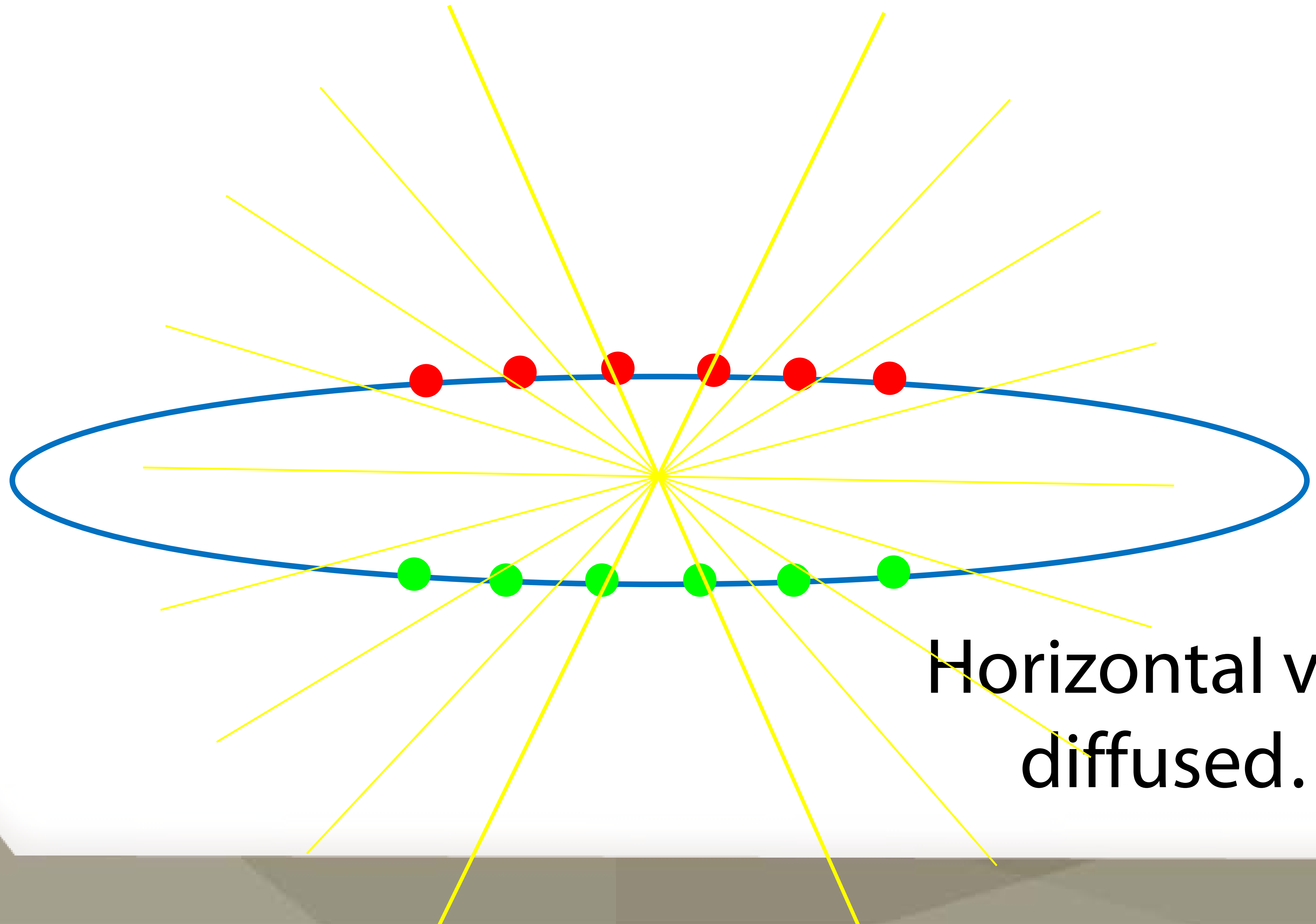
Need to weight sample pairs by the inverse of the distance between them



Vertical votes concentrated…

Weighting Samples

Need to weight sample pairs by the inverse of the distance between them

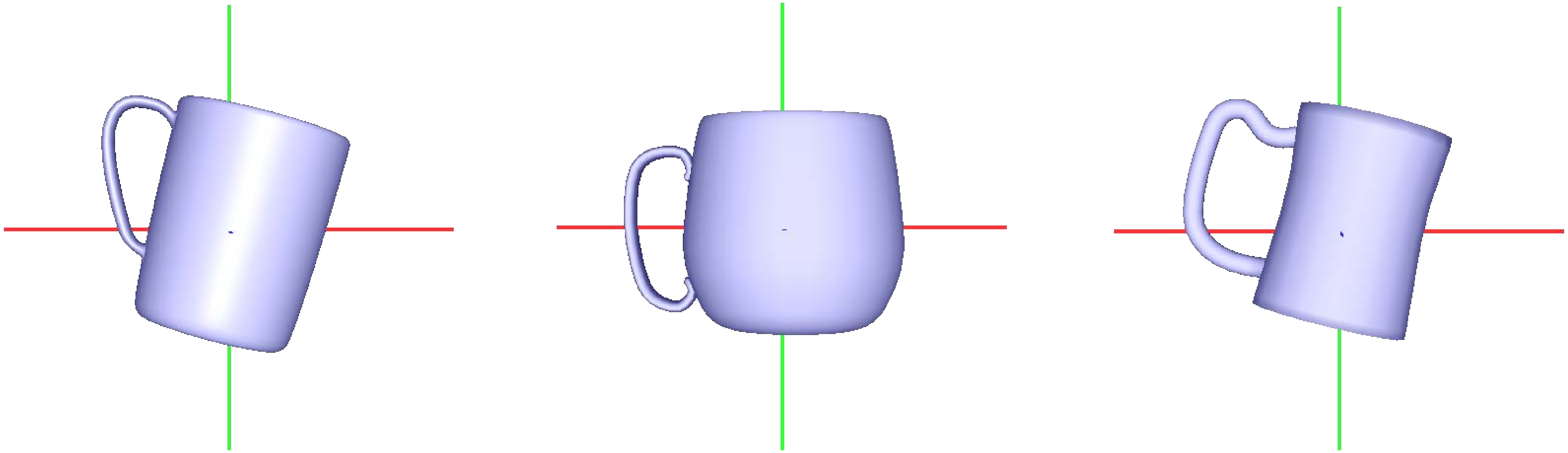


Horizontal votes diffused…

Application: Alignment

Motivation:

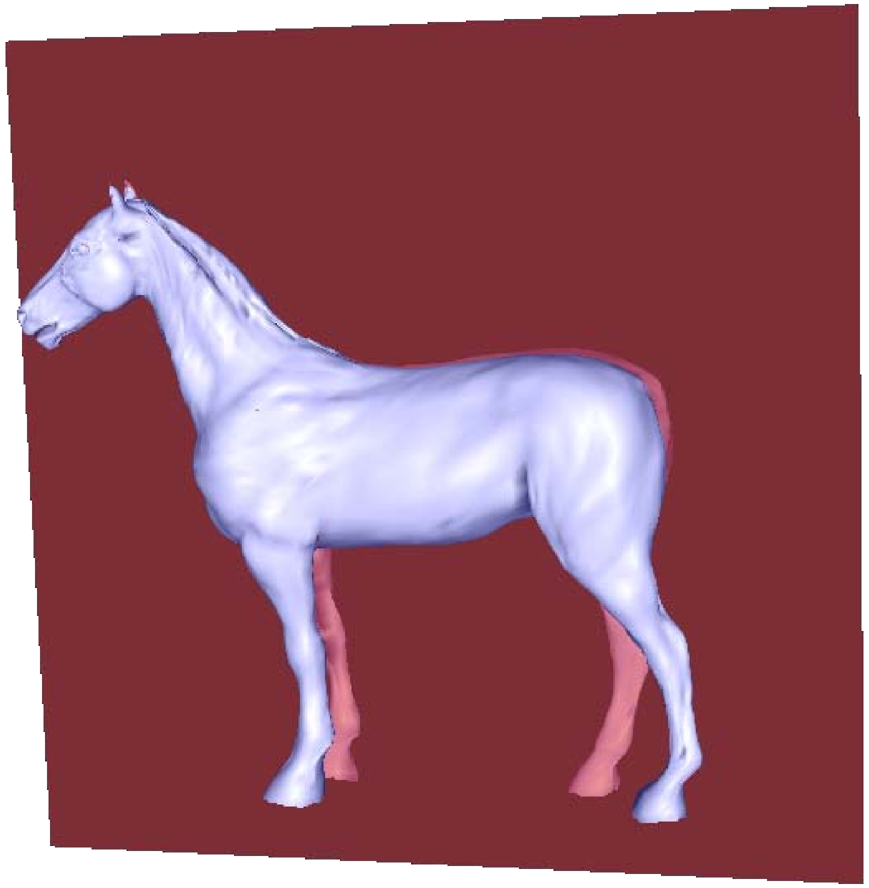
- Composition of range scans
- Morphing



PCA Alignment

Approach:

Perpendicular planes with the greatest symmetries create a symmetry-based coordinate system.



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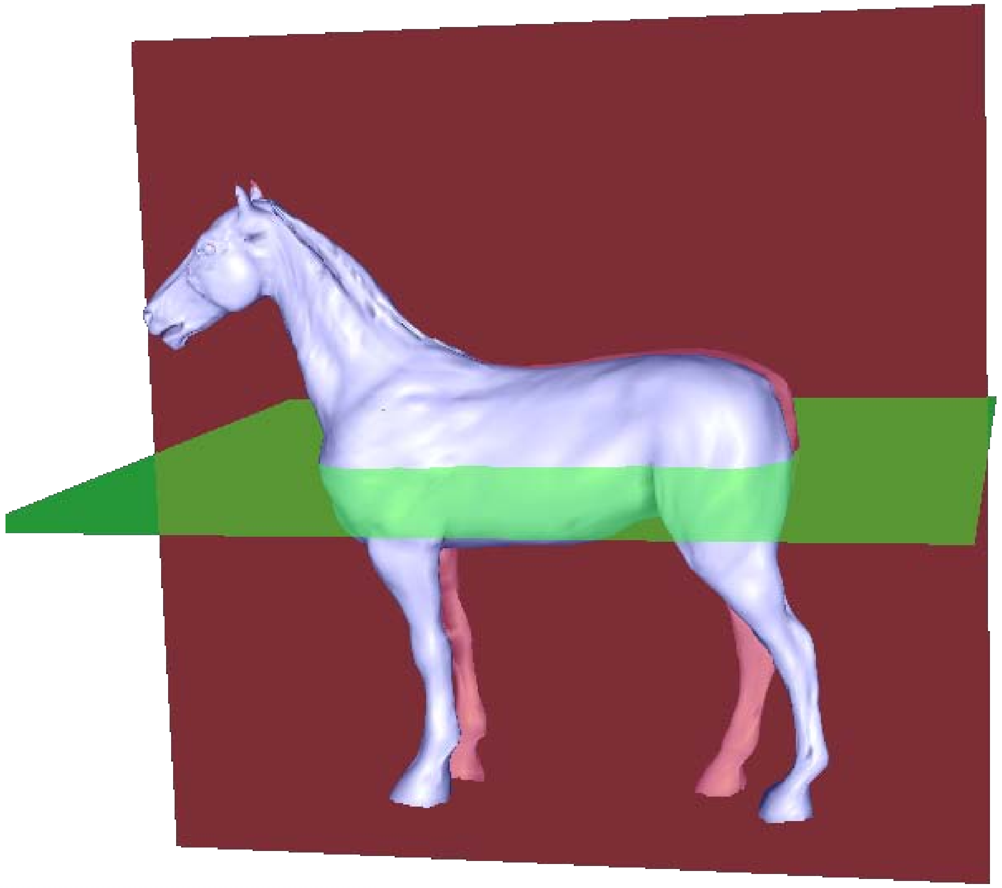

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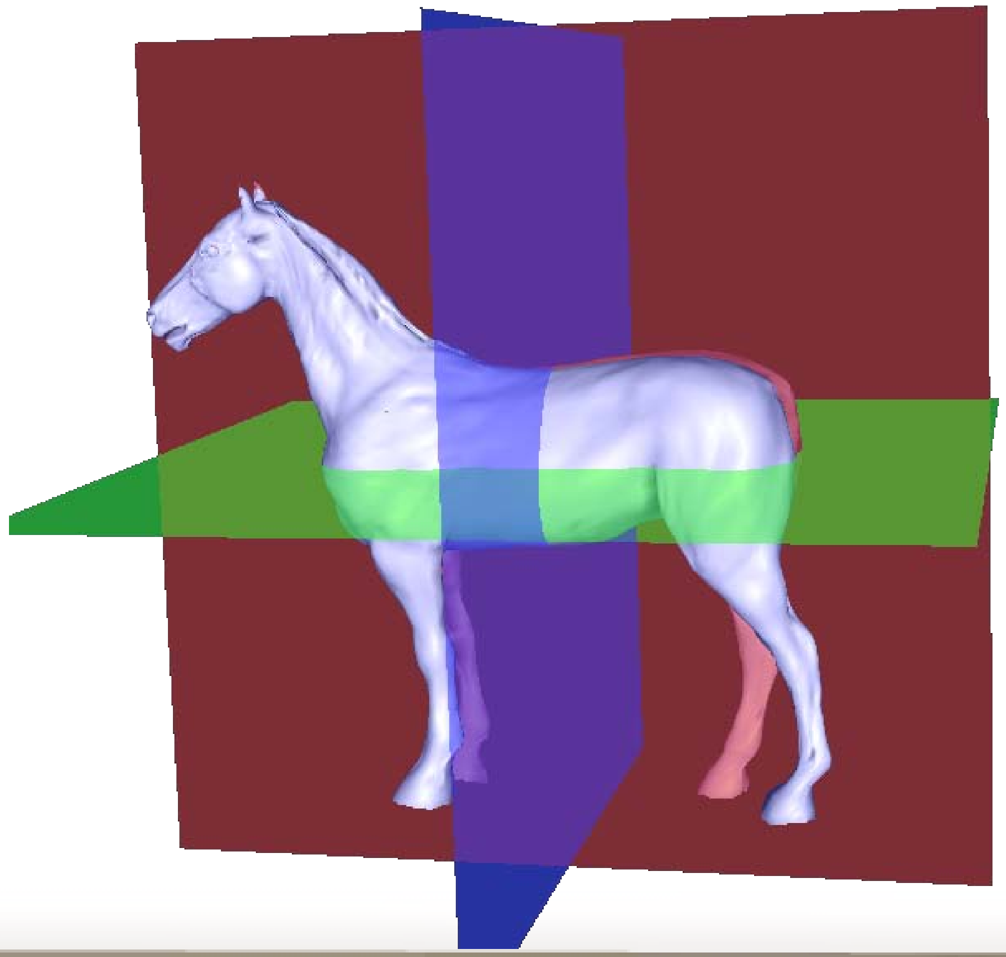

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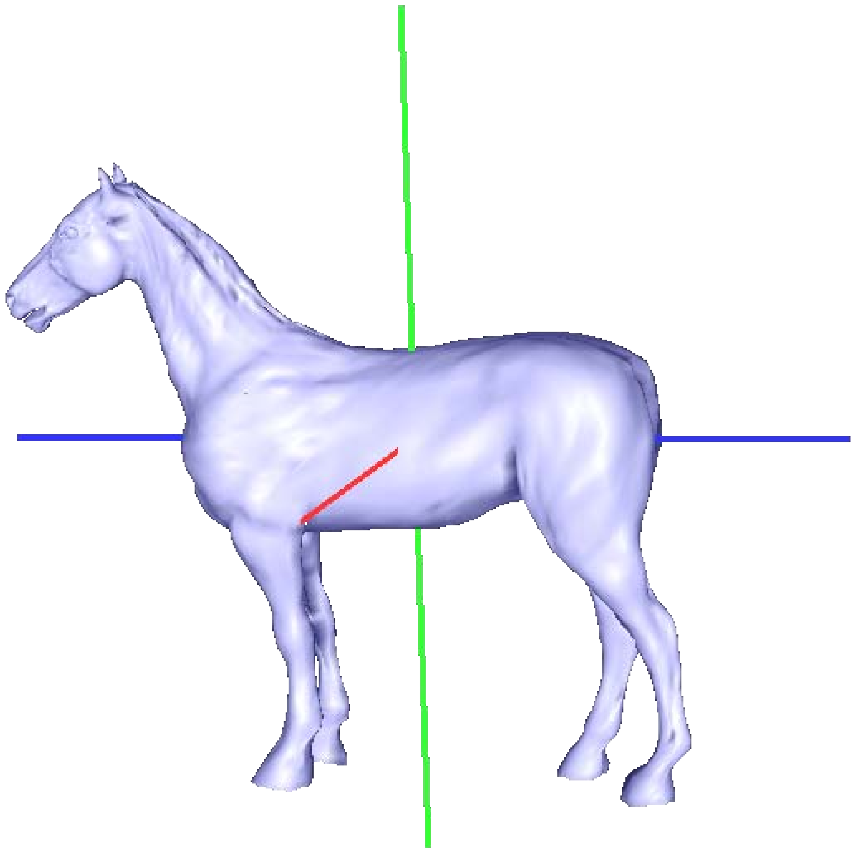
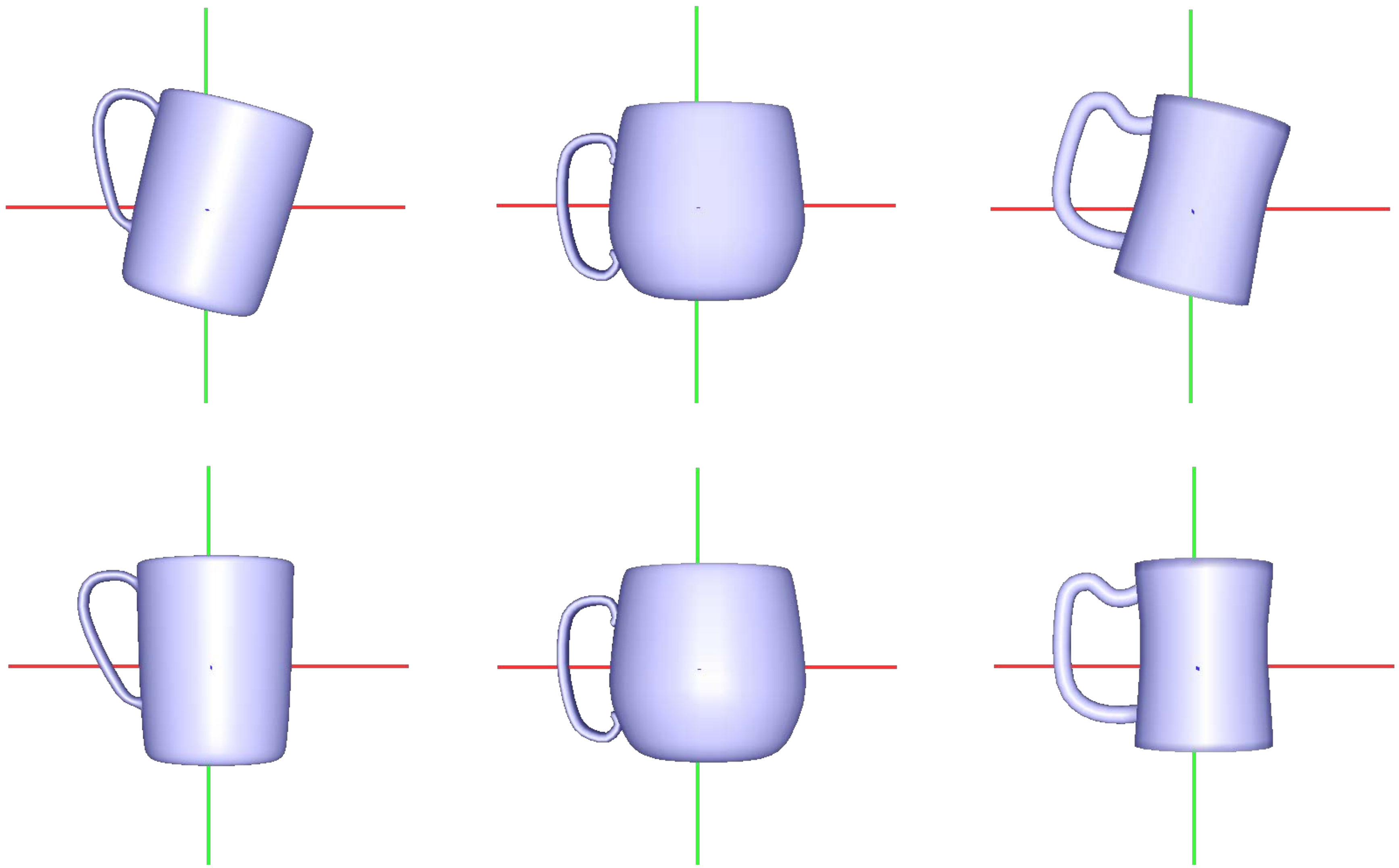


Results:

PCA Alignment

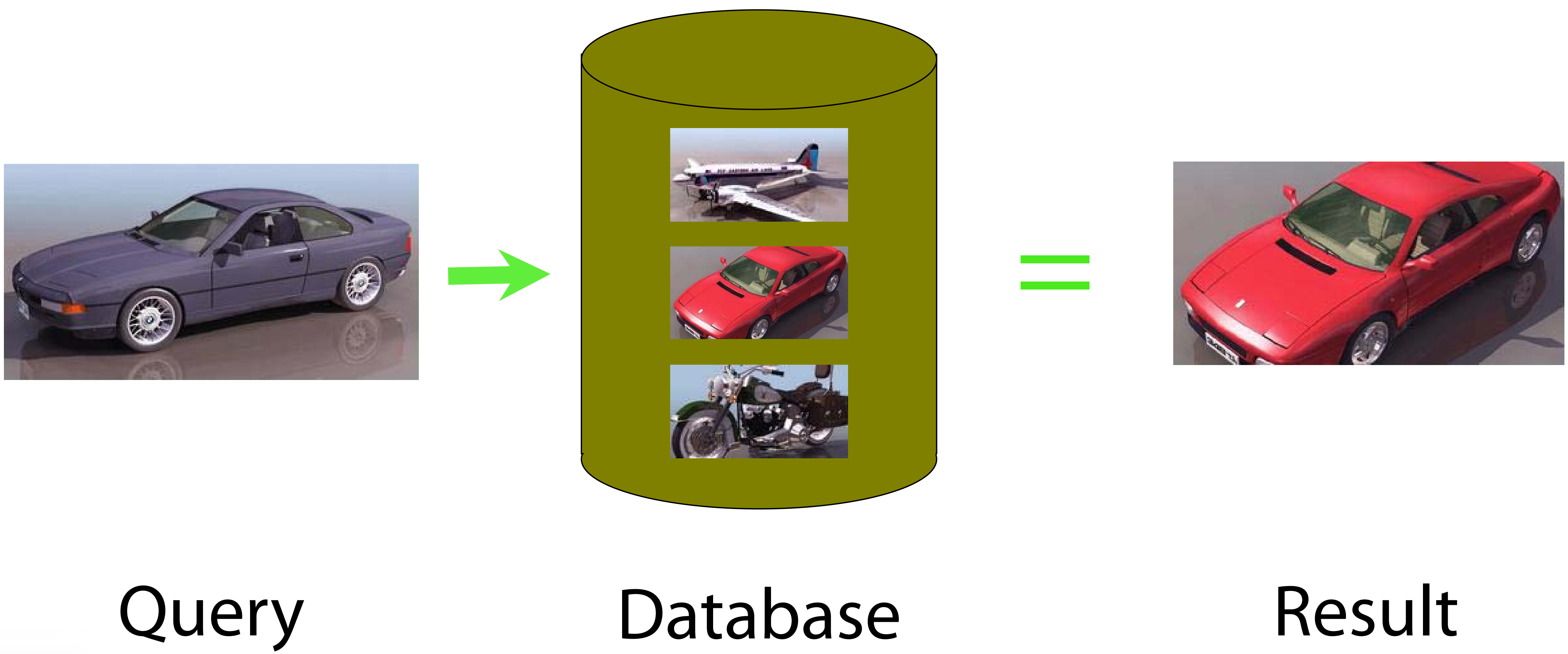
Symmetry Alignment



Application: Matching

Motivation:

Database searching



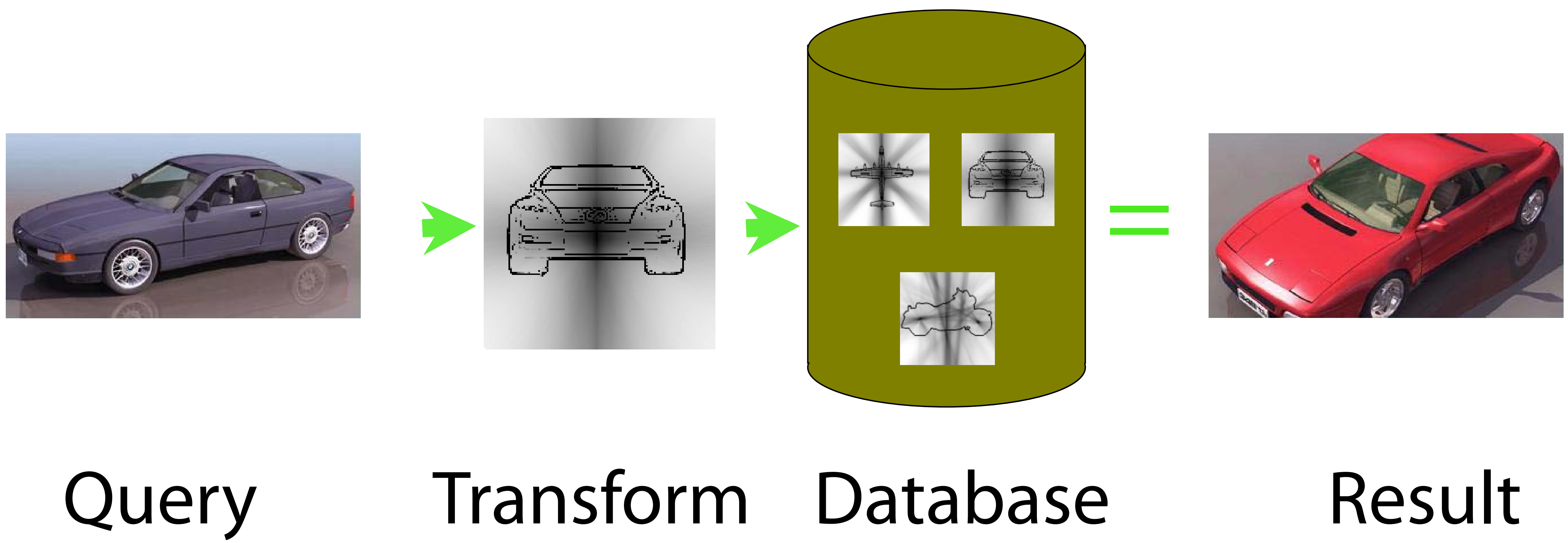
Application: Matching

Observation:

All chairs display similar principal symmetries

Approach:

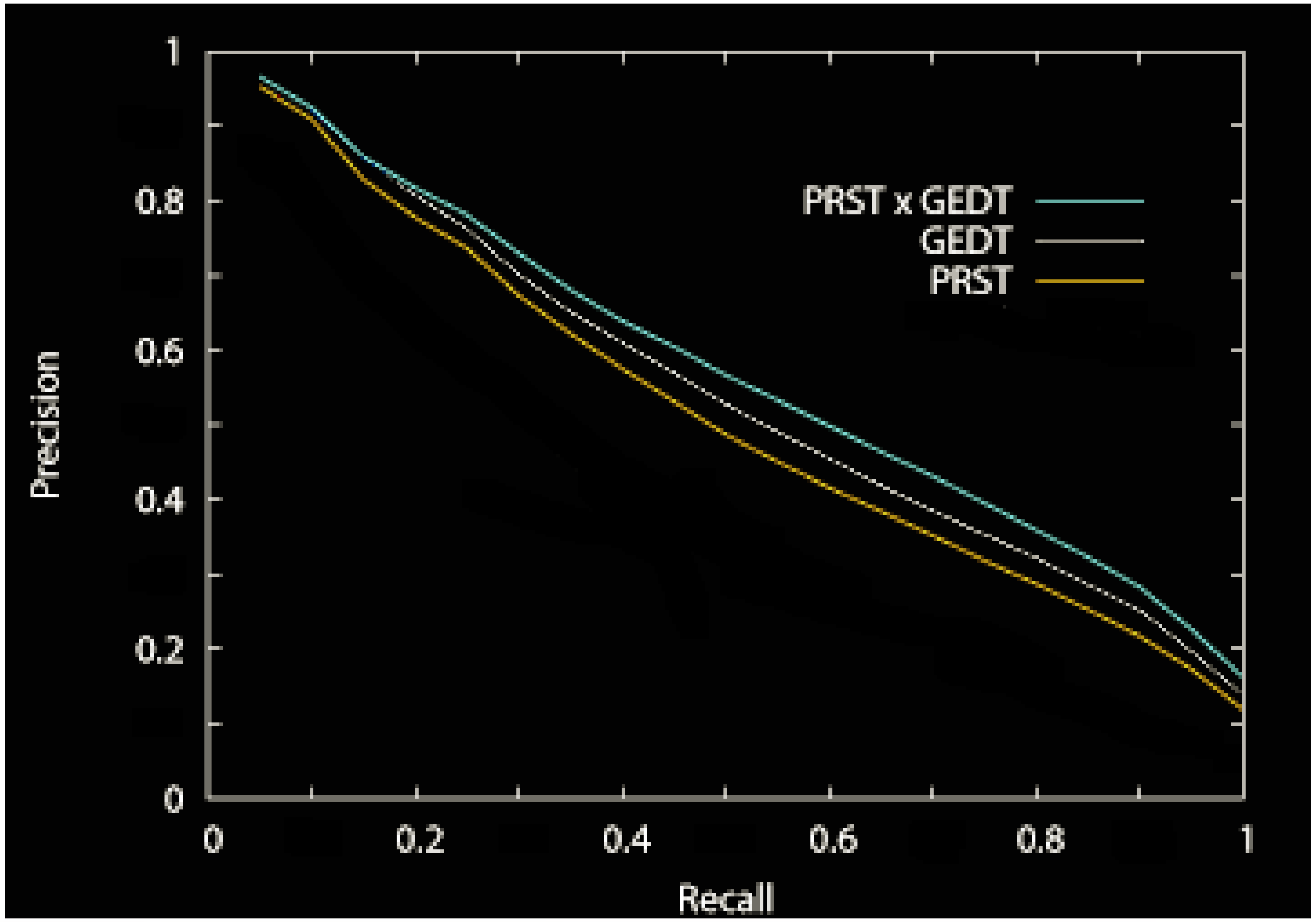
Use Symmetry transform as shape descriptor



Application: Matching

Results:

Symmetry provides orthogonal information about models and can therefore be combined with other descriptors



Summary

Planar-Reflective Symmetry Transform

Captures degree of reflectional symmetry about all planes

Monte Carlo computation

Applications: alignment, search, completion, segmentation, canonical viewpoints, …