Seminar: Computational Geometry and Geometric Computing
Kickoff-Meeting

Eric Berberich    Ben Galehouse    Michael Sagraloff

Max Planck Institute for Informatics

April 14, 2010
Outline
Computational Geometry

- **Design and analyse** algorithms and data structures for geometric settings
- Combinatorial problems
  - how to reduce the overall number of operations
  - sparse data structures
- Algebraic problems
  - fundamental layer for many algorithms in CG
  - in particular, for non-linear objects
- **Examples**: Convex hull of points (in plane/space), nearest-neighbor queries, Voronoi diagrams, arrangement computations, topology of curves and surfaces, polynomial system solving,...
Geometric Computing

- **Implement** algorithms and data structures for geometric settings

- doubles (fixed precision floating-points) are evil - usually
  - sufficient for many instances in linear scenarios, but
  - for non-linear objects we have to deal with larger errors in floating point computations

- “algorithm engineering”: filter techniques, parallel evaluation
What about?  
Who we are?  
Organisatorial  
Maaaaaaaaaaaaaaaaaaaany Topics  

Geometric Computing

- **Implement** algorithms and data structures for geometric settings
- doubles (fixed precision floating-points) are evil - usually sufficient for many instances in linear scenarios, but for non-linear objects we have to deal with larger errors in floating point computations
- “algorithm engineering”: filter techniques, parallel evaluation
CG vs. GC

Computational Geometry

- leads to theoretically best algorithms
- ignores constant factors, relies on unimplemented earlier work
- often makes assumption: General position, real-RAM (exact computation with real values)

Geometric Computing

- reformulate algorithms to handle any input
- "real-RAM" implementation
- algebraic methods in the literature are either numerical (fast, not certifying) or symbolic (slow, exact)
- combining both worlds to increase efficiency
The Group

- Department 1: Headed by Kurt Mehlhorn
- 5 subgroup leaders
- about 20 PostDocs, about 20 PhD students, 2 secretaries
- many HiWis (research assistant), B.Sc.-, M.Sc.-students
- Subgroups:
  - Foundations and Discrete Maths
  - Combinatorial Optimiziation
  - Bio-inspired computing
  - Algorithmic Game Theory
  - Geometric Computing
Involvement of the subgroup

- Research in Computational Geometry was always there (various number of people)
- Geometric Computing (really compute on PC)
- LEDA\(^1\) has a geometry kernel since early 90s - for linear objects (more recent: circles)
- 1996: CGAL\(^2\) was founded by research sites across Europe (one is D1)
- Goal: Make theory available as software!
- several EU-Research projects

---

\(^1\)Library of Efficient Data Structures and Algorithms
\(^2\)Computational Geometry Algorithms Library
Involvement of the subgroup - cont’d

- since 2001: focus on non-linear objects in GC-setting: circles, conics, and beyond
- 2001-2004: Effective Computational Geometry (ECG)
- 2001: D1 founded EXACUS-project\(^3\)
- since 2006: EXACUS merges into CGAL (merge not complete, but no more dev in EXACUS)

\(^3\)Library for Efficient and eXact Algorithms for CUrves and Surfaces
Eric Berberich

- Grew up in Saarland, joined Saarland Uni 1999
- 2001: Seminar Effective Computational Geometry
- 2002: Lecture Effective Computational Geometry
- 2002: Fopra about Arrangements of Conics
- 2004: Diplom about Quadric intersection curves
- 2008: PhD on 2.5 dimensional arrangements (of algebraic objects)
- 2009: PostDoc in Tel-Aviv on arrangements and Minkowski deconstruction
- 2010: PostDoc at MPI - running a seminar
Eric Berberich

- Grew up in Saarland, joined Saarland Uni 1999
- 2001: Seminar Effective Computational Geometry
- 2002: Lecture Effective Computational Geometry
- 2002: Fopra about Arrangements of Conics
- 2004: Diplom about Quadric intersection curves
- 2008: PhD on 2.5 dimensional arrangements (of algebraic objects)
- 2009: PostDoc in Tel-Aviv on arrangements and Minkowski deconstruction
- 2010: PostDoc at MPI - running a seminar
Eric Berberich

- Grew up in Saarland, joined Saarland Uni 1999
- 2001: Seminar Effective Computational Geometry
- 2002: Lecture Effective Computational Geometry
- 2002: Fopra about Arrangements of Conics
- 2004: Diplom about Quadric intersection curves
- 2008: PhD on 2.5 dimensional arrangements (of algebraic objects)
- 2009: PostDoc in Tel-Aviv on arrangements and Minkowski deconstruction
- 2010: PostDoc at MPI - running a seminar
Ben Galehouse

- Grew up in Ohio
- PhD at New York Universiy 2009 (Subdivision based meshing)
- Focus so far is on subdivision based meshing, but starting to diversify.
- Started Postdoc at MPII, November 2009
Michael Sagraloff

- Grew up in Bavaria, joined University in Bayreuth 1998 (Math/Algebraic Geometry)
- 2002-2005: PhD at Saarland University (Algebraic Geometry: classification of algebraic curves)
- 2005-now: Postdoc at MPII - heading the Geometric Computing group
  - main interest in the "algebraic stuff"
  - complexity analysis of root solvers/topology computation
  - combination of numerical/symbolic methods
  - adaptive algorithms
Now: Who are you?

- Please introduce yourself with name, studies, previous courses, interests
- What attracts you in our course? Expectations?
- Use a few sentences!
3 Goals

- Cover a specific exciting area of research in various details, i.e., with the help of (recent) scientific articles
- Make you familiar with (scientific) work: reading, understanding, discussion, presentation, writing
- Join our group as HiWi or for a thesis
Weekly Meeting

- There won’t be Pizza every week.
- We meet for 2hours
  - 60-75min talk
  - 10-15min questions
  - 10-15min discussion

Issues

- Problem 1: Find a time that suits all attendees
- Problem 2: When is the first usual meeting?
Weekly Meeting

- There won’t be Pizza every week.
- We meet for 2 hours
  - 60-75 min talk
  - 10-15 min questions
  - 10-15 min discussion

Issues

- Problem 1: Find a time that suits all attendees
- Problem 2: When is the first usual meeting?
Grading rubric

7 CreditPoints

- Mandatory: Participation
- 75% Talk
- 25% Summary of up to 4 pages (\LaTeX)
Supervision

We are happy to

- preview your slides (strongly encouraged)
- comment on draft versions of your topic summary

Contact us by email to schedule meetings.
Literature

General:

- Book: de Berg, Cheong, van Kreveld, Overmars: *Computational Geometry: Algorithms and Applications*
- Book: Basu, Pollack, Roy: *Algorithms in Real Algebraic Geometry*
  Available online at http://www.math.purdue.edu/~sbasu/
- Book: LaValle: *Motion Planning*
  Available online at http://planning.cs.uiuc.edu/
- Website: www.cgal.org
- CGGC-Lecture Winter Term 2009/2010 with a lot of material:
  http://www.mpi-inf.mpg.de/departments/d1/teaching/ws09_10/CGGC

Topic:

- Main references provided by us
- Using more is encouraged
Literature

General:
- Book: de Berg, Cheong, van Kreveld, Overmars: *Computational Geometry: Algorithms and Applications*
- Book: Basu, Pollack, Roy: *Algorithms in Real Algebraic Geometry*
  Available online at [http://www.math.purdue.edu/~sbasu/](http://www.math.purdue.edu/~sbasu/)
- Book: LaValle: *Motion Planning*
  Available online at [http://planning.cs.uiuc.edu/](http://planning.cs.uiuc.edu/)
- Website: [www.cgal.org](http://www.cgal.org)

Topic:
- Main references provided by us
- Using more is encouraged
Minkowski Sums and Offsets

Minkowski sum
For sets $X, Y$: $M = X \oplus Y := \{x + y | x \in X, y \in Y\}$

$X, Y$ polygonal
- Convex decomposition
- Convolution cycle

$X$ polygonal, $Y$ a disk = offset
- Exact constructions
- Approximation

Extension: Real-time offsets of triangle soups on the GPU
Minkowski Sums and Offsets

Minkowski sum

For sets $X, Y$: $M = X \oplus Y := \{ x + y | x \in X, y \in Y \}$

$X, Y$ polygonal

- Convex decomposition
- Convolution cycle

$X$ polygonal, $Y$ a disk = offset

- Exact constructions
- Approximation

- Extension: Real-time offsets of triangle soups on the GPU
Minkowski Sums and Offsets

Minkowski sum

For sets $X, Y$: $M = X \oplus Y := \{ x + y | x \in X, y \in Y \}$

$X, Y$ polygonal

- Convex decomposition
- Convolution cycle

$X$ polygonal, $Y$ a disk = offset

- Exact constructions
- Approximation

- Extension: Real-time offsets of triangle soups on the GPU
Offset Deconstruction

Reverse direction

Given a polygonal shape $P$, two real parameters $r, \varepsilon > 0$. Is $Q$ a close approximation (Hausdorff-distance $\leq \varepsilon$) of the $r$-offset of another (unknown) polygonal shape $P^\star$. If so, find a nice-looking $P$.

- decision algorithm
- greedy construction for convex input
Offset Deconstruction

Reverse direction

Given a polygonal shape $P$, two real parameters $r, \varepsilon > 0$. Is $Q$ a close approximation (Hausdorff-distance $\leq \varepsilon$) of the $r$-offset of another (unknown) polygonal shape $P^\star$. If so, find a nice-looking $P$.

- decision algorithm
- greedy construction for convex input
Offset Deconstruction

Reverse direction

Given a polygonal shape $P$, two real parameters $r, \varepsilon > 0$. Is $Q$ a close approximation (Hausdorff-distance $\leq \varepsilon$) of the $r$-offset of another (unknown) polygonal shape $P^*$. If so, find a nice-looking $P$.

- decision algorithm
- greedy construction for convex input
Voronoi Diagram and Medial Axis

**Voronoi Diagram**

Given a set of sites in the plane, compute a decomposition of the plane such that each site owns one cell and is closest site to all points of the cell it owns.

- New algorithm that relates to edges of a VD to the medial axis of an augmented planar domain
- Can be also used for computing offsets
Voronoi Diagram and Medial Axis

Voronoi Diagram

Given a set of sites in the plane, compute a decomposition of the plane such that each site owns one cell and is closest site to all points of the cell it owns.

- New algorithm that relates to edges of a VD to the medial axis of an augmented planar domain
- Can be also used for computing offsets
2D Straight Skeleton

Straight Skeleton (simplified definition)

The straight skeleton of a polygon is given by moving the edges at a fixed rate, and watching where the vertices go.

- similar to *medial axis*
- not a voronoi diagram!
- interesting applications: “roofing”, terrain reconstructions, origami folding, planar motion planning, offset construction
Robot Motion Planning (Sampling)

Moving pieces

Is it possible to move a piece in a space from position $A$ to position $B$ without colliding with any obstacle?

- Definition of Configuration Space
- Techniques: Rapid Exploring Dense Tree, Visibility Roadmap
Nef Polyhedra (in 2d and 3d)

Nef-Polyhedra

A Nef-polyhedron in dimension $d$ is a point set $X \subset \mathbb{R}^d$ generated from a finite number of open halfspaces by set complement and set intersection operations.
Towards “Nef”-Quadrics

Quadric

Algebraic surface of degree 2: Ellipsoids, Paraboloid, Hyperboloid, ...

- Given a set of intersecting quadrics: Compute how they are connected, and which volumes they induced (bounded by faces, edges, vertices)
- Two steps: (1) Adjacency graph (2) Neighborhood maps
Towards “Nef”-Quadrics

Quadric

Algebraic surface of degree 2: Ellipsoids, Paraboloid, Hyperboloid, ...

- Given a set of intersecting quadrics: Compute how they are connected, and which volumes they induced (bounded by faces, edges, vertices).
- Two steps: (1) Adjacency graph (2) Neighborhood maps
Towards “Nef”-Quadrics

Quadric

Algebraic surface of degree 2: Ellipsoids, Paraboloid, Hyperboloid,…

- Given a set of intersecting quadrics: Compute how they are connected, and which volumes they induced (bounded by faces, edges, vertices)
- Two steps: (1) Adjacency graph (2) Neighborhood maps
Subdivision

- One approach to analyzing implicit surfaces is to subdivide a finite domain into regions until each region is “simple”.
- Using interval arithmetic system, it is possible to handle non-singular problems in an essentially numeric way.
- Literature:
  - Simon Plantinga and Gert Vegter “Isotopic Meshing of Implicit Surfaces”, 2005 — A classic and very well studied algorithm.
Numerical solvers

- Finding roots of polynomials using numerical (floating point) computation is one of the oldest problems in computer science literature.
- Typically, these methods combine some variant of Newton iteration with various theoretic results and guarantee convergence when all roots are simple.
- Literature:
  - Dario Andrea Bini “Numerical computation of polynomial zeros by means of Aberth’s method”, 1995
Shortest Path among Discs

- The problem is to find the shortest path between two points in the plane which avoids a given collection of discs.
- Problem is interesting because exact solutions can be found, even though the problem is in a sense transcendental.
- Literature:
Snap Rounding

- One approach to the analysis of exact geometric structures is to find and compute using a low precision approximation of them.
- The result is a solution to an approximated problem, but certain properties can be preserved.
- Literature:
  - Dan Halperin and Eli Packer “Iterated snap Rounding”, 2001
Controlled Perturbation

**Problem**

Exact computation is often very challenging: you have to deal with degeneracies and costly symbolic computations. But: Can we compute the exact result for nearby input, at least, without this additional effort?

- framework for CP.
- general analysis: how much precision is needed for certain geometric predicates.
- application to well-known algorithms
- many research problems: so far, CP has only been applied to “linear problems”.

April 14, 2010
Eval - a real root solver

Problem

Given a polynomial $f \in \mathbb{R}[x]$, determine a set of disjoint intervals (boxes) that contain all real (complex) roots of $f$.

- “simplest” approach checks whether an interval contains a root of $f$ or its derivative $f'$.
- If an interval $I$ contains no root of $f'$, then it suffices to check endpoints of $I$.
- Testing for roots with interval arithmetic
- Efficiency in comparison to more sophisticated methods (Sturm/Descartes)
- Generalization to complex roots and polynomials with approximate coefficients.
Quadratic interval refinement

**Problem**

Given a polynomial $f \in \mathbb{R}[x]$ and an isolating interval for a root of $f$, how can we get arbitrary good approximations of the root in an efficient way.

- simple binary research is not good enough (only linear convergence)
- Newton like methods (approximation of the polynomial by a linear function) lead to quadratic convergence.
- complexity of approximating a root up to a certain precision.
Algebraic Curve Analysis via RUR

Problem

Given a polynomial \( f \in \mathbb{Z}[x, y] \), describe the set \( C \) of all points \((x, y) \in \mathbb{R}^2\) with \( f(x, y) = 0 \). Compute a linear isotopic approximation of \( C \).

- CAD approach vs. RUR approach
- determine critical points and connect them in appropriate manner
Surface Analysis/Triangulation

Problem
Given a polynomial $f \in \mathbb{Z}[x, y, z]$, describe the set $S$ of all points $(x, y, z) \in \mathbb{R}^3$ with $f(x, y, z) = 0$. Compute a triangular mesh which is isotopic to $S$.

- $f = z^4 + (-5) \cdot z^2 + (y^4 + (-5) \cdot y^2 + (x^4 + (-5) \cdot x^2 + 10))$
- “generalization” of topology computation for curves
- new ideas for adjacency computation/vertical lines
- open problems: arrangements of surfaces, complexity of triangulation
Surface Analysis/Triangulation

Problem

Given a polynomial $f \in \mathbb{Z}[x, y, z]$, describe the set $S$ of all points $(x, y, z) \in \mathbb{R}^3$ with $f(x, y, z) = 0$. Compute a triangular mesh which is isotopic to $S$.

- $f = z^4 + (-5) \cdot z^2 + (y^4 + (-5) \cdot y^2 + (x^4 + (-5) \cdot x^2 + 10))$
- “generalization” of topology computation for curves
- new ideas for adjacency computation/vertical lines
- open problems: arrangements of surfaces, complexity of triangulation
... and now: Problem 3

The Assignment

- There are research papers about this problem!
- We do ... something sophisticated on the whiteboard :-(
... and now: Problem 3

The Assignment

- There are research papers about this problem!
- We do ... something sophisticated on the whiteboard :-)

April 14, 2010