Optimization Summer 2010	Homework 2	N. Megow, K. Mehlhorn J. Mestre

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This assignment is **due on Apr 29/30** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1: Let $P \subseteq \mathcal{R}^n$ be a polyhedron and be $\pi_k(P)$ the projection of P onto its first k coordinates. Assume $\mathbf{x} \in P$. For each of the following statements either disprove with a counter example or provide a short proof.

i) If $\mathbf{x}$ is an extreme point of P then $\pi_k(\mathbf{x})$ is an extreme point of $\pi_k(P)$.

ii) If $\pi_k(\mathbf{x})$ is an extreme point of $\pi_k(P)$ then $\mathbf{x}$ is an extreme point of P.

Problem 2: Is it the case that every non-empty polyhedron has at least one extreme point? Justify your answer.

Problem 3: Consider the polyhedron $\{\mathbf{x} \in R^3 \mid \mathbf{Ax} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0}\}$ where

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 4 \end{bmatrix} \qquad b = (0, -2)$$

List all its bases and their associated basic solutions specifying which are feasible.

Problem 4 (extra credit): Let $\mathbf{A_1}, \ldots, \mathbf{A_m}$ be vectors in R^n and assume $m > n$. The convex hull of these vectors is

$$C = \left\{ \lambda_1 \mathbf{A_1} + \cdots + \lambda_m \mathbf{A_m} \;\middle|\; \sum_{i=1}^{m} \lambda_i = 1 \text{ and } \lambda_i \geq 0 \text{ for } i = 1, \ldots, m \right\}.$$

Show that each $\mathbf{y} \in C$ can be written as a convex combination of just $n+1$ of these vectors.