Homework 3

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This assignment is **due on May 6/7** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1: Consider the following linear program.

$$\begin{aligned} \text{minimize} \quad & -5x_1 - 4x_2 - 3x_3 \\ \text{subject to} \quad & 2x_1 + 3x_2 + x_3 \leq 5 \\ & 4x_1 + x_2 + 2x_3 \leq 11 \\ & 3x_1 + 4x_2 + 2x_3 \leq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

i) Convert this program into another equivalent LP in standard form.

ii) Run the simplex algorithm. Start at $x_1 = x_2 = x_3 = 0$. Show the tableau associated with each basic feasible solution the algorithm goes through.

Problem 2: Consider the following linear program.

$$\begin{aligned} \text{minimize} \quad & -0.75x_1 + 20x_2 - 0.5x_3 + 6x_4 - 3 \\ \text{subject to} \quad & 0.25x_1 - 8x_2 - x_3 + 9x_4 \leq 0 \\ & 0.5x_1 - 12x_2 - 0.5x_3 + 3x_4 \leq 0 \\ & x_3 \leq 1 \\ & x_1, x_2, x_3, x_4 \geq 0 \end{aligned}$$

i) Convert this program into another equivalent LP in standard form by introducing slack variables.

ii) Run the simplex algorithm until it cycles. Start at $x_1 = x_2 = x_3 = x_4 = 0$. Show the tableau associated with each basic feasible solution the algorithm goes through. Use the following pivoting rules in case of ties:

  a) Select the nonbasic variable with the most negative reduced cost to enter the basis.

  b) Select the basic variable with the smallest subscript to leave the basis.

Problem 3: Show that the following version of Farkas Lemma implies the version of Farkas Lemma shown in the class:

> Let $\mathbf{A} \in R^{m \times n}$ and $\mathbf{b} \in R^{m \times 1}$. Exactly one of the following statements is true:
>
> - The system $\mathbf{A}\mathbf{x} = \mathbf{b}$, $\mathbf{x} \geq \mathbf{0}$ is feasible.
> - There exists a vector $\mathbf{y} \in \mathcal{R}^{m \times 1}$ such that $\mathbf{y}'\mathbf{A} \geq \mathbf{0}$ and $\mathbf{y} \cdot \mathbf{b} < 0$.