# Homework 4

N. Megow, K. Mehlhorn
J. Mestre

---

This assignment is **due on May 13/14** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

**Problem 1:** Derive the dual of the following program

$$\begin{aligned} \text{minimize} \quad & 2x_1 + 3x_2 - 5x_3 \\ \text{subject to} \quad & 2x_1 - 4x_3 \geq 4 \\ & x_1 + 2x_2 \leq 7 \\ & x_2 + x_3 = 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Find a pair of optimal primal and dual solutions. Verify that they have the same cost and that complementary slackness conditions hold.

**Problem 2:** Consider a two-player zero-sum game defined by a skew symmetric matrix $\mathbf{D}$; that is, $\mathbf{D} = -\mathbf{D}'$. Prove that for such games we always have $\max_{\mathbf{x}} \min_{\mathbf{y}} \mathbf{x}' \mathbf{D} \mathbf{y} = 0$. Do you know any game that has this property?

**Problem 3:** Consider the linear program on the right. Transform it into an "equivalent" linear program in standard form by introducing a slack variable for each constraint. (By equivalent we mean that a feasible solution for the first LP can be transformed into a feasible solution for the second LP with the same cost and vice-versa.) Show that the dual of the original program and the dual of the new program are also "equivalent".

$$\begin{aligned} \text{minimize} \quad & \mathbf{c}'\mathbf{x} \\ \text{subject to} \quad & \mathbf{A}\mathbf{x} \geq \mathbf{b} \\ & \mathbf{x} \geq \mathbf{0} \end{aligned}$$

**Problem 4 (extra credit):** Prove that the system $\mathbf{A}\mathbf{x} = \mathbf{0}$ and $\mathbf{x} > \mathbf{0}$ is infeasible if and only if the system $\mathbf{p}'\mathbf{A} \leq \mathbf{0}'$ and $\mathbf{p}'\mathbf{A} \neq \mathbf{0}'$ is feasible. (Here $\mathbf{x} > \mathbf{0}$ means every coordinate of $\mathbf{x}$ is strictly positive.) *Hint: Come up with a pair of primal and dual programs such that the first condition is true if and only if the primal is bounded and the second condition is true if and only if the dual is feasible.*