Optimization
Summer 2010

Homework 7

N. Megow, K. Mehlhorn
J. Mestre

This assignment is **due on June 10/11** in your tutorial session. You are allowed (even encouraged) to discuss these problems with your fellow classmates. All submitted work, however, must be *written individually* without consulting someone else's solutions or any other source like the web.

Problem 1 **(6 points)**

Let $s \in \mathbb{N}$ and

$$A := \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 2^s & 1 \end{pmatrix} \quad \text{and} \quad b := \begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}.$$

Furthermore, let $P := \{x \in \mathbb{R}^2 \mid Ax \leq b\}$. Find a feasible solution with the ellipsoid method for $s = 0$ and $s = 1$.

Problem 2 **(4 points)**

Consider the polyhedron $P = \{(x, y) \in \mathbb{R}^2 | y \leq \sqrt{2}x\}$. Show that its integer hull $P_I = conv.hull(P \cap \mathbb{Z}^2)$ is not a polyhedron, that is, it cannot be described by a finite number of inequalities.

Problem 3 **(4 points)**

Let $\mathcal{P} := conv.hull\{(0,3), (2,2), (0,0), (3,0)\}$ and consider the following two linear representations:

$$\begin{aligned} x + 2y &\leq 6 \\ 2x + y &\leq 6 \\ x, y &\geq 0 \end{aligned} \qquad \text{and} \qquad \begin{aligned} x + 2y &\leq 6 \\ 2x + y &\leq 6 \\ x + y &\leq 4 \\ x &\leq 3 \\ y &\leq 3 \\ x, y &\geq 0 \end{aligned}$$

They clearly define the same polyhedron. Prove that one is TDI but the other one is not.

Problem 4 **(4 points)**

Prove that adding a feasible constraint does not destroy TDI-ness: If $Ax \leq b$ is TDI and $a^T x \leq \beta$ is a valid inequality for $\{x \mid Ax \leq b\}$, then $Ax \leq b, a^T x \leq \beta$ is also TDI.