



Computer Algebra
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Exercise 1

Casting out nines

Consider the following test to "check" whether the product c of two integers a and b is actually the product: We compute the *checksums* s_a , s_b and s_c of a , b and c (sum up the digits in an iterative manner), respectively. For instance, if $a = 1378$, then

$$1378 \rightarrow 19 \rightarrow 10 \rightarrow 1,$$

hence $s_{1378} = 1$. Show: If the checksum of the product of s_a and s_b differs from s_c , then $a \cdot b \neq c$.

Karatsuba multiplication

Show that using the Karatsuba method, we can multiply two integers of digit length n with $O(n^{\log 3})$ primitive operations.

Doubles

What is the largest number representable as a double, the smallest positive number, the smallest normalized positive number?

Some Computations are Exact

Let $a, b \in F$ with $\frac{1}{2} \leq \frac{a}{b} \leq 2$. Show that $a \ominus b = a - b$. This was first observed by Sterbenz.

A High Precision Computation of the Euler number e

Show how to compute e with an error less than 2^{-20} .

Have fun with the solution!