



Problem Set 2 Topological Methods in Geometry

SS 2011

Problem 1 (Gluing Lemma).

Let X and Y be a topological spaces and let $X = X_1 \cup X_2$ where X_1 and X_2 are closed subsets of X. Let $f : X_1 \mapsto Y$ and $g : X_2 \mapsto Y$ be continuous functions such that for any $x \in X_1 \cap X_2$, f(x) = g(x). Let h be a function which maps any $x \in X$ to f(x) if $x \in X_1$ and to g(x) otherwise. Prove that h is continuous.

Problem 2 (Simplicial Complex of a Simplex).

Prove that a simplex and all its lower dimensional faces form a simplicial complex. What about all the faces of a simplex of dimension at most k, for some integer k? Do they form a simplicial complex?

Problem 3 (Moment Curve).

Consider the curve $C = \{(t, t^2, t^3, ..., t^d) : t \in \mathbb{R}\}$ in \mathbb{R}^d . Let h be the hyperplane $a_1x_1 + a_2x_2 + \cdots + a_dx_d = c$, where a_1, \ldots, a_d and c are real numbers. Prove that h intersects C at at most d points. Prove that any d + 1 distinct points on C are affinely independent.

Reading Exercise: Basics of Measure Theory

Get acquainted with the basic notions of Measure Theory. For example, read the first chapter of www.mathematik.uni-kl.de/~seifried/eng/teaching/teach_08ws/measure.pdf