Problem Set 5

Topological Methods in Geometry

SS 2011

Problem 1 (No Retraction Theorem)**.**
Prove that the following theorem is equivalent to the Brouwer's fixed point theorem: There is no continuous function $f : B^n \mapsto \partial B^n$ such that for each $x \in \partial B^n, f(x) = x$.

Problem 2.
In the lecture, we proved the following version of the Borsuk Ulam theorem: For any continuous function $f : I^n \mapsto \mathbb{R}^n$, where $I = [-1, 1]$, which is antipodal on ∂I^n, there exists some $x \in I^n$ s.t. $f(x) = 0$. Prove that if we replace I^n by B^n, the statement remains true.

Problem 3 (Radon' theorem in $\mathbb{R}^2$)**.**
Let σ^n denote an n-dimensional simplex. Let $f : \sigma^3 \mapsto \mathbb{R}^2$ be an affine map. Show that there are two points in σ^3 with disjoint supports that are mapped to the same point by f.

Problem 4.
Give an example of a pseudomanifold whose boundary is not a pseudomanifold. Let M^n be an n-dimensional pseudomanifold whose boundary is a pseudomanifold. Prove that $\partial\partial M^n = \emptyset$.