Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 3 Deadline: Thursday, May 5, 2011

Rules: There are 4 regular exercise problems. The first two serve as preparation for the test conducted in the exercise class. You should solve them, therefore, but you do not need to hand them in. The test yields 8 points. The remaining two problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. Each problem yields 4 points. You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Exercise 1 (*oral homework, in total 8 points via test*)

(a) Read, learn by heart, and understand all definitions in Sections 1.4 through 1.6 of the Diestel book.

(b) Draw a graph G that has edge-connectivity $\lambda(G) = 8$ and (vertex-)connectivity $\kappa(G) = 2$.

(c) Let G have at least 2 vertices. What do you need to show in order to prove that $\lambda(G) \leq 25$? Do not use the word 'ℓ-edge-connected' for $\ell \geq 2$ in your answer.

(d) Spell out what it means for a graph *not* to be maximally acyclic (see (iv) in Exercise 3 below).

(e) Prove, using the classic 'consider a longest path' argument, that every tree has a leaf.

(f) Read and understand the proof of Corollary 1.5.3 in the Diestel book.

(g) Let T be a tree. Is it true that every graph G that satisfies $\delta(G) \geq |V(T)| - 1$ and has girth at least $|V(T)| + 1$ contains an *induced* copy of T?

(h) True or false: Each 4-partite graph is 5-partite.

Exercise 2 (*oral homework, in total 8 points via test*)

Read and fully understand the proof of Theorem 1.4.3 in the Diestel book.

Exercise 3 (*written homework, 4 points*)

Prove Theorem 1.5.1 in the Diestel book. I.e., show that the following assertions are equivalent for a graph T:

(i) T is a tree.

(ii) Any two vertices of T are linked by a unique path in T.

(iii) T is a minimally connected, i.e. T is connected but $T-e$ is disconnected for every edge $e \in T$.

(iv) T is maximally acyclic, i.e. T contains no cycle but $T+xy$ does, for any two non-adjacent vertices $x, y \in T$.

Exercise 4 (*written homework, 4 points*)

(a) Show that in a 2-connected graph every vertex is contained in a cycle. [2P.]

(b) Let G be a graph. Show that G and its complement $\overline{G}$ cannot have both diameter larger than 3. [2P.]