Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 5 Deadline: Thursday, May 19, 2011

Rules: The first two problems serve as preparation for the test conducted in the exercise class. You should solve them, therefore, but you do not need to hand them in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Occasionally, there might be bonus problems, which yield additional bonus points. They are, typically, more difficult or not that closely related to that week's content of the lecture.

Exercise 1 (*oral homework, in total 8 points via test*)

(a) What is the easy direction of Tutte's characterization of perfect matchings? Explain why this direction follows.

(b) Read carefully and understand Corollary 2.2.2 and its proof in the Diestel book.

(c) Show how to construct K_4 (the complete graph on four vertices) by starting from a cycle, and successively adding H-paths to graphs already existing.

(d) Prove that if H is a 2-connected graph, P an H-path, and $G = H \cup P$, then G is also 2-connected.

(e) Prove that the block graph $B(G)$ of any graph G is a forest.

(f) What are the blocks of a tree?

Exercise 2 (*written homework, 2 points*)

Prove that any $2k$-edge-connected $(2k+1)$-regular graph has a perfect matching (1-factor). (Hint: Do exercise 1(b) before solving this problem.)

Exercise 3 (*written homework, 2 points*)

Let G be a graph. *Subdividing* an edge $\{u,v\} \in E(G)$ results in a graph G' with $V(G') = V(G) \cup \{x\}$, $x \notin V(G)$, and $E(G') = (E(G) \setminus \{u,v\}) \cup \{\{u,x\},\{x,v\}\}$. In other words, subdividing the edge $\{u,v\}$ replaces this edge with a path of length 2 with u and v as its endpoints.

(a) If G is 2-connected, then subdividing any of its edges also results in a 2-connected graph. [1P]

(b) Prove or give a counterexample to the following statement: Subdividing any edge in G results in a graph G' with the same number of blocks as G. [1P]

Exercise 4 (*written homework, 4 points*)

Let G be a graph with at least 3 vertices.

(a) Show that if every pair of vertices belong to some cycle of G, then G is 2-connected. [1P]

(b) Show that if G is 2-connected, and $\{u,v\} \in E(G)$, then u and v belong to some cycle of G. [1P]

(c) Use (b) to show that if G is 2-connected, and $u, v \in V(G)$, then u and v belong to some cycle of G. [2P]

Observe that (a) and (c) gives us another characterization of 2-connected graphs: A graph is 2-connected iff every pair of vertices belong to some cycle of the graph.