Universität des Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 6 Deadline: Thursday, May 26, 2011

Rules: The first two problems serve as preparation for the test conducted in the exercise class. You should solve them, therefore, but you do not need to hand them in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Occasionally, there might be bonus problems, which yield additional bonus points. They are, typically, more difficult or not that closely related to that week's content of the lecture.

Exercise 1 (*oral homework, in total 8 points via test*)

(a) Read carefully and understand Menger's Theorem (local (3.3.1) and global (3.3.6) versions) in Diestel's book.

(b) Let G be a graph, and let $A,B \subseteq V(G)$. If there is an A-B separator in G of size k, can there be more than k disjoint A-B paths in G? Why?

(c) If X is an A-B separator in G, is any A-X separator in G also an A-B separator of G? Why?

(d) What is the graph obtained by contracting an arbitrary edge in K_5?

(e) Read carefully and understand all the notions and definitions given in Section 4.2 of Diestel's Book.

(f) Draw a planar drawing of $K_{2,3}$.

(g) What are the faces of a plane graph?

(h) What is Euler's Formula?

Exercise 2 (*written homework, 2 points*)

Suppose that G_1 and G_2 are k-connected graphs with $V(G_1) \cap V(G_2) \geq k$. Prove that $G = G_1 \cup G_2$ is also k-connected.

Exercise 3 (*written homework, 3 points*)

Let G be a k-connected graph for some $k > 1$.

(a) Show that for any cycle C in G, and any vertex $v \in V(G) \setminus V(C)$, there are $\min\{k, |V(C)|\}$ paths from v to $V(C)$ such that none of these paths intersect except on v. [1P]

(b) Use (a) to show that any set of k vertices $X \subseteq V(G)$ is contained on some cycle of G. (Hint: Start with a cycle that contains as many vertices of X as possible, and use (a) to arrive at a contradiction if this cycle does not contain all of X). [2P]

Exercise 4 (*written homework, 3 points*)

Show that any graph can be drawn in $\mathbb{R}^3$ with no edge crossings (in the same sense that any planar graph can be drawn in the plane with no edge crossings).