Universität
des
Saarlandes
FR 6.2 Informatik

Prof. Dr. Benjamin Doerr, Dr. Danny Hermelin, Dr. Reto Spöhel **Summer 2011**

Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 7 Deadline: Friday, June 3, 2011

Rules: The first problem serves as a preparation for the test conducted in the exercise class. You should solve it, but you do not need to hand it in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

General notation: For a pair of positive integers i and j, we use $K_{i,j}$ to denote the complete bipartite graph with i vertices on one side and j vertices on the other, and K_i to denote the complete graph on i vertices.

Exercise 1 (*oral homework, in total 8 points via test*)

(a) Read carefully and understand all definitions and claims in Section 1.7 of Diestel's book, along with the proof of Lemmas 4.4.2 and 4.4.3.

(b) Draw a subdivision of $K_{3,3}$ that has 10 vertices.

(c) What is the definition of a minor?

(d) Is the following true: If G contains $K_{3,3}$ as a minor, then it also contains $K_{3,3}$ as a topological minor. If so informally explain why, otherwise give a counter example.

(e) Give a characterization of minors in terms of graph operations. That is, describe when a graph H is a minor of another graph G using graph operations.

(f) In the proof of Lemma 4.4.3, why do we need the assumption that G is 3-connected? Why do we need the assumption that G does not contain $K_{3,3}$ and K_5 as minors, rather than topological minors?

Exercise 2 (*written homework, 5 points*)

A *forbidden minor characterization* for a set of graphs $\mathcal{G}$ is a set of graphs $\mathcal{F}$ such that for any graph G the following holds: $G \notin \mathcal{G}$ iff F is a minor of G for some $F \in \mathcal{F}$. A *forbidden topological minor characterization* is defined similarly using topological minors instead of minors. We say that a set of graphs $\mathcal{G}$ is *closed under minors* if $G \in \mathcal{G}$ implies that any minor of G is also in $\mathcal{G}$.

(a) One of the most celebrated results in graph theory is Robertson and Seymour's Graph Minor Theorem: The graph minor order has no infinite antichains. That is, in any infinite set of graphs $\mathcal{G}$, there is always a pair of distinct graphs $H, G \in \mathcal{G}$ such that H is a minor of G. Use this theorem to show that any set of graphs which is closed under minors has a finite forbidden minor characterization. [2P]

(b) Prove that any set of graphs which is closed under minors has a finite forbidden topological minor characterization. [3P]

Exercise 3 (*written homework, 3 points*)

A graph is *outerplanar* if it has a planar drawing with all vertices lying on the boundary of the outer face.

(a) Prove the a graph G is outerplanar iff adding a new vertex to G that is connected to all of its vertices results in a planar graph. [1P]

(b) Prove that a graph G is outerplanar iff it does not contain K_4 or $K_{2,3}$ as topological minors. [2P]