Universität
des
Saarlandes
FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 8 Deadline: Thursday, June 9, 2011

Rules: The first problem serves as a preparation for the test conducted in the exercise class. You should solve it, but you do not need to hand it in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

General terminology: A *vertex-coloring* of a graph $G = (V,E)$ is a map c from V to some (finite) set S (the 'colors', usually $S = \{1,\dots,k\}$). A vertex-coloring $c : V \to S$ is called *proper* if $c(u) \neq c(v)$ for any two adjacent vertices u and v. (This is different from the Diestel book.)

Exercise 1 (*oral homework, in total 8 points via test*)

(a) Read carefully and understand all material in Section 4.4 of the Diestel book.

(b) Why is every maximal planar graph with at least four vertices 3-connected?

(c) Read carefully and understand the material in Section 5.0 and 5.2 (up to 5.2.2.) of the Diestel book.

(d) Explain why Corollary 5.2.3 in the Diestel book follows.

(e) Give an example of a graph whose chromatic number equals its chromatic index.

(f) Give an example of a graph whose chromatic number is strictly higher than its chromatic index.

(g) Give an example of a graph whose coloring number equals its chromatic number.

(h) Give an example of a graph whose coloring number is strictly higher than its chromatic number.

(i) Determine the chromatic index of the complete graph on 4 vertices.

(j) Determine the chromatic index of the complete graph on 5 vertices.

Exercise 2 (*written homework, 2 points*)

Let G_1 and G_2 be two graphs with $|V(G_1) \cap V(G_2)| < k$, and let X be a k-connected graph. Prove that if $G := G_1 \cup G_2$ contains a subdivision H of X, then all branch vertices of H must belong to either $V(G_1)$ or $V(G_2)$. (Note that this is a generalization of the observation we used throughout the proof of Lemma 4.4.4 in the lecture.)

Exercise 3 (*written homework, 2 Points*)

Show that the chromatic number of a graph G is exactly the maximum of the chromatic numbers of the blocks of G.

Exercise 4 (*written homework, 4 Points*)

a) Show that every finite graph G has a vertex-ordering for which greedy coloring uses exactly $\chi(G)$ colors. [1P.]

b) Explain why what you showed in (a) does *not* imply that $\chi(G) = \mathrm{col}(G)$ for every graph G, and give an example of a graph G with $\chi(G) < \mathrm{col}(G)$. [1P.]

c) Show that for every integer k there is a graph G with $\chi(G) = 2$ and a vertex-ordering for which greedy coloring uses k colors. [2P.]