Universität des Saarlandes

FR 6.2 Informatik

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Exercises for Graph Theory

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss11/graph_theory/

Assignment 10 Deadline: Friday, June 24, 2011

Rules: The first problem serves as a preparation for the test conducted in the exercise class. You should solve it, but you do not need to hand it in. The test yields 8 points. The remaining problems have to be handed in *nicely written up as you would do in a thesis, as the Diestel does, ...* in the Thursday lecture. These homework problems yield 8 points in total.

You need to collect at least 50% of all these points (tests and written homework) from (i) the first three exercise sessions, (ii) the first six sessions, and (iii) the whole term.

Occasionally, there might be bonus problems, which yield additional bonus points. They are, typically, more difficult or not that closely related to that week's content of the lecture.

Notice that due to the holiday, the deadline for submission is next Friday, rather than next Thursday.

Exercise 1 (*oral homework, in total 8 points via test*)

Recall the definition of line graphs. Read carefully and understand the material in Sections 5.4 up to 5.5.1. Learn by heart the definitions of interval graphs and comparability graphs (exercises 39 and 40 in chapter 5). Read and understand Fulkerson's proof of Dilworth's Theorem (see course website).

Exercise 2 (*written homework, 3 points*)

Give a complete characterization of graphs which are isomorphic to their line graphs. Prove that this characterization is complete and correct.

Exercise 3 (*written homework, 3 points*)

Use König's Theorem (2.1.1 in the Diestel) to prove that the complement of any bipartite graph is perfect. You are not allowed to use Theorem 5.5.4 in the Diestel. (**Hint:** For a bipartite graph G, consider what $\omega(\overline{G})$ corresponds to in G, and what $\chi(\overline{G})$ corresponds to in G.)

Exercise 4 (*written homework, 2 points*)

Prove that a graph G is perfect iff any induced subgraph $H \subseteq G$ has an independent set $I \subseteq V(H)$ such that $\omega(H - I) < \omega(H)$.