Universität des Saarlandes
FR 6.2 Informatik

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Graph Theory: Test 3 (Monday, May 2, 2011)

Time: 20 Minutes

Name: ____

Exercise 1+2 (*total 8 points*)

Answer each of the following questions. If proofs are needed, a short sketch of the main argument is sufficient. If counterexamples are needed, it suffices to give the example (unless it is not obvious why this is a counterexample). All questions can be answered in about two lines. Each item is worth one point, unless otherwise indicated.

a) Define what it means that a graph $G = (V, E)$ is bipartite. (Do not use the word 'colorable' or similar words in your answer.)

b) Draw a graph G with (vertex-)connectivity $\kappa(G) = 1$ and edge-connectivity $\lambda(G) = 3$.

c) Let G have at least 2 vertices. What do you need to show in order to prove that $\lambda(G) \geq 13$? (Do not use the words 'ℓ-edge-connected' for $\ell \geq 2$ or 'cut' in your answer.)

d) Draw a 4-partite graph that is not 3-partite, or argue why no such graph exists.

e) Draw two non-isomorphic trees on six vertices which both have exactly four leaves.

f) Prove that every tree with at least one edge has a leaf.

g) **(2P)** Assume that the graph H satisfies $|E(H)|/|V(H)| \geq \gamma$ and is minimal (with respect to subgraph inclusion) with that property. Show that $\delta(H) > \gamma$.
(A similar argument was used in the proof of Theorem 1.4.3, which you read as part of the oral homework.)

Feedback:

How many hours did you spend working on the assignment sheet?

The material covered last week was [ ] easy, [ ] fine, [ ] difficult, [ ] very difficult.

Comments?