Universität des Saarlandes
FR 6.2 Informatik

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Graph Theory: Test 8 (Monday, June 6, 2011)

Time: 20 Minutes

Name: _____

Exercise 1+2 (*total 8 points*)

Answer each of the following questions. If proofs are needed, a short sketch of the main argument is sufficient. If counterexamples are needed, it suffices to give the example (unless it is not obvious why this is a counterexample). All questions can be answered in about two lines. Each item is worth one point.

General terminology: A *vertex-coloring* of a graph $G = (V,E)$ is a map c from V to some (finite) set S (the 'colors', usually $S = \{1,\dots,k\}$). A vertex-coloring $c : V \to S$ is called *proper* if $c(u) \neq c(v)$ for any two adjacent vertices u and v. (This is different from the Diestel book.)

a) Let G be a graph. What do you need to show in order to prove that $\chi(G) \geq 7$?

b) Let G be a graph. What do you need to show in order to prove that $\mathrm{col}(G) \geq 7$?

c) What is the *chromatic number* of K_ℓ (the complete graph of size ℓ), and what is its *coloring number*?

d) What is the *chromatic number* of C_ℓ (the cycle of length ℓ), and what is its *coloring number*?

e) Give an example of a graph whose *chromatic number* is strictly higher than its *chromatic index*. State the values of the two parameters explicitly.

f) Give an example of a graph with chromatic number 3 and coloring number 101. (Explain why the coloring number of your example is 101.)

g) True or false: The *coloring number* of any tree T with at least one edge equals $\Delta(T)+1$ (where as usual $\Delta(T)$ denote the maximum degree of T). Give a proof or provide a counterexample.

h) True or false: The *coloring number* of any tree T with at least one edge equals its chromatic number $\chi(T)$. Give a proof or provide a counterexample.

Feedback:

How many hours did you spend working on the last assignment sheet?

The material covered last week was [ ] easy, [ ] fine, [ ] difficult, [ ] very difficult.

Comments?