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Summer 2011

Graph Theory: Test 13 (Monday, July 11, 2011)

Time: 20 Minutes

Name: _____

Exercise 1+2 (total 8 points)

Answer each of the following questions. If proofs are needed, a short sketch of the main argument is sufficient. If counterexamples are needed, it suffices to give the example (unless it is not obvious why this is a counterexample). All questions can be answered in about two lines. Each item is worth one point.

Notation: By $G_{n,p}$ we denote a random graph on n vertices obtained by including each edge with probability p independently. All random variables occurring are assumed to be on finite probability spaces.

- a) What is the expected number of copies of K_5 in $G_{n,p}$?

- b) Give an example of a sequence of non-negative integer random variables X_n , $n \in \mathbb{N}$, with $\mathbb{E}[X_n] \rightarrow \infty$ and $\Pr[X_n = 0] \rightarrow 1$ as $n \rightarrow \infty$, or argue why no such example exists.

- c) Give an example of a sequence of non-negative integer random variables X_n , $n \in \mathbb{N}$, with $\mathbb{E}[X_n] \rightarrow 0$ and $\Pr[X_n = 0] \rightarrow 0$ as $n \rightarrow \infty$, or argue why no such example exists.

- d) Let X be a random variable with $\Pr[X = 1] = p$, $\Pr[X = 0] = 1 - p$. Compute $\text{Var}[X]$, the variance of X .
- e) Let X_n , $n \in \mathbb{N}$, be a sequence of non-negative random variables. State the asymptotic condition that is needed to infer that $\lim_{n \rightarrow \infty} \Pr[X_n > 0] = 1$ by the Second Moment Method.
- f) What is a threshold probability (threshold function) of a graph property $\mathcal{P}$ of $G_{n,p}$?
- g) How do you use the first and second moment methods to show that p is a threshold probability for the appearance of a triangle in $G_{n,p}$?
- h) True or false: For every integer k there exists a graph G that has chromatic number at least k for which every subgraph $H \subseteq G$ on at most k vertices has chromatic number less than 100. (Justify your answer!)

Feedback:

How many hours did you spend working on the last assignment sheet?

The material covered last week was [] easy, [] fine, [] difficult, [] very difficult.

Comments?