*

L_{i+1} is planar want to argue

$3|L_{i+1}|-6$ at most edges in L_{i+1}

$3|L_{i+1}|-6 \geq 4|L_i|$

$\Rightarrow |L_{i+1}| \geq \frac{4}{3}|L_i| + 2$

Static Set of Orientations:

Maintain 14-orientations (did) the algor. from prev. lec. [Seq. of contractions & deletions]

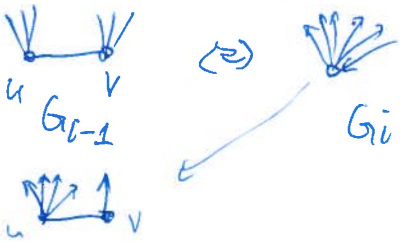
~~Lemma~~: (Consider k opers on G_0 (G) (starting), Let $G_1, .., G_k$ be the generated graphs)

Lemma:
There are 5-orientations $O_0, .., O_k$ of G_0 (G), .., G_k s.t. for every i
O_{i-1} and O_i differ in at most $3 \log n$ edges.

Proof: ~~Const. orient(5) (Do it by Euler formula)~~ backward $G_k \to G_{k-1} \to \dots G_0$

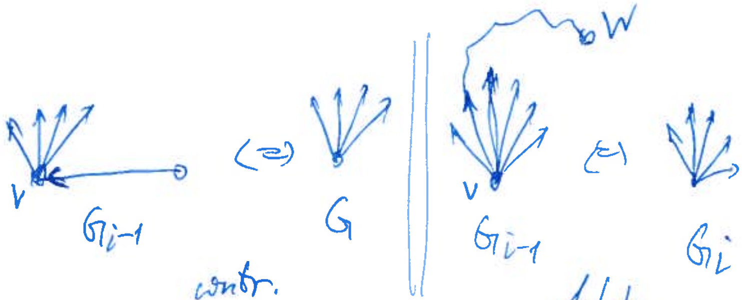
Find O_k: Fix mapping $G_i \to G_{i-1}$ give orientation.

Case 1 Obtain O_k by contraction of a none leaf edge ~~(outdeg(u) > 1~~ ~~outdeg(v) > 1~~ (u,v)



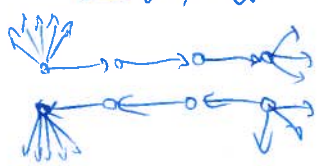
Contraction does not introd. multiple edge

Case 2: delete an edge or contract a leaf edge



take orientation of G_i
$\exists$ path (small) from v to a vertex with outdeg ≤ 3 reverse all edges among the path

By "telescoping" on inner vertices we might have introduce at most one conflict in last vertex w but outdeg(w) ≤ 3 $\Rightarrow$ ✓



Count # of reorientations:

Thm: As G is transformed from G_0 to G_k, the total number of reorientations is $O(n + k\log n)$

$\hat{O}$ - orientation maintained by algorithm — Dyn. changing G

$O(G)$ - stable 5-orientation of G

Define potential func. $\phi(G, \hat{O}) = |\hat{O} - O(G)| \le m$ — # of diff orient. between $\hat{O}$ & $O(G)$

Edge is good if it has the same orient. in both $\hat{O}, O(G)$ else bad.

Effect of deletion or contraction on $O(G)$:

ϕ increases by at most $3\log n$

Effect of — " — $\hat{O}$ per operation of while-loop

good edges turning bad ≤ 5

good $\ge 15 - 5 = 10$

total effect on ϕ is decreased by ≥ 5

Total decrease of ϕ: initial $\le m$; $m + k \cdot 3\log n$

Total increase of $\le 3\log n$

k - total # of iterations

$\Rightarrow \le \frac{m + k \cdot 3\log n}{5}$ # total number of iterations of while loop

Total number of reorientations

from good to bad: $m + k\, 3\log n$

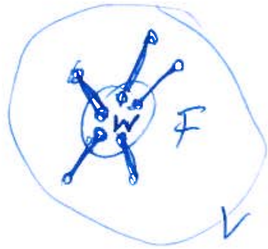
from bad to good: $\le m + m + k\, 3\log n = O(n + k\log n)$

Existence of 5-orientation (its not computed) but it can be by Euler Formula.

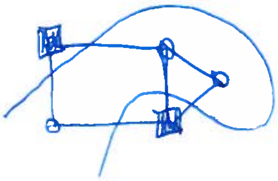
Used as comparison wrt $\hat{O}$ (just measure it.).

Max-Cut NP-C

For $W \subseteq V$, $\delta(W) = \{uv \in E \mid u \in W, v \notin W\}$ set of outgoing edges



Cut = subset F of E s.t. there is a $W \subseteq V$ with $\delta(W) = F$



Cut edges does not form Δ of edges
No odd cycles (bipartition)

No cut, as cut-edges form bipartite graph
If all edge-weights negative $\Rightarrow$ $\in P$ by min-cut (Max Cut is NP-C)

2-Approx.

For every vertex $v \in V$, assign v to W with prob. $1/2$

Thm. The expected size of cut is $m/2$ $[E(\text{size of cut}) = m/2]$

Consider $\forall$ edge (u,v): $\forall e \in E$, there are 4 outcomes:

	u	v
edge is not in cut	W	W
	$\overline{W}$	$\overline{W}$
edge in cut	W	$\overline{W}$
	$\overline{W}$	W

each has same prob.

$\Rightarrow P[e \text{ in cut}] = 1/2 = E\left[\begin{array}{l}\text{contrib. of } e \\ \text{to the weight in} \\ \text{the cut}\end{array}\right]$

$$E[\text{size of cut}] \overset{*}{=} \sum_{e \in E} 1/2 = m/2$$

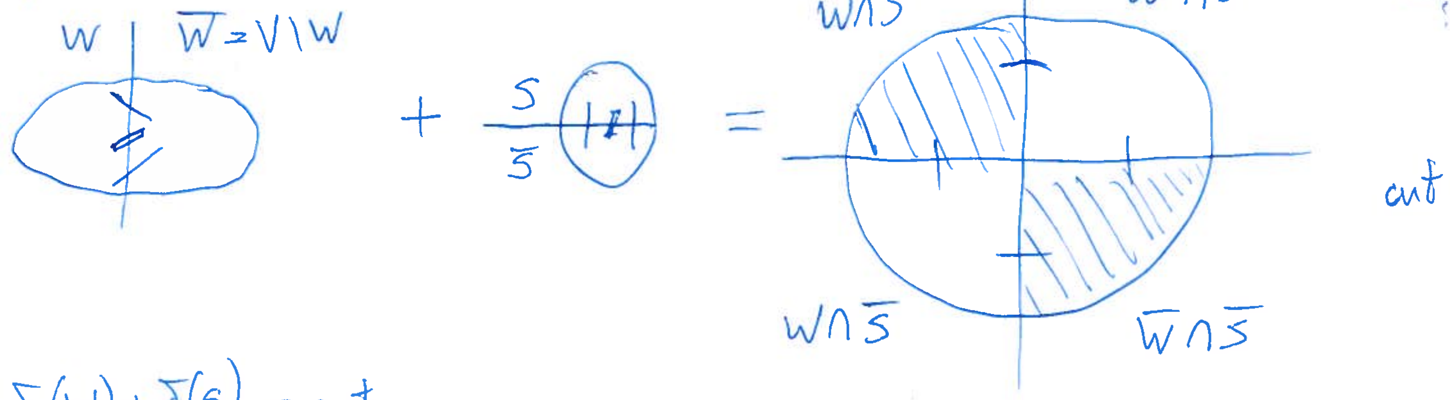
(*) lin. of expect.

Max Cut in Planar cuts: $\exists$ Alg with $O(n^k)$

Cut represented as a bit vector of edges (1 if edge is cut edge, 0 else)
Dim. of cut vector is $|E|$ ~~[illegible]~~
Set of all possible cut vectors = Cut Space $(\mathbb{Z}_2)$

Thm: The cut-space is a vector space
- neutral element 0
- Inverse element wrt addition over $\mathbb{Z}_2$ of V: $V + V = 0$
- commutative
- closed under multiplication
- closed under addition

Have a cut



$\delta(W) + \delta(S) = \text{cut}$