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Exercises for Optimization

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT

Exercise sheet 1 Due: **Wednesday, May 2, 2012**

*Normally homework is to be handed in on Tuesday in the lecture. As there is no lecture on Tuesday, May 1, the first homework is to be handed in on Wednesday in the tutorial (**not** on Thursday in the lecture as previously announced!).*

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*15 points*) Consider the following optimization problem:

$$\begin{aligned} \text{maximize} \quad & -x_1 + x_2 \\ \text{s.t.} \quad & -x_1 + 2x_2 \leq 10 \\ & x_1 + x_2 \leq 6 \\ & 2x_1 - x_2 \geq -8 \\ & x_2 \geq 0 \end{aligned}$$

a) Plot the feasible region, and determine the optimal solution of the problem.

b) Write the problem in what we defined to be "general form"; i.e., find a matrix A and vectors b and c of appropriate dimensions such that the problem is equivalent to "minimize $c^T x$ s.t. $Ax \geq b$". Give an optimal solution x of the modified problem. Is it unique?

c) Bring the problem into standard form; i.e., find a matrix A and vectors b and c of appropriate dimensions such that the problem is equivalent to "minimize $c^T x$ s.t. $Ax = b$ and $x \geq 0$". Give an optimal solution x of the modified problem. Is it unique?

Exercise 2 (*15 points*) Prove that every polyhedron is a convex set.

Hint: Show first that any intersection of convex sets is convex.

Try first to prove this without help – it's not that hard. If you cannot manage, check pages 44f. of the textbook, then close the book and write down the proof in your own words.

Exercise 3 (*10 points*) Let P be a polyhedron in $\mathbb{R}^3$ which is the intersection of three half-spaces $S_i := \{x \in \mathbb{R}^3 \mid a_i^T x \geq b_i\}$, $i = 1, 2, 3$, where $a_1, a_2, a_3 \in \mathbb{R}^3 \setminus \{0\}$ and $b_1, b_2, b_3 \in \mathbb{R}$.

Note that the vectors a_i are required to be nonzero vectors; I might have forgotten to state this in the lecture.

a) Assume that a_1, a_2, a_3 are linearly independent. How can P look like?

b) Assume that a_1, a_2, a_3 are *not* linearly independent, but that any two of them are linearly independent. What does that mean geometrically for the vectors a_1, a_2, a_3? How can P look like?

c) Assume that a_2 is a multiple of a_1 (so a_1 and a_2 are linearly dependent), and that a_3 is *not* a multiple of a_1. How can P look like?