max planck institut informatik

SAARLAND UNIVERSITY

COMPUTER SCIENCE

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Exercises for Optimization

`http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT`

Exercise sheet 10 Due: **Tuesday, July 3, 2012**

You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.

Exercise 1 (*10 points*)

Consider a network flow problem and assume that there exists at least one feasible solution. We wish to show that the optiaml cost is $-\infty$ if and only if there exists a negative cost directed cycle such that every arc on the cycle has infinite capacity.

(a) Provide a proof based on the flow decomposition theorem.

(b) For uncapacitated problems, provide a proof based on the network simplex method.

Exercise 2 (*10 points*)

One way to get an initial feasible solution for the network simplex algorithm is to build a minimum spanning tree (and route all flow exclusively on the tree, as discussed in the lecture). Give an example where this is not the optimal solution.

Exercise 3 (*20 points (BT 7.25)*) The scaling method for maximum flow

Consider a maximum flow problem Π. Let n be the number of nodes, let u_{ij} be the capacity of arc (i, j), assumed integer, and let v be the value of a maximum flow. We construct a scaled problem Π_s in which the capacity of each arc (i, j) is $\lfloor u_{ij}/2 \rfloor$ and we let v_s be the corresponding optimal value.

(a) Consider an optimal flow for the problem Π_s and multiply it by 2. Show that the result is a *feasible* flow for the original problem Π.

(b) Show that $2v_s \leq v \leq 2v_s + n^2$.

(c) Consider running the Ford-Fulkerson problem on problem Π, starting with the feasible flow described in (a). How many flow augmentations will be needed, and what is the total computational effort?

(d) Show how to solve the maximum flow problem with a total of $O(n^4 \log U)$ arithmetic operations, where U is an upper bound on the capacities u_{ij}.