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## Exercises for Optimization

<http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/OPT>

Exercise sheet 12

Due: **Tuesday, July 17, 2012**

*You need to collect at least 50% of all points on the first six exercise sheets, and at least 50% of all points on the remaining exercise sheets. You are allowed to hand in homework in teams of two.*

### Exercise 1 (10 points (Disjunctive constraints))

We saw in class a method to model the situation where at least one of the constraints  $a^T x \geq b$  and  $c^T x \geq d$  should hold, for the case where  $a \geq 0$  and  $c \geq 0$ . Show that this method fails if  $a$  or  $c$  may have negative entries by giving an example where a feasible problem is turned into an infeasible problem by introducing the binary variable  $y$ . Keep your example as simple as possible.

### Exercise 2 (5 points)

Consider the potential reduction algorithm that we discussed in class. Show that to achieve a polynomial running time, it would be sufficient to decrease the potential by some value  $\eta = O(n^{-c})$  in every step, where  $c$  is a constant. Assume that the running time per step does not change.

### Exercise 3 (15 points (BT 10.1))

Suppose that we are given  $m$  constraints  $a_i^T x \geq b_i$ ,  $i = 1, \dots, m$ , but without the restriction  $a_i \geq 0$ . Model the requirement that at least  $k$  of them are satisfied. Assume that there exists a number  $f$  such that  $a_i^T x \geq f$  for  $i = 1, \dots, m$  and for all feasible  $x$ .

### Exercise 4 (10 points (BT 10.5))

Suppose that you are planning to move out to your new house. You have  $n$  items of size  $a_j$ ,  $j = 1, \dots, n$ , that need to be moved. You have rented a truck that has size  $Q$  and you have bought  $m$  boxes. Box  $i$  has size  $b_i$ ,  $i = 1, \dots, m$ . Formulate an integer programming problem in order to decide whether the move is possible.