Assignment 1 (2pts each) Show the following.

(a) For all total computable functions $f$ there is a $g$ total computable such that
\[ \forall^\infty x : g(x) = \min(\text{range}(f)); \]

(b) There is a $g$ total computable such that, for all $e$, $\forall^\infty x : g(x, e) = \min(\text{range}(\varphi_e));$

(c) There is a $g$ total computable such that, for all $e$, $\forall^\infty x :$
\[ g(e, x) = \begin{cases} 1, & \text{if } 5 \in \text{range}(\varphi_e); \\ 0, & \text{otherwise}. \end{cases} \]

Assignment 2 (2pts) Suppose $A$ is a set of natural numbers such that, for all $e, e'$ with $\varphi_e = \varphi_{e'}$ and $e \in A$, we have $e' \in A$. Show that $A$ is infinite.

Note that the third exercise is postponed by one week.

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1The quantifier $\forall^\infty$ means “for all but finitely many.”
2Note that this is the effective or constructive version of (a).