Timo Kötzing **SS 12**

Exercises for Limits of Computational Learning

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss12/learning/

Assignment 5 Deadline: Wed 30.5.2012, 10am

Exercise 1 *(2pts) Let $\mathcal{S} \subseteq \mathcal{R}$. Show that the following two statements are equivalent.*

(a) *There is $f \in \mathcal{R}$ such that $\mathcal{S} = \{\lambda x \,.\, f(n, x) \mid n \in \mathbb{N}\}$.*

(b) *There is $r \in \mathcal{R}$ such that $\mathcal{S} = \{\varphi_{r(n)} \mid n \in \mathbb{N}\}$.*

Exercise 2 *(2pts) Let $r \in \mathcal{R}$ and $\mathcal{S} = \{\varphi_{r(n)} \mid n \in \mathbb{N}\}$ be such that $\{\langle n, x, y \rangle \mid \varphi_{r(n)}(x) = y\}$ is decidable.*

(a) *Show that $\mathcal{S} \cap \mathcal{R}$ is **GEx**-learnable.*

(b) ***Extra credit (2pts):** Give an example $\mathcal{S}$ which is* not *uniformly computable (a class as in Exercise 1).*

Exercise 3 *(4pts) Show the following two statements.*

(a) *$\tau(\mathbf{T})\mathbf{GEx} \nsubseteq \mathbf{GFin}$.*

(b) *$\mathbf{GFin} \nsubseteq \tau(\mathbf{T})\mathbf{GEx}$.*

Exercise 4 *(4pts) Show that*
$$\mathcal{R}\mathbf{GM} = \tau(\mathbf{T})\mathbf{GEx}.$$

For the following two exercises, use *self-learning* sets of functions. Let me know if you need help for defining those. You need only solve one of them, the other is **extra credit**.

Exercise 5 *(4pts) Show that*
$$\mathbf{GTEx} \subset \mathbf{G}\mathcal{R}\mathbf{M}.$$

Exercise 6 *(4pts) Show that*
$$\mathbf{G}\mathcal{R}\mathbf{M} \subset \mathbf{GEx}.$$