The first exercise of this sheet concerns *optimal identification*. We make the following definition.

Given a learner $h$ and a learnee $g$, we let $\text{conv}(h, g)$ be the least $t$ such that, for all $t' > t$, $h(g[t']) = h(g[t])$ (the point where $h$ on $g$ has converged). Note that $\text{conv}$ is not computable. A learner $h$ is said to be *optimal* iff, for all learners $h'$ learning every function $\text{GEx}$-learned by $h$, if there is $g \in \text{GEx}(h)$ with $\text{conv}(h', g) < \text{conv}(h, g)$, then there is a $g' \in \text{GEx}(h)$ with $\text{conv}(h, g') < \text{conv}(h', g')$; in other words, if $h'$ is strictly better than $h$ on some target function, then $h'$ is strictly worse than $h$ on some other target function.

**Exercise 1** (8pts, 4pts for each direction) Let $h \in \mathcal{P}$ show that $h$ is optimal iff

(a) $h$ is consistent (on what it learns);
(b) $h$ is prudent (on what it learns); and
(c) $h$ is strongly non-U-shaped (on what it learns).

**Exercise 2** (8pts, 4pts for each direction) Let $S \subseteq \mathcal{R}$. Show that $S \in \text{GFin}$ is equivalent to the existence of $p, d \in \mathcal{R}$ such that

(a) $S \subseteq \{\varphi_{p(i)} \mid i \in \mathbb{N}\}$; and

(b) For all $i, j$ with $i \neq j$ we have

$$\exists x \leq d(i) : \varphi_{p(i)}(x) \neq \varphi_{p(j)}(x).$$

Explicitly show that $\text{GFin}$ allows for learning by enumeration.

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1Note that $d(i)$ thus give a bound independent of $j$. 