

- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. Let  $G$  be a graph with  $n$  vertices and  $m$  edges. How many vertex- and edge-induced subgraphs does  $G$  contain, respectively?
2. Determine the number of edges  $m$ , minimum degree  $\delta(G)$ , average degree  $d(G)$ , connectivity  $\kappa(G)$  and edge connectivity  $\lambda(G)$  for the following graphs:
  - (a) The path of length  $m$ ,  $P_m$ .
  - (b) The cycle on  $n$  vertices,  $C_n$ .
  - (c) The complete graph on  $n$  vertices,  $K_n$ .
  - (d) Let  $d \in \mathbb{N}$  and  $V := \{0, 1\}^d$ ; thus,  $V$  is the set of all 0-1 sequences of length  $d$ . The graph on  $V$  in which two such sequences form an edge if and only if they differ in exactly one position is called the  $d$ -dimensional cube.
3. Show that every connected graph  $G$  contains a path of length at least  $\min\{2\delta(G), |G| - 1\}$ .