

- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. What is the biggest number n such that K_n is embedable on the torus?
2. The estimation $m \leq 3n - 6$ on the number of edges for a planar graph G is tight only when G is a triangulation.
Show a better bound on number of edges (as good as you can), when you additionally know that each cycle of G has at least k edges. Generalise this bound to an arbitrary surface of Euler characteristic ϵ .
3. Prove Hardwiger's conjecture for $r = 4$.