- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. Show that if G has two edge-disjoint spanning trees, it has a connected spanning subgraph whose degrees are all even.

2. Find the flaw in the following simple "proof" of the tree packing theorem: Assume k edge-disjoint spanning forests $F_1, \dots, F_k$ in G such that $E(F_1 \cup \dots \cup F_k)$ is maximal. If every F_i is a tree, the claim is true. Otherwise, there is a forest F_j that is not connected. As F_j is spanning, there is an edge $e \in G$ that is not in F_j. We add e to F_j. This links precisely two trees of F_j, which implies that our new forests have one edge more than $F_1, \dots, F_k$, contradicting the maximality-assumption.

3. Derive the marriage theorem (Hall's theorem) from Tutte's theorem. As a reminder, here are the two theorems:

   (a) Tutte's theorem: A graph G has a 1-factor if and only if $q(G - S) \leq |S|$ for all $S \subseteq V(G)$, where $q(G - S)$ is the number of odd components of the graph $G - S$.

   (b) Hall's theorem: Let G be a bipartite graph with $\{A, B\}$ its bipartition. G contains a matching of A if and only if $|N(S)|/geq|S|$ for all $S \subseteq A$.

   You need not prove the "trivial" direction of Hall's theorem , just the interesting one.

4. (Optional) (Erdős-Szekeres) Find a graph theoretic proof of the following theorem: A sequence of $rs+1$ integers contains an increasing subsequence of $r+1$ integers or a decreasing subsequence of $s+1$ integers.