

- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. Let $k \geq 2$. Show that in a k -connected graph any k vertices lie on a common cycle.
2. Let G be a k -connected graph, and let xy be an edge of G . Show that G/xy is k -connected if and only if $G - \{x, y\}$ is $(k-1)$ -connected.
3. Find the error in the following ‘simple proof’ of Menger’s theorem. Let X be an $A - B$ separator of minimum size. Denote by G_A the subgraph of G induced by X and all the components of $G - X$ that meet A , and define G_B correspondingly. By the minimality of X , there can be no $A - X$ separator in G_A with fewer than $|X|$ vertices, so G_A contains k disjoint $A - X$ paths by induction. Similarly, G_B contains k disjoint $X - B$ paths. Together, all these paths form the desired $A - B$ paths in G .