- This homework set has *three* questions, each one with increasing difficulty. You must work in pairs to determine the solutions.
- Every member of the team must be able to explain how you arrived at the answer.
- You may be asked to present your answer on the blackboard.

1. A k-chromatic graph is called critically $k-chromatic$, or just $critical$, if $\chi(G-v) < k$ for every $v \in V(G)$. Show that every k-chromatic graph has a critical k-chromatic induced subgraph, and that any such subgraph has a minimum degree at least $k-1$.
2. Determine the critical 3-chromatic graphs. .
3. Show that the following statements are equivalent for a graph G:
   1. $\chi(G) \leq k$
   2. G has an acyclic orientaton without directed paths of length k.
4. **Show that the following statements are equivalent for a graph G:
   1. $\chi(G \leq k$
   2. G has an orientaton without directed paths of length k
   3. G has an acyclic such orientation (one without directed cycles).