

Problem Set 1

Due: May 30

General Remarks for Problem Sets:

- Send the pdf version to `hsun@mpi-inf.mpg.de`, or come to Room 325 of the MPII Building to submit your homework before the deadline.
- Please work on these problems individually.
- Typesetting your solutions with Latex is highly recommended.

Problem 1. A DNF (disjunctive normal form) formula over boolean variables x_1, \dots, x_n is defined to be a logical OR of terms, each of which is a logical AND of literals (x_i or $\neg x_i$). Given a DNF formula φ and an integer k , we ask if it is possible to delete at least k terms so that the remaining formula is equivalent to φ . Show that this problem is in Σ_2 .

Problem 2. Let X be a random variable. Show that for any deterministic function f it holds that $\mathbf{H}(f(x)) \leq \mathbf{H}(X)$.

Problem 3. For every $n, k, m \in \mathbb{N}$, every $\varepsilon > 0$ and every flat k -source X , let Ext be a function chosen randomly from

$$\mathcal{H} \triangleq \{f | f : \{0, 1\}^n \mapsto \{0, 1\}^m\}$$

where $m = k - 2 \log(1/\varepsilon) - O(1)$. Show that $\text{Ext}(X)$ is ε -close to \mathcal{U}_m with probability $1 - 2^{-\Omega(K\varepsilon^2)}$, where $K = 2^k$ and \mathcal{U}_m is the uniform distribution over $\{0, 1\}^m$.

Problem 4. Suppose the feasible set of the LP

$$\begin{array}{ll} \text{maximize} & b^T z \\ \text{subject to} & A^T z \leq c \end{array}$$

is nonempty and bounded, with $\|z\|_\infty < \mu$ for all feasible z . Show that any optimal solution of the problem

$$\begin{array}{ll} \text{minimize} & c^T x + \mu \|Ax - b\|_1 \\ \text{subject to} & x \geq 0 \end{array}$$

is also an optimal solution of the LP

$$\begin{array}{ll} \text{minimize} & c^T x \\ \text{subject to} & Ax = b \\ & x \geq 0. \end{array}$$