

Problem Set 2

Due: June 28

General Remarks for Problem Sets:

- Send the pdf version to `hsun@mpi-inf.mpg.de`, or come to Room 325 of the MPII Building to submit your homework before the deadline.
- Please work on these problems individually.
- Typesetting your solutions with Latex is highly recommended.

Problem 1. Let

$$(x_1, c_1), (x_2, c_2), \dots, (x_m, c_m), \dots$$

be a data stream, where every $x_i \in [n]$ and every c_i is '1' or '2'. Moreover, every item $x_i \in [n]$ appears at most K times. Let X be the set of x_i s where $c_i = 1$, and Y be the set of y_i s where $c_i = 2$. (1) Design a data streaming algorithm to compute the number of distinct items in $(X \setminus Y) \cup (Y \setminus X)$. (2) Analyze the space complexity of the algorithm.

Problem 2. Let G be an expander graph with n vertices and vertex expansion $A \in \mathbb{R}^+$. Show that the diameter of G is $O(\log n)$.

Problem 3. Let G be a d -regular connected graph with n nodes, and \mathbf{A} be the adjacency matrix of G with eigenvalues $\lambda_1 \geq \dots \geq \lambda_n$. Prove that G is bipartite iff $\lambda_1 = -\lambda_n$.

Problem 4. Prove that $\text{PCP}(\log n, 1) \subseteq \text{NP}$.