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Assignment 4 for
Approximation Algorithms and Hardness of Approximation

Discussion:
Thursday, 5 June 2014, 14 pm

Assignment 1 *(Integrality gap for LP approach to* $R||C_{\max}$ *)*

Consider the LP relaxation for minimum makespan scheduling on unrelated machines discussed in the lecture: For a given value T, we used

$$\begin{array}{lll} \max & \text{(no objective)} & \\ \text{s.t.} & \sum_{i\in M} x_{ij} \geq 1 & \forall j \in J \\ & \sum_{j\in J} x_{ij}p_{ij} \leq T & \forall i \in M \\ & x_{ij} = 0 & \forall i \in M, j \in J : p_{ij} > T \\ & x_{ij} \geq 0 & \forall i \in M, j \in J. \end{array}$$

Find an instance (given by values p_{ij} and T) for which a feasible fractional solution with makespan T exists, but any integral assignment ($x_{ij} \in \{0,1\}$) induces a makespan of at least $(2 - \frac{1}{m})T$.

Assignment 2 *($\frac{3}{2}$-approximation on* $R|p_j \in \{\frac{1}{2}, 1, \infty\}|C_{\max}$ *)*

Show how to get a $\frac{3}{2}$-approximation algorithm for minimum makespan scheduling on unrelated machines, where all processing times are in $\{\frac{1}{2}, 1, \infty\}$.

Hint: Try to get a $\frac{3}{2}$-approximation for the case that (1) $\mathrm{OPT} = \frac{1}{2}$, (2) $\mathrm{OPT} = 1$, (3) $\mathrm{OPT} = \frac{3}{2}$ and (4) $\mathrm{OPT} \geq 2$. (4) is solved by an algorithm from the lecture, for (1)-(3) matchings in a suitable bipartite graph might be useful.

Assignment 3 *(Greedy approach for MIS in unit-disk graphs)*

In the lecture, we proved that the GreedyDiskIS achieves a 5-approximation on the maximum independent set in unit-disk graphs. Suppose we sort the disks in increasing x-coordinate of their centers and then apply GreedyDiskIS. Does this sorting step improve the approximation performance? If yes, prove a better upper bound. If no, give a tight example for GreedyDiskIS.

Assignment 4 *(MIS in unit-square graphs)*

Suppose we are given squares of side length 1 instead of disks. Can you find a simple constant-factor approximation on the maximum independent set? Which constant do you get?

Assignment 5 *(Independent set in general disk graphs)*

Let n disks $D_1, \ldots, D_n$ with radii $r_1, \ldots, r_n$ be given. Can you adapt the greedy algorithm for unit-disk graphs to approximate the maximum independent set of $D_1, \ldots, D_n$? What approximation performance do you achieve?