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Tutorials: Marvin Künnemann

Assignment 6 for Approximation Algorithms and Hardness of Approximation

Discussion:
Thursday, 3 July 2014, 14 pm

Assignment 1 *(Iterative Rounding for minimum-cost perfect matching on bipartite graphs)*

Let $G = (V, E)$ be a bipartite graph. In the lecture, we used the iterative rounding approach to the linear program

$$\begin{aligned} \text{maximize} \quad & \sum_{e \in E} w_e x_e & \\ \text{subject to} \quad & \sum_{e \in \delta(v)} x_v \leq 1 & \forall v \in V \\ & x_e \geq 0 & \forall e \in E. \end{aligned}$$

Adapt this approach to the following LP and prove that minimum-cost perfect matching is polynomially solvable on bipartite graphs:

$$\begin{aligned} \text{minimize} \quad & \sum_{e \in E} w_e x_e & \\ \text{subject to} \quad & \sum_{e \in \delta(v)} x_v = 1 & \forall v \in V \\ & x_e \geq 0 & \forall e \in E. \end{aligned}$$

Assignment 2 *(Iterative Rounding for VC on bipartite graphs)*

Given a bipartite graph $G = (V, E)$, show how to solve vertex cover optimally by using the iterative rounding approach. Consider the following LP.

$$\begin{aligned} \text{minimize} \quad & \sum_{v \in V} c_v x_v & \\ \text{subject to} \quad & x_u + x_v \geq 1 & \forall e = \{u, v\} \in E \\ & x_v \geq 0 & \forall v \in V. \end{aligned}$$

Show that you can always find a vertex v with $x_v = 0$ or $x_v = 1$ and recurse on an easier subproblem.

Hint: Delete isolated vertices. What properties on the extremal points can you derive from the rank lemma? In particular, show that any extremal point x of the LP with $x_v > 0, d_E(v) \geq 1 \forall v \in V$ gives a set $F \subseteq E$ with (i) $x_u + x_v = 1 \forall \{u, v\} \in F$, (ii) $|V| = |F|$, and (iii) the vectors $\{\chi(\{u, v\}) \mid \{u, v\} \in F\}$ are linearly independent. Show that F is acyclic.

Assignment 3 *(Survivable Network Design)*

In the lecture, the following claim was left unproven: Let $r_{uv} \in \mathbb{N}$ for $u, v \in V$ with $r_{uv} = r_{vu}$ be given and define $r(S) := \max_{u \in S, v \notin S} r_{uv}$. Show that r is a skew supermodular function, i.e, for any $S, T \subseteq V$, one of the following statment holds:

1. $r(S) + r(T) \leq r(S \cup T) + r(S \cap T)$, or,
2. $r(S) + r(T) \leq r(S \setminus T) + r(T \setminus S)$.