Universität des Saarlandes
FR 6.2 Informatik

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Excercises
Online Algorithms

http://www.mpi-inf.mpg.de/departments/d1/teaching/ss14/OnlineAlgos/

Sheet 2 Deadline: 15.05.2014

Rules: Until the end of the semester you have to reach 50% of the achievable points to be admitted to the exam.

Exercise 1 (*10 points*)
Prove that when the total number of pages is $k+1$, then algorithm MARK is H_k-competitive.

Exercise 2 (*10 points*)
Prove that MARK is not H_k-competitive in general.

Hint: There exists a counterexample with $k = 2$ and a total number of 4 pages.

Exercise 3 (*8 points*)
Show that the greedy algorithm for the k-server problem has an unbounded competitive ratio. The greedy algorithm always uses the server that is closest to the request.

Exercise 4 (*6+6 points*)
Consider the k-server problem on the real line metric space. The *Double Coverage (DC)* algorithm is defined as follows:

- if the next request r is on one side of all the servers, then the server nearest to r is moved to serve the request.
- else, request r is between two servers s_i and s_{i+1}. Start moving both servers s_i and s_{i+1} at the same speed towards r and stop moving them when a server reaches r.

The goal in this exercise is to prove that DC is k-competitive for the real line metric space: Let at any point $s_1, s_2, \ldots s_k$ and $a_1, a_2, \ldots a_k$ be the locations of DC's and OPT's servers ordered from left to right. Define the potential function $\Phi = k \cdot M + \Theta$, where $M := \sum_{i=1}^{k} d(s_i, a_i)$ is the cost of a minimum weight matching in the bipartite graph between $s_1, s_2, \ldots s_k$ and $a_1, a_2, \ldots a_k$, and $\Theta := \sum_{i<j} d(s_i, s_j)$ is the sum of all pairwise distances between DC's servers.

(i) Prove that Φ satisfies the following properties:

  (a) At all times $\Phi \geq 0$,

  (b) When the adversary increases its cost by x, then the change in the potential $\Delta\Phi \leq k \cdot x$, and

  (c) When DC increases its cost by x', then the change in potential $\Delta\Phi \leq -x'$.

(ii) Prove that DC is k-competitive.